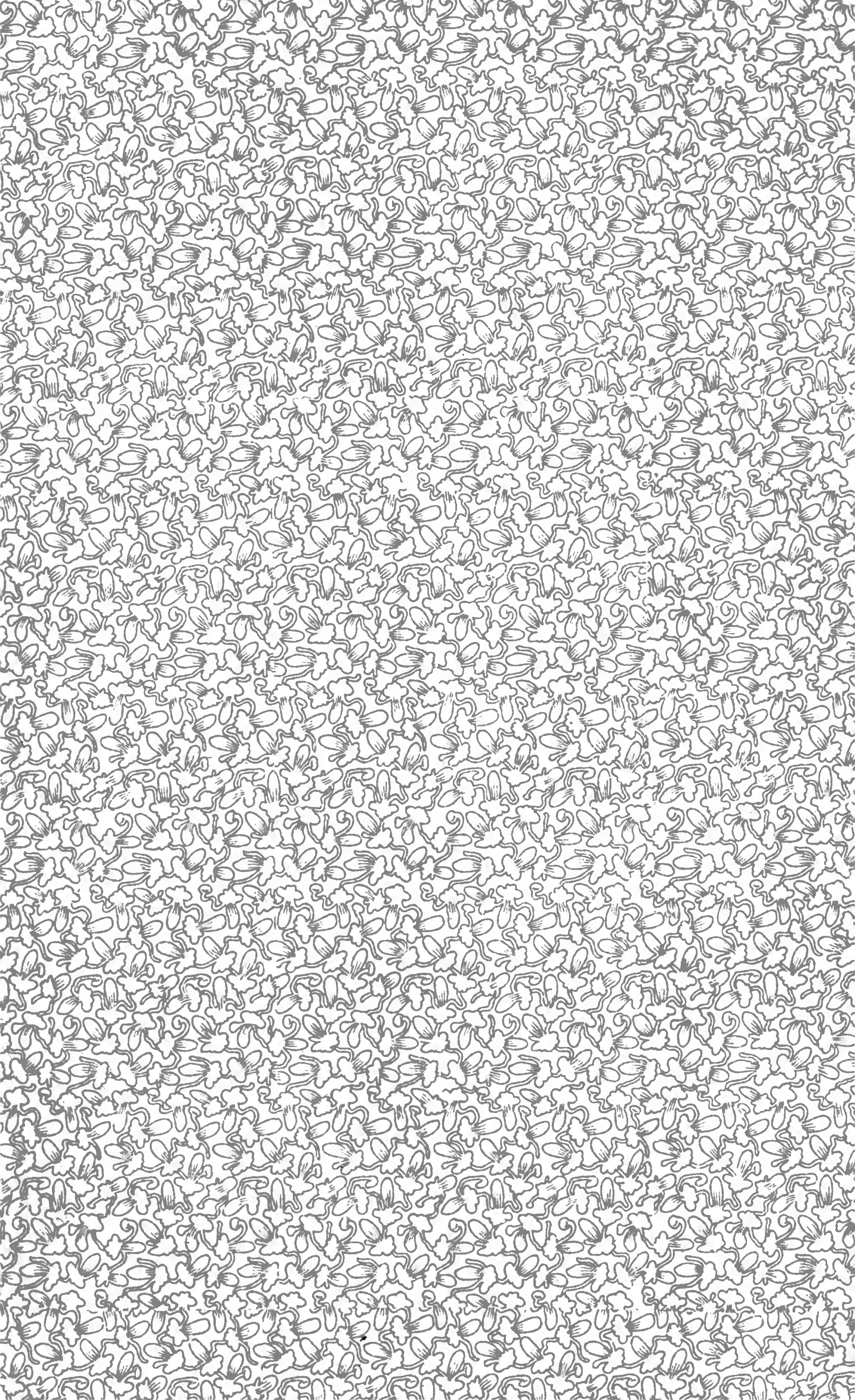

# MACHINE DESIGN

## HOISTS, DERRICKS, CRANES

BY

H. D. HESS, M.E.

PROFESSOR OF MACHINE DESIGN, SIBLEY COLLEGE, CORNELL UNIVERSITY. FORMERLY DESIGNER AND COMPUTER FOR THE MECHANICAL DEPARTMENT OF THE PENCOYD IRON WORKS AND THE AMERICAN BRIDGE COMPANY. MEMBER OF THE AMERICAN SOCIETY OF MECHANICAL ENGINEERS

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# PREFACE

The author believes that no class of problems afford as good general training and practice as may be had in the design of cranes. The larger portion of crane details being the simplest machine elements are common to a wide field of machine design. The stresses both in the frames and machinery are readily determined by the elementary principles of mechanics and the design thus easily carried out on a theoretical rather than on an empirical basis. The book is intended to aid the work of machine design in technical schools and colleges and it is hoped it may prove useful in drawing-rooms where a general field of machine design is covered. The problems have been selected with this in view rather than that of appealing to the specialist in crane design.

The engineering literature drawn upon includes the works of Bach, Bethmann and Ernst, and Machinery and Engineering Record among the periodicals. Through the kindness of several crane builders, catalogues, photographs, specifications and drawings, illustrating American practice, have been placed at our disposal.

Ithaca, N. Y., June, 1912.

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# ELEMENTS OF
# MACHINE DESIGN

## PART I.—INTRODUCTION

### MATERIALS.

The following materials are the ones most largely used in this class of machine design:

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Air-furnace iron and chilled iron.
Wrought-iron.
Rolled steel, hot rolled, cold rolled and drawn.
Steel castings.
Crucible steel.
Steel-wire.

#### Bearing Materials.

Cast-iron.
Brass castings.
Bronze, gun-metal, and phosphor bronze.
Babbitt metal.

#### Electrical Conductors.

Copper (rolled).

#### Timber.

Oak, hemlock, pine (long leafed, Southern, yellow), spruce and white pine.

#### Foundations.

Concrete.

Cast-Iron.—Ordinary cast-iron is obtained by remelting pig-iron or a mixture of pig-iron and cast scrap in a cupola. The metal and the fuel are charged together. As the entire charge is never melted and mixed at one time the product is a varying one, depending upon the time during the heat that the metal in any casting is tapped.

Cast-iron for machine parts should be soft to permit of being readily machined. Its ultimate strength per square inch in tension should reach 16,000 to 20,000 pounds. It has no distinctly defined elastic limit, but this can be assumed for general purposes at 8,000 pounds per square inch. In compression when no bending is introduced either in beam or column action its ultimate strength will range from 90,000 to 100,000 pounds per square inch. The resilience of cast-iron is very low. As resilience measures the ability of a material to resist shock, cast-iron is but a poor material where it would be subjected to shock.

AIR-FURNACE IRON.—A more uniform cast-iron is obtained when the pig-iron and the scrap are melted on the hearth of an air-furnace. Here the furnace has a hearth of sufficient capacity to hold the entire charge when melted; this permits the entire bath to be mixed before pouring. The metal is melted by the hot gases passing over the charge.

This metal is cleaner than metal melted in a cupola. Iron of this class will have an ultimate strength of 30,000 to 40,000 pounds per square inch.

CHILLED IRON.—The strength and hardness of cast-iron depend chiefly upon the amount of carbon combined with it, and also upon the form of this carbon, whether free as graphite or chemically combined carbon. If cast-iron which is high in carbon has a part of it suddenly chilled from the molten state, the chilled portion will contain a large amount of chemically combined carbon, while the portion slowly cooled will retain its carbon largely as graphite. The result of this is that the chilled portion will be extremely hard and only capable of being machined with greatest difficulty.

The remaining part will be soft and easily machined. The chilling is effected by using iron sections faced with a light coating of loam at the points to be chilled. Iron car-wheels are made in this way, thus giving a chilled tread to resist wear, while the hub is soft, permitting of easy boring for the axle.

The thickness of the chilled portion may vary from 1/8 inch to 5/8 inch or more, depending upon the iron mixture and the method of chilling.

WROUGHT-IRON.—Wrought-iron is refined pig-iron. In pig-iron the carbon may reach 3 to 4 per cent., while in wrought-iron it will be under 1/10 of 1 per cent. The other impurities in the pig-iron, namely, silicon, manganese, and sulphur, are correspondingly reduced. The refining is done by oxidizing the above impurities by contact with an oxidizing flame and additions of oxidizing agents to the molten bath. The process is assisted by puddling or thoroughly stirring the metal in the bath. The average ultimate strength of wrought-iron when the force acts parallel with the fibers will range from 47,000 to 57,000 pounds per square inch. With the force applied at right angles to the fibers this strength becomes 40,000 to 50,000 pounds. The elastic limit is approximately ½ the ultimate strength.

The resilience of wrought-iron is high and consequently it is well adapted to resisting shock. Wrought-iron can be welded, that is, two pieces brought to a white heat, having their ends lapped and hammered, unite to make a single piece, the joint being practically as strong as the solid bar.

Steel.—Ordinary steel differs from wrought-iron in the per cent. of carbon it contains and in the process of manufacture. Steel is made from pig-iron by the Bessemer process and from pig-iron and wrought-iron or steel scrap by the open-hearth process. In the Bessemer process the carbon, silicon, etc., are burnt out by the forcing of air through the molten metal. In the open-hearth process the oxidization is done by the hot gases and by oxidizing charges added to the bath of molten metal. In the steel furnace the boiling of the metal replaces the agitation by the puddlers as it is required in the manufacture of wrought-iron. In both processes the impurities, including carbon, are first burnt out and the bath then recarbonized.

The ordinary structural and merchant steels range in carbon from 0.08 of 1 per cent. to 0.3 of 1 per cent.; and are classed as "mild or soft" up to 15 points carbon, and as "medium" from 15 to 30 points carbon. "Hard" steel has more than 30 points carbon. A point is ¹⁄₁₀₀ of 1 per cent. The ultimate strength will vary with the carbon, ranging from 48,000 to 70,000 pounds per square inch. Owing to its high resilience steel is admirably adapted to resisting shock. From the grades of steel just mentioned the following shapes are rolled (hot): rounds, squares, flats, plates, billets, blooms, structural shapes, including I-beams, channels, angles, Z-bars, etc.

Cold-Rolling.—These grades of steel are rolled hot until close to the required dimension and then the final rolling is done when cold. This produces a section accurately of the desired dimension, and as it is burnished it requires no further machining for a considerable amount of work. Cold-rolling increases both tensile and transverse strength, also the elastic limit is raised.

The elastic resilience is also greatly increased. The ductility is somewhat reduced, but in every other respect the metal is improved.

Open-hearth steel is made by both the acid and the basic process. The basic process differs from the acid process in permitting the addition of lime, by which the bulk of the phosphorus contained in the metal bath is eliminated. This allows the use of low grade pig-iron in the manufacture of steel. In the Bessemer process only acid linings are used in this country; this necessitates the use of pig-iron low in phosphorus. High sulphur makes steel hot-short, *i.e.*, brittle during rolling, while high phosphorus makes it cold-short or brittle in use.

Steel Castings.—This steel is made by the open-hearth process similarly to that for rolling. The steel is then poured into molds in the same manner as iron castings, excepting that owing to the much higher temperature of molten steel than of molten cast-iron, it is necessary to have the mold frames much more substantial and the molds must be thoroughly dried. The very high temperature causes excessive contraction, while cooling results in great internal stress where the cooling is not uniform or the piece being cast is improperly designed. To insure relieving this stress all steel castings should be carefully annealed.

The ultimate strength is 50,000 pounds per square inch or more, depending upon the carbon contents. The resilience is high, the material being far better able to resist shock than similar castings of iron. Steel castings, however, are higher in price and cost more to machine on account of the necessity of lighter cuts and slower cutting speeds.

Crucible Steel.—This is high carbon or tool steel; it is made by adding the proper amount of charcoal to a crucible charged with wrought-iron; the absorption of this carbon by the iron goes on with great rapidity when the metal is molten and thus the iron is transformed to steel; this is then poured into ingots.

High carbon open-hearth steel has replaced much steel formerly made by this process. The ultimate strength varies with the carbon, but exceeds 100,000 pounds per square inch. The elastic limit should exceed 75,000 pounds. Its resilience is high.

STEEL-WIRE.—Steel for wire can be made by either the open-hearth or the Bessemer process. The large amount of work done upon it insures a high-grade material. For the several grades of wire rope see description under "Wire Rope."

NICKEL-STEEL.—This is a steel containing about 3½ per cent. nickel. It has an ultimate strength of 80,000 to 90,000 pounds per square inch and its elastic limit is about 75 per cent. of its ultimate strength. The elongation approximates 20 per cent. in 8 inches.

This material, like the other special steel, finds little use in crane design; it is largely used for important engine forgings.

BEARING MATERIALS.—Cast-iron bushings are sometimes used for bearings and when so used the iron glazes and offers a good bearing surface.

BRASS CASTINGS.—Yellow brass is a composition of copper and zinc. The approximate proportions are 2 of copper to 1 of zinc. Its tensile strength is about 22,000 pounds per square inch.

BRONZE.—Red bronze, 83 per cent. copper, 17 per cent. tin, to 82 per cent. copper, 16 per cent. tin, and 2 per cent. zinc. Phosphor bronze contains from ½ to 1 per cent. phosphorus. Ordinary phosphor bronze has an ultimate strength of 90,000 pounds per square inch and an elongation of 72 per cent. Gun-metal contains 10 per cent. tin.

BABBITT'S METAL.—This is an alloy of copper, tin and antimony, the proportions being about 4 copper, 8 antimony, and 96 tin. This metal is cast in the boxes around the journal, thus making the bearing surface for the journal.

COPPER.—Copper weighs 560 pounds per cubic foot, and melts at 1930° F.

Rolled or forged copper has an ultimate strength of 30,000 pounds per square inch between 60 and 300° F. The strength decreases rapidly with increases of temperature. Electrolytic copper drawn into wire has an ultimate strength of 47,000 pounds per square inch.

TABLE OF PHYSICAL PROPERTIES OF METALS

| Material | Modulus of Elasticity | | Ultimate Strength | | Elastic Strength | |
|---|---|---|---|---|---|---|
| | Tension Compression | Torsion | Tension | Shear | Tension | Shear |
| Cast-iron (Cupola) | 10,700,000<br>15,000,000 | 4,000,000<br>6,000,000 | 16,000<br>20,000 | 16,000<br>20,000 | 8,000 | 8,000 |
| Cast-iron (Air-fur.) | ........<br>........ | ........<br>........ | 30,000<br>40,000 | .....<br>..... | .....<br>..... | ....<br>.... |
| Wrought-iron | 28,000,000 | 11,000,000 | 47,000<br>57,000 | 35,000<br>43,000 | .....<br>..... | ....<br>.... |
| Steel 0.15 Carbon | 30,000,000 | 11,800,000 | 60,000 | 45,000 | 40,000 | .... |
| Steel 0.25 Carbon | 30,000,000 | 11,800,000 | 70,000 | 52,000 | 45,000 | .... |
| Crucible Steel (high carbon) | 31,000,000 | 12,100,000 | 100,000 | ..... | 75,000 | .... |
| Steel castings | 30,600,000 | 11,800,000 | 50,000<br>100,000 | 30,000<br>60,000 | .....<br>..... | ....<br>.... |
| Copper castings | 12,000,000 | ........ | 22,000 | ..... | 6,000 | .... |
| Copper, rolled | 16,000,000 | ........ | 28,500<br>33,000 | .....<br>..... | .....<br>..... | ....<br>... |
| Brass cast., yel. | 11,400,000 | ........ | 22,000 | ..... | ..... | .... |
| Brass cast., yel., red | 12,800,000 | ........ | 28,500 | ..... | ..... | .... |
| Gun-metal | 15,400,000 | ........ | 42,800 | ..... | ..... | .... |
| Phosphor bronze | ........ | ........ | 57,000 | ..... | 24,000 | .... |

NOTE.—The ultimate strength of cast-iron in compression is 90,000 to 100,000 pounds per square inch. Its elastic strength in compression can be assumed as 25,000 pounds per square inch. The ultimate compressive strengths of the other materials can be taken as equal to their ultimate tensile strengths without appreciable error.

## Working Fiber Stress.

The selection of a proper working fiber stress is influenced by the physical properties of the material, *i.e.*, ultimate strength, elastic limit, elongation, reduction of area, modulus of elasticity and resilience. The physical properties may in themselves be influenced by the method of manufacture; material worked by rolling is superior to the same material cast. The working stress is also affected by the nature of the loading, whether dead or live load, and, if a live load, by the manner applied, *i.e.*, with or without shock. The stress at which failure may occur is still further influenced by repetition of stress. Experiments made by Wohler and several formulæ deduced therefrom give approximately the following results:

1. DEAD LOAD.—Failure produced by stress equal to the ultimate strength.

2. LIVE LOAD.—Stress in tension or compression only; the failure would be produced by an innumerable repetition of a stress of $\frac{6}{10}$ of the ultimate strength.

3. LIVE LOAD.—Stress changing from tension to compression; innumerable repetitions of a stress equal to $\frac{1}{3}$ of the ultimate strength would produce failure.

To obtain the same degree of security, when considerations of shock are neglected, Case 2 would require a working fiber stress of $\frac{6}{10}$ and Case 3, $\frac{1}{3}$ the stress allowed in Case 1.

Shock.—Loads applied instantly produce twice the fiber stress that would be occasioned by the same load applied gradually. In loads applied with impact the stress is still further increased by the necessity of the material absorbing the kinetic energy of the impinging body. The common practice is not to make elaborate calculations to determine what working fiber stress shall be used; the designer gradually becomes acquainted with the generally allowed working stresses in his particular sphere of design and then when extending his work bases his selection of working stresses upon the data at his command.

In bridge design and in the design of gear-teeth formulæ are used which decrease the working fiber stresses as the velocity of the train load or speed of the gears increases.

### Ordinary Working Fiber Stresses.

| Character of Stresses or Loading | | Wrought-Iron | Soft Steel | | Mild Steel | | Steel Castings | | Cast-Iron | Rolled Copper |
|---|---|---|---|---|---|---|---|---|---|---|
| | | | from | to | from | to | from | to | | |
| Tension | I | 12,500 | 12,500 | 17,000 | 17,000 | 21,000 | 8,500 | 12,500 | 4,300 | 8,500 |
| | II | 8,500 | 8,500 | 11,200 | 11200 | 14,500 | 5,600 | 8,500 | 2,800 | 4,300 |
| | III | 4,300 | 4,300 | 5,600 | 5,600 | 7,100 | 2,800 | 4,300 | 1,400 | ..... |
| Compression | I | 12,500 | 12,500 | 17,000 | 17,000 | 21,000 | 12,500 | 17,000 | 12.500 | ..... |
| | II | 8,500 | 8,500 | 11,200 | 11,200 | 14,300 | 8,500 | 12,500 | 8,500 | ..... |
| Bending | I | 12,500 | 12,500 | 17,000 | 17,000 | 21,000 | 10,500 | 15,000 | See | ..... |
| | II | 8,500 | 8,500 | 11,200 | 11,200 | 14,300 | 7,000 | 10,000 | Note | ..... |
| | III | 4,300 | 4,300 | 5,600 | 5,600 | 7,100 | 3,600 | 5,000 | 1 | ..... |
| Shearing | I | 10,000 | 10,000 | 14,000 | 14,000 | 17,000 | 6,500 | 12,000 | 4,300 | ..... |
| | II | 6,500 | 6,500 | 9,000 | 9,000 | 11,000 | 4,500 | 8,000 | 2,800 | ..... |
| | III | 3,500 | 3,500 | 4,600 | 4,600 | 5,600 | 2,300 | 4,000 | 1,400 | ..... |
| Torsion | I | 5,000 | 8,500 | 12,000 | 12,000 | 17,000 | 6,500 | 12,000 | ..... | ..... |
| | II | 3,400 | 5,700 | 8,000 | 8,000 | 11,000 | 4,500 | 8,000 | See Note | |
| | III | 1,700 | 3,000 | 4,000 | 4,000 | 5,600 | 2,500 | 4,000 | 2 | ..... |

NOTE 1.—According to Hütte the allowable stress in bending upon cast-iron varies with the section and is as follows:

Rectangle. . . 1.70 × corresponding working tensile stress
Circle. . . . . . . 2.05 × corresponding working tensile stress
I . . . . . . . . . 1.45 × corresponding working tensile stress

The allowable stress on any section is given by

$$p_b = \mu \, p_t \sqrt{\frac{e}{z_o}}$$

where $p_b$ = allowable unit working stress in bending.
$\mu = 1.20$ to $1.33$.
$p_t$ = allowable unit working stress in tension.
$e$ = distance neutral axis to extreme fibers.
$z_o$ = distance from the neutral axis to the center of gravity of the portion of the section to one side of the neutral axis.

NOTE 2.—Torsional stress also varies with the section.

Circle. . . . . . . . . . . . . . . . . . . . . . . . . . $p_r = p_t$.
Circular ring and hollow ellipse. . . $p_r = 0.8$ to $1.0\ p_t$.
Ellipse. . . . . . . . . . . . . . . . . . . . . . . . . $p_r = 1.0$ to $1.25\ p_t$.
Square. . . . . . . . . . . . . . . . . . . . . . . . . $p_r = 1.4\ p_t$.
Rectangle. . . . . . . . . . . . . . . . . . . . . . . $p_r = 1.4$ to $1.6\ p_t$.
Hollow rectangle. . . . . . . . . . . . . . . . . $p_r = 1.0$ to $1.25\ p_t$.
I, [, + and L. . . . . . . . . . . . . . . . . . . $p_r = 1.4$ to $1.6\ p_t$.

$p_r$ = working fiber stress in torsion.

I. Dead load.
II. Stress varies from zero to a maximum but does not pass through zero.
III. Stresses vary from positive to negative through zero; that is a reversal of stress.

These figures make no allowance for shock; they, however, provide against an innumerable number of applications of the load.

## Properties of Timber.

Wooden beams are designed in the same way as metal beams. They are much more liable to fail by horizontal shearing and they should therefore be calculated to resist this action. The working stress in shear (lengthwise) can be assumed at one-fifth the ultimate shearing resistance given in the tables.

The formula for bending is

$$M = p\frac{I}{e}$$

M = bending moment in inch-pounds.
I = moment of inertia. Inches.
$e$ = the distance from the neutral axis to the extreme fibers, in inches.
$p$ = the working fiber stress, in bending. Pounds per square inch.

Timber beams being commonly rectangular, this becomes

$$M = \frac{pbd^2}{6}$$

$b$ = width of the beam in inches.
$d$ = depth of the beam in inches.

To guard against horizontal shear $p_s = \frac{3R}{2bd}$ where

$p_s$ = fiber stress in shear, pounds per square inch.
R = end reaction in pounds.
$b$ and $d$ the same as above.

## Columns.

The following formula is suggested by the U. S. Government reports on timber:

$$p_c = P_c \times \frac{700 + 15c}{700 + 15c + c^2}$$

$p_c$ = ultimate compressive strength of the column in pounds per square inch.
$P_c$ = ultimate crushing strength of short timber column, pounds per square inch.

$$c = \frac{l}{d} \text{ where}$$

$l$ = length of the column in inches.
$d$ = least diameter in inches.

The percentage of this load which it is safe to carry under ordinary conditions varies from 20 per cent. when the wood contains 18 per cent. of moisture and is used in the open to 30 per cent. when it contains 10 per cent. or less of moisture and is used inside of heated buildings. The following values of "F" can be assumed:

| Name of wood. | Pounds per sq. in. |
|---|---|
| White oak, Southern long leafed pine | 5000 |
| Short leafed yellow pine (Georgia) | 4500 |
| Hemlock, chestnut and spruce | 4000 |
| White pine and cedar | 3500 |

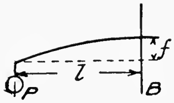
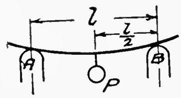
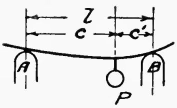
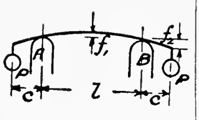
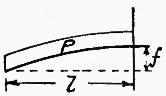
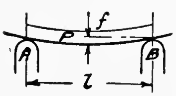
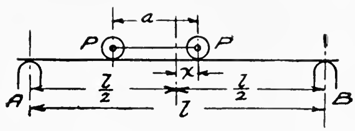
### Table of Properties of Timber.

| Name of Wood | Ultimate Resistance to | | | | | Elastic Limit | Modulus of Elasticity | Modulus of | | Ordinary Working Stresses | | | Weight, pounds per cubic feet |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Tension | Compression | | Shear | | | | | | | | | |
| | | Length-wise | Cross-wise | Length-wise | Cross-wise | | | Ultimate Bending | Elastic Bending | Tens'n | Comp. | Bend'g | |
| Ash | 17,000 | 7,200 | 1,900 | 1,100 | 6,280 | 7,900 | 1,640,000 | 10,800 | 7,900 | 2,000 | 1,000 | 1,200 | 39 |
| Cedar | ...... | 5,200 | 700 | 400 | 1,370 | 5,800 | 910,000 | 6,300 | 5,800 | 1,200 | 600 | 800 | 23 |
| Chestnut | 11,500 | 5,300 | ..... | ..... | 1,530 | ...... | 1,140,000 | 8,100 | ...... | 1,400 | 600 | 900 | 41 |
| Douglas Spruce | 13,000 | 5,700 | 800 | 500 | ..... | 6,400 | 1,680,000 | 7,900 | 6,400 | 1,400 | 700 | 1,000 | 32 |
| Hemlock | 8,700 | 5,700 | ..... | 400 | 2,750 | ...... | ......... | 7,100 | ...... | ..... | ..... | 750 | 25 |
| Oak (White) | 13,600 | 8,500 | 2,200 | 1,000 | 4,400 | 9,600 | 2,090,000 | 13,100 | 9,600 | 1,700 | 1,000 | 1,500 | 50 |
| Pine (White) | 10,000 | 5,400 | 700 | 400 | 2,500 | 6,400 | 1,390,000 | 7,900 | 6,400 | 1,200 | 700 | 900 | 24 |
| Pine So. Yellow-long leafed | 13,000 | 8,000 | 1,260 | 835 | 5,600 | 10,000 | 2,070,000 | 12,600 | 9,500 | 1,600 | 1,000 | 1,500 | 38 |
| Pine Cuban | 13,000 | 8,700 | 1,200 | 770 | ..... | 11,100 | 2,370,000 | 13,600 | 10,640 | ..... | ..... | ..... | .. |
| Pine Lobolly | 13,000 | 7,400 | 1,150 | 800 | ..... | 9,200 | 2,050,000 | 11,300 | 9,400 | 1,600 | 900 | 1,200 | 33 |
| Spruce (Northern) | 11,000 | 6,000 | ..... | 400 | 3,250 | ...... | 1,400,000 | 8,000 | ...... | 1,200 | 700 | 900 | 26 |
| Spruce Pine | 12,000 | 7,300 | 1,200 | 800 | ..... | 8,400 | 1,640,000 | 10,000 | 8,400 | 1,200 | 700 | 900 | 30 |

Basis.—United States Department of Agriculture, Forestry Division, Moisture 12 per cent. less.

All quantities are in pounds per square inch.

BENDING MOMENTS, DEFLECTIONS, ETC., FOR BEAMS OF UNIFORM SECTION

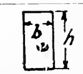
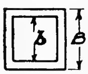
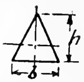
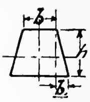
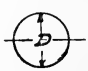
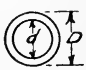
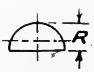
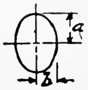
| Form of Support and Load | End Reactions A., B. Bending Moment M. | Relation between Load "P" and Moment of Resistance. | Maximum Deflection. |
|---|---|---|---|
|  | $B=P$<br>$M=Px$<br>$M_{max.}=Pl$ | $P=\frac{pW}{l}$<br>$W=\frac{Pl}{p}$ | $f=\frac{P}{EI}\cdot\frac{l^3}{3}$<br>$=\frac{1}{3}\frac{p.l^2}{E.e}$ |
|  | $A=B=\frac{P}{2}$<br>$M=\frac{Px}{2}$<br>$M_{max.}=\frac{Pl}{4}$ | $P=4\frac{pW}{l}$<br>$W=\frac{Pl}{4p}$ | $f=\frac{P}{EI}\cdot\frac{l^3}{48}$<br>$f=\frac{1}{12}\frac{p.l^2}{E.e}$ |
|  | $A=\frac{Pc^1}{l}, B=\frac{Pc}{l}$<br>$M_{max.}=\frac{Pcc^1}{l}$ | $P=\frac{pWl}{cc^1}$<br>$W=\frac{Pcc^1}{l.p}$ | $f=\frac{P}{EI}\cdot\frac{l^3}{3}\cdot\frac{c^2}{l^2}\cdot\frac{c_1^2}{l^2}$ |
|  | $A=B=P$<br>Between A & B<br>$M=P.c=\text{const.}$ | $P=\frac{pW}{c}$<br>$W=\frac{Pc}{p}$ | $f_1=\frac{P}{EI}\cdot\frac{l^3}{8}\cdot\frac{c}{l}$<br>$f_1=\frac{1}{8}\frac{p}{E}\cdot\frac{l^2}{e}$<br>$f_2=\frac{P}{EI}\left(\frac{c^3}{3}+\frac{c^2l}{2}\right)$ |
|  | $B=P$<br>$M=\frac{Px^2}{2l}$<br>$M_{max.}=\frac{Pl}{2}$ | $P=2\frac{pW}{l}$<br>$W=\frac{Pl}{2p}$ | $f=\frac{P}{EI}\cdot\frac{l^3}{8}$<br>$f=\frac{1}{4}\frac{pl^2}{Ee}$ |
|  | $A=B=\frac{P}{2}$<br>$M=\frac{Px}{2}\left(1-\frac{x}{l}\right)$<br>$M_{max.}=\frac{Pl}{8}$ | $P=\frac{8pW}{l}$<br>$W=\frac{Pl}{8p}$ | $f=\frac{P}{EI}\cdot\frac{5l^3}{384}$<br>$f=\frac{5}{48}\frac{pl^2}{Ee}$ |
|  | | $A=P\left(1-\frac{2x}{l}+\frac{a}{l}\right)$<br>$B=P\left(1+\frac{2x}{l}-\frac{a}{l}\right)$ | |

The bending moment is a maximum under load 1 when $x=\frac{a}{4}$

$$\text{Mmax.}=\frac{Pl}{2}\left(1-\frac{a}{2l}\right)^2$$

The letters used have the following significance and for convenience they should be expressed in the units stated:

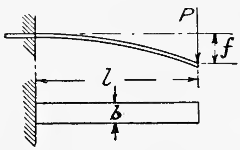
A and B = end reactions, pounds.
P = load, pounds.
$p$ = extreme fiber stress, pounds per sq. in.
$l$ = span of beam in inches.
W = resistance = $\frac{I}{e}$
I = moment of inertia, in.
$e$ = distance neutral axis to extreme fibers having fiber stress $p$, inches.
E = modulus of elasticity, pounds per sq. in.
$c$, $c^1$ = portions of span, inches.
$x$ = left end to section where bending is wanted, in.
$f$ = deflection, inches.

| Shape of Section | Moment of Inertia | Resistance | Distance Base to Center of Gravity | Least Radius of Gyration |
|---|---|---|---|---|
|  | $\frac{bh^3}{12}$ | $\frac{bh^2}{6}$ | $\frac{h}{2}$ | $\frac{\text{Lesser side}}{3.46}$ |
|  | $\frac{B^4-b^4}{12}$ | $\frac{1}{6}\ \frac{B^4-b^4}{B}$ | $\frac{B}{2}$ | $\sqrt{\frac{B^2+b^2}{12}}$ |
|  | $\frac{bh^3}{36}$ | $\frac{bh^2}{24}$ | $\frac{h}{3}$ | The smaller $\frac{h}{4.24}$ or $\frac{b}{4.9}$ |
|  | $\frac{6b^2+6bb_1+b_1^2}{36(2b+b_1)}.h^3$ | | $\frac{1}{3}\cdot\frac{3b+b_1}{2b+b_1}.h$ | |
|  | $\frac{D^4}{20.38}$ | $\frac{D^3}{10.19}$ | $\frac{D}{2}$ | $\frac{D}{4}$ |
|  | $0.049\ (D^4-d^4)$ | $0.098\ \frac{D^4-d^4}{D}$ | $\frac{D}{2}$ | $\frac{1}{4}\sqrt{(D^2+d^2)}$ |
|  | $0.11\ R_4$ | $W_1=0.191R^3$<br>$W_2=0.259R^3$ | $0.424\ R$ | $0.07\ R^2$ |
|  | $0.7854\ ba^3$ | $0.7854\ ba^2$ | | |

## Springs.

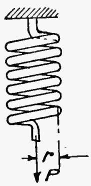
Nomenclature—

$l$ = length of spring, inches.
P = load in pounds.
$d$ = diameter of spring wire, inches.
$f$ = deflection of spring, inches.
$r$ = mean radius of spring helix.
$n$ = number of coils of helical spring.
I = moment of inertia of section from which spring is made.
E = modulus of elasticity of spring material, direct.
G = modulus of elasticity of spring material, transverse.
$p_b$ = extreme fiber stress bending, pounds per square inch.
$p_s$ = extreme fiber stress shear, pounds per square inch.



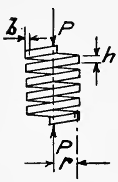

$$P=\frac{bh^2}{6}\frac{p_b}{l};\quad I=\frac{bh^3}{12}$$

$$f=\frac{P}{EI}\frac{l^3}{3}=4\frac{l^3 P}{bh^3 E}=\frac{2}{3}\frac{l^2 p_b}{hE}$$




$$P=\frac{\pi d^3}{16}\frac{p_s}{r}=0.1963\frac{d^3}{r}p_s$$

$$f=\frac{64nr^3}{d^4}\cdot\frac{P}{G}=\frac{4\pi nr^2}{d}\times\frac{p_s}{G}$$




$$P=\frac{2}{9}b^2h\times\frac{p_s}{r}$$

$$f=7.2\pi nr^3\frac{b^2+h^2}{b^3h^3}\frac{P}{G}$$

$$f=1.6\pi nr^2\frac{b^2+h^2}{bh^2}\frac{p_s}{G}$$

## Belts.

Where a number of belts are in service the latest and best scheme for the design and maintenance is that proposed by Mr. Carl Barth. (See Transactions American Society of Mechanical Engineers, January, 1909.) In making his calculations, besides considering the usual factors used in belt design, he has included the influence of belt creep and belt maintenance, demanding an economical initial tension. The belts are supposed to be installed with the required initial tension and the belt removed and tightened when the tension has dropped a predetermined amount. This method necessitates measuring the tension with balances when putting it up.

Other interesting papers which formed the basis of Mr. Barth's calculations are:

Transactions American Society Mechanical Engineers, 1894, a paper by Mr. F. W. Taylor and a paper in the proceedings of 1886 by Mr. Wilfred Lewis.

The belting calculations usually met in hoisting problems differ from the above in being subjected to very intermittent and quite variable loads, and they are usually isolated cases. They are therefore unlikely to be maintained in accordance with Mr. Barth's most admirable plan. The belt velocities are also usually lower than those affected to an appreciable extent by centrifugal force.

BELTS.—Leather belts are made from selected strips or oxhide taken from the back of the animal. These strips are then lap jointed, the joints being cemented and either copper riveted or stitched. Care should be taken that the sections are of uniform thickness or are made so. The belts are usually "single" and made of single strips of hide about 0.22 inch thick, or "double" and made of two thicknesses of hide, the belt becoming about 0.35 inch thick.

Tests of leather belting show a tensile strength ranging from 2,000 to 5,900 pounds per square inch.

The coefficient of friction between leather and cast-iron varies with the velocity from 0.26 to 0.52.

RUBBER BELTS.—These are made by cementing together several plies of canvas with rubber and also facing the belt with rubber. The rubber makes an efficient friction face in contact with the pulley.

The following table gives the belt pull $p$ per inch of belt measured in pounds:

| "D" | Velocity in feet per minute | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 600 | | 1000 | | 2000 | | 3000 | | 4000 | | 5000 | |
| *Inches* | *a* | *b* | *a* | *b* | *a* | *b* | *a* | *b* | *a* | *b* | *a* | *b* |
| 4 | 11 | . . | . . | . . | 17 | . . . | 17 | . . . | 19 | . . . | 19 | . . . |
| 8 | 17 | . . | 22 | . . | 28 | . . . | 31 | . . . | 33 | . . . | 36 | . . . |
| 20 | 28 | 45 | 39 | 50 | 45 | 56 | 50 | 61 | 56 | 67 | 61 | 73 |
| 40 | 33 | 56 | 47 | 67 | 56 | 78 | 61 | 90 | 67 | 95 | 73 | 100 |
| 80 | 39 | 67 | 56 | 84 | 67 | 112 | 73 | 123 | 78 | 135 | 84 | 140 |

NOTE.—*a*, single belts; *b*, double belts.

This table is based upon a table given by Hütte. For driving electrical machinery the values should be taken from one-third to one-half the values given, while for machinery doing intermittent work the values can be increased 50 per cent.

The horse-power transmitted by a belt is given by

$$\text{H. P.} = \frac{p \times b \times D \times n}{126100}$$

$p$ = belt pull in pounds per inch of belt width. Value to be taken from the table.

$b$ = width of belt, inches.

D = diameter of pulley, inches.

$n$ = revolutions per minute of pulley.

Problem.—A 3-inch double belt is driven by a 24-inch pulley making 300 revolutions per minute. What horse-power is transmitted?

**Solution.**—Velocity of the belt in feet per minute

$$\frac{24 \times 3.14 \times 300}{12} = 1884'$$

From the table $p$ is found to be approximately 60 pounds. Now substituting in the equation,

$$\text{H. P.} = \frac{60 \times 3 \times 24 \times 300}{126100} = 10.3$$

The following empirical rules are sometimes used for belting:

1. A single belt 1 inch wide running 800 feet per minute will transmit 1 horse-power.

2. A double belt 1 inch wide running 500 feet per minute will transmit 1 horse-power.

Reversing by Belts.—Reversing being commonly required in this type of machinery, it is accomplished by using an open and a crossed belt.

Belts must be shifted by pressing on the edge of the belt as it runs on a pulley; a slight force exerted here suffices to make the belt travel across the pulley.

## Belt Pulleys.

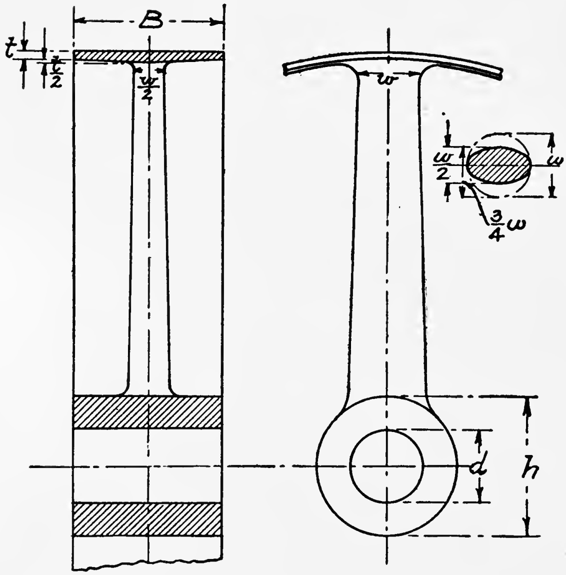



FIG. 1.

D = diameter of the pulley in inches.
$b$ = width of belt in inches.
B = width of pulley = $1.1b + \frac{1}{8}$ inch.

$t$ = thickness of rim. Single belt $= \frac{D}{200} + \frac{1}{8}$ inch.

Double belt $= \frac{D}{200} + \frac{1}{4}$ inch.

$w$ = width of arm at rim $= 0.04 + \frac{7}{8}$ inch.
$v$ = thickness of arm at rim $= \frac{1}{2}w$.
$d$ = diameter of shaft in inches.
$h$ = diameter of hub $= 1\frac{7}{8} \times d$.
$e$ = length of hub $= \frac{2}{3}$ B to B.

### Arms.

The number of arms can be made $i = \frac{B.D}{150} + 3$.

Each side along width of arm tapered $\frac{3}{8}$ inch in 12 inches.
Each side along thickness of arm tapered $\frac{3}{16}$ inch in 12 inches.

## Gearing.

For crane and general work the 15 degree Involute is practically the only tooth used. Its advantages over the cycloidal teeth in spur gears are:

1. Gears of the same pitch are interchangeable.
2. Involute gears operate satisfactorily if the distance center to center is slightly increased over that originally intended.
3. Involute teeth are stronger than cycloidal teeth of the same pitch.

## Drawing Involute Teeth.

To construct a theoretically correct tooth, through the point *o* draw *fh*, making an angle of 15 degrees with the horizontal, draw the circle *ebcd* tangent to the line *fh* and produce the involute by unwrapping the line *ebcd* from the circumference of the circle *ebcd*.

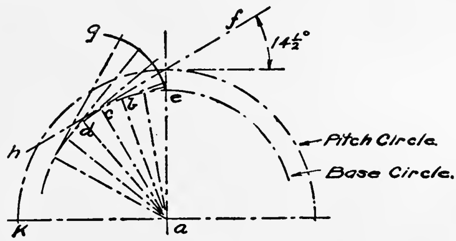


FIG. 2.

When a pattern is to be made for the gear, including the teeth, the following approximate method is useful.

In Fig. 3 draw *bg*, making an angle of 15 degrees with the horizontal; draw the arc *fa* tangent to it, take *cd* equal to ⅓ of *ac* and

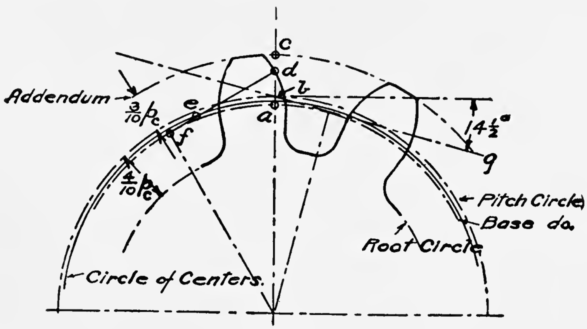


FIG. 3.

through *d* draw *fd* tangent to the arc *fa*; *ed* is ¾ of *fd* and is the radius of the approximate tooth face, the center being at *e*.

Gear teeth are commonly designated by either circumferential or diametral pitch, the latter method gaining very much in favor. Circumferential pitch is measured on the pitch circle and is the distance from a point on one tooth to a corresponding point on the adjacent tooth. Diametral pitch is the number of teeth per inch of diameter, or the total number of teeth divided by the pitch diameter in inches. The relation between circumferential and diametral pitch is expressed algebraically by

$$p_c \times p_d = \pi = 3.1416$$

$p_c$ = circumferential pitch in inches.
$p_d$ = diametral pitch.

DIAMETRAL PITCHES AND CORRESPONDING CIRCUMFERENTIAL PITCHES.

| D. Pitch | C. Pitch | D. Pitch | C. Pitch | D. Pitch | C. Pitch |
|---|---|---|---|---|---|
| | *Inches* | | *Inches* | | *Inches* |
| .... | ..... | 2 | 1.571 | 5 | 0.628 |
| .... | ..... | 2¼ | 1.396 | 6 | 0.524 |
| 1 | 3.142 | 2½ | 1.257 | 7 | 0.449 |
| 1¼ | 2.513 | 2¾ | 1.142 | 8 | 0.393 |
| 1½ | 2.094 | 3 | 1.047 | 9 | 0.349 |
| 1¾ | 1.795 | 4 | 0.785 | 10 | 0.314 |

Common proportions for gear teeth are as follows:

Height of tooth above pitch circle ........... $\frac{3}{10}\,p_c$
Depth of tooth below pitch circle ........... $\frac{4}{10}\,p_c$
Width of tooth on pitch circle, cast teeth . $\frac{19}{40}$ to $\frac{39}{80}\,p_c$

Width of space between teeth, measured on the pitch circle, cast teeth $\frac{21}{40}$ to $\frac{41}{80}\,p_c$.

For cut teeth the proportions are:

Width of tooth = width of space = ½ $p_c$.

Height of tooth above pitch circle 1 ÷ diametral pitch.

## Gear Tooth Design.

The method most commonly used for the design of gear teeth is that of Mr. Wilfred Lewis. (See *Proceedings Engineers Club of Philadelphia*, 1893.) This method is based upon the strength of the actual tooth profile, and assumes the force acting on the tooth as applied at the top of the tooth but distributed across its face.

This formula for 15 degrees Involute is

$$W = sp_cf\left(0.124 - \frac{0.684}{n}\right)$$

where W = load transmitted by the teeth in pounds.
$s$ = allowed working stress on the material.
$p_c$ = circular pitch in inches.
$f$ = tooth face in inches.
$n$ = number of teeth on the weaker wheel of the pair being considered.

When the gear and pinion are both of the same material and neither is shrouded, the pinion will be the weaker, otherwise both should be examined for strength.

To allow for the effect of shock, Mr. Lewis gives a table of allowable working stresses which approximate the values given by the following formulæ. In these formulæ $v$ is the velocity at the pitch circle in feet per minute, and $f$ is the working fiber stress in pounds per square inches.

$$\text{Cast-iron, } s = 8{,}000\left(\frac{600}{600+v}\right)$$

$$\text{Steel (rolled), } s = 20{,}000\left(\frac{600}{600+v}\right)$$

$$\text{Steel castings, } s = 14{,}000\left(\frac{600}{600+v}\right)^*$$

It is sometimes convenient to find the smallest pinion, having a given number of teeth, that can be used in a particular place. When the twisting moment is known the circular pitch can be found from the following formula:

$$p_c = \sqrt[3]{\frac{6.28\,M_t}{n.s.c.\left(0.124 - \frac{0.684}{n}\right)}}$$

here $c = \frac{f}{p_c}$

If the horse-power transmitted is known the formula becomes,

$$p_c = \sqrt[3]{\frac{396000 \times \text{H.P.}}{n.s.c.\text{N}\left(0.124 - \frac{0.684}{n}\right)}}$$

where N = revolution per minute of gear or pinion.

The only difficulty experienced in using these formulæ is in deciding upon the pitch velocity in order to estimate the allow-

* Not given by Lewis.

able working fiber stress. It frequently happens, however, that one desires to limit the fiber stress to an amount considerably below that recommended by Mr. Lewis and under these circumstances the fiber stress can be assumed with sufficient accuracy.

An example will illustrate: the twisting moment upon a shaft is 100,000 inch-pounds, the gear reduction between it and the pinion shaft is 2:1; what is the smallest 15 tooth pinion of steel casting allowing 8,000 pounds fiber stress?

Twisting moment on pinion shaft is

$$\frac{M_t}{\text{gear ratio}}=\frac{100000}{2}=50000 \text{ inch-pounds.}$$

$$c=\frac{f}{p_c}=2.5$$

$$p_c=\sqrt[3]{\frac{6.28\ M_t}{n.s.c\left(0.124-\frac{0.684}{n}\right)}}$$

$$=\sqrt[3]{\frac{6.28\times 50000}{15\times 8000\times 2.5\left(0.124-\frac{0.684}{15}\right)}}=2.377 \text{ inches } c.p.$$

$$\text{diametral pitch}=\frac{\pi}{\text{circular-pitch}}=\frac{3.14}{2.377}=1.32 \frown 1\tfrac{1}{4} \text{ diametral pitch.}$$

The pitch diameter of the pinion then is:

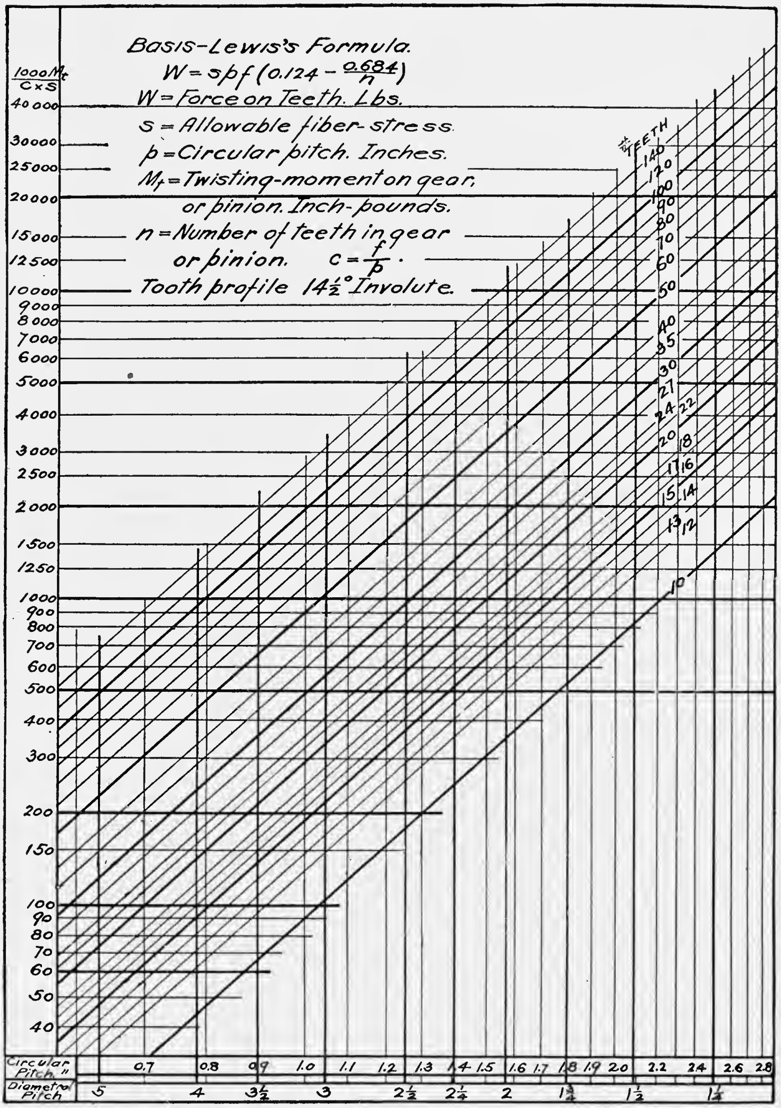
$$D=\frac{\text{number of teeth}}{\text{diameter pitch}}=\frac{15}{1.25}=12 \text{ inches.}$$

The solution of gear problems by this method will be facilitated by the use of the curves on page 21. The ordinates are

$$\frac{1000\ M_t}{c\times s}$$

while the abscissas are the circular or diametral pitches. Both ordinates and abscissas are laid off upon logarithmic bases. The diagonal lines are drawn for varying numbers of teeth.

At velocities of 100 feet per minute and less Mr. Lewis uses 8,000 pounds for cast-iron and 20,000 pounds for steel as the allowable working stress. Where quiet operation is desired the velocity should be kept under 2,400 feet per minute.




The minimum number of teeth in the pinion is preferably 12 to 15; when a smaller number than this is used the teeth should be laid out and the profiles examined for interference. In the case of gears operated by hand the minimum number should be 10 to 12.

In gears with cast teeth the face width is preferably not over twice the circumferential pitch. This provides for the possibility of the force acting on a corner of the tooth instead of entirely across the face. In the case of cut gears the face width need not be limited in this way, although it does not generally exceed 4 times the circumferential pitch.

## Strengthened Gear Teeth.

Spur gear teeth may be strengthened by shrouding. Shrouding is a band cast or forged at the ends of the teeth in order that if the teeth fail they must shear at the shrouding.

The strengthening effect will vary with the tooth width, but with usual proportions it will be about 100 per cent. stronger than

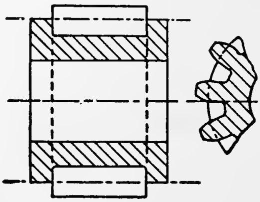


Fig. 4.

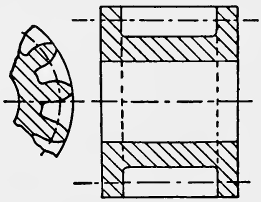


Fig. 5.

the plain tooth. Fig. 4 illustrates "half shrouding." Fig. 5 shows "full shrouding." With half shrouding both gears can be shrouded, as the shrouding extends to the pitch circle only. In the case of "full shrouding" only one gear can be shrouded; this will always be the one with the weaker tooth; that is, the pinion.

Frequently both gears are not made of the same materials; thus the pinion whose tooth form is weaker than that of the gear is made of steel, while the gear may be cast-iron.

In cut gearing the pinion tooth may be strengthened by making it of rolled steel and making its face slightly wider than that of the gear tooth. In this case it is desirable that the pinion shaft be allowed some play along its axis to distribute the wear over the full width of the pinion.

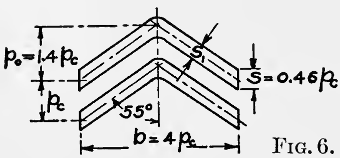



Fig. 6.

$S_1 = 0.377\ p_c$ at 55°.

When particularly smooth running is desired the gears may be made with "herring-bone teeth." According to Hütte the usual proportions of such teeth are those given in Fig. 6.

Where the pinion has a small number of teeth $b$ is sometimes made 5. $p_c$; then $t_o = 1.75p_c$.

These gears are generally "half shrouded" and their strength may be assumed as that of shrouded spur gears of the same pitch and width of face. The height of the tooth above the pitch circle is 3/10 circular pitch; the depth to the root circle from the pitch circle is 4/10 circular pitch.

## Proportions of Gear Arms and Rims.

The stresses in other portions of gears than the teeth are too complicated to lend themselves to very satisfactory analysis; the result is that the design is most frequently accomplished by using proportions that have given satisfaction.

The following method is similar to those generally used where calculations are made. The arms are assumed as cantilevers fixed at the hub and carrying the rim load at the other end; the tooth load is considered as being carried by all the arms. The width of the arms is calculated at the hub. All dimensions in inches.

R = radius of the pitch circle.

W = force on tooth in pounds.

$$c = \left(0.124 - \frac{0.684}{n}\right)$$

$f$ = face of the tooth.

$i$ = number of arms.

$s$ = fiber stress pounds per square inch.

B = width of the arm, measured at the hub (inches).

$$I/e = \text{resistance of an elliptical section} = \frac{B^3}{20}$$

$$I/e = \text{resistance of a cross arm section} = \frac{pB^2}{12}$$

The number of arms is usually an even number, and is about

$$i = \tfrac{3}{4}\sqrt{D}$$

D = pitch diameter in inches.

$p_c$ = circular pitch.

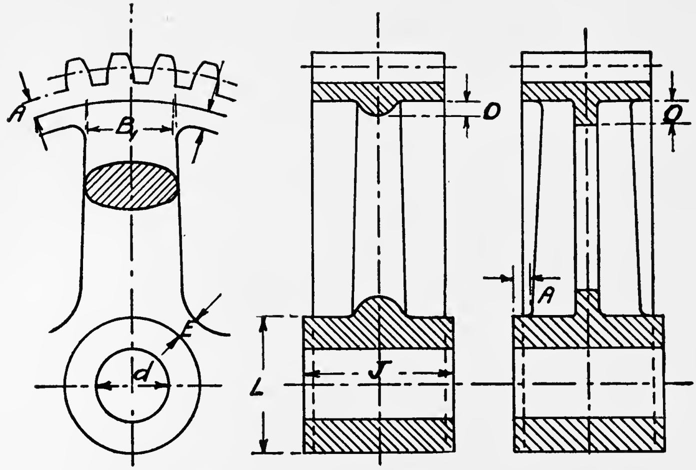
$$M = \frac{Wy}{i} = \frac{SI}{ei} \text{ and } W = spfc$$

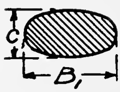
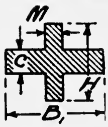
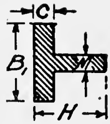
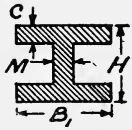
For elliptical arms $B = \sqrt[3]{\dfrac{20\,fcRp}{i}}$  For cross arms $B = \sqrt[2]{\dfrac{12fcR}{i}}$

$n$ = number of teeth in gear.

## Gearing.

Fig. 7.




| |  |  |  |  |
|---|---|---|---|---|
| A | $\frac{p}{2}$ | $\frac{p}{2}$ | $\frac{p}{2}$ | $\frac{p}{2}$ |
| $B_1$, (at rim) | $2.1 \times p$ | 2.3p | $2.3 \times p$ | $2.4 \times p$ |
| C | $\frac{B_1}{2}$ | $\frac{p}{2}$ | $\frac{p}{2}$ | $\frac{p}{2}$ |
| O | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ |
| E | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ | $\frac{B_1}{4}$ |
| H | $f-p$ | $f-p$ | $f-p$ | $-\frac{p}{2}$ |
| J | $f+\frac{O}{40}$ | $f+\frac{O}{40}$ | $f+\frac{O}{40}$ | $f+\frac{O}{40}$ |
| L | 2d | 2d | 2d | 2d |
| M | — | $\frac{p}{3}$ | $\frac{p}{3}$ | $\frac{p}{3}$ |

The sections at head of columns are taken at rim.

Taper of arm widths ¾ inch per foot.
Taper of arm thicknesses ⅜ inch per foot.
The face width $f$ is from 2 to 3½ times the circular pitch.

$D$ = pitch diameter. $d$ = bore of the hub.

$p$ = circular pitch in inches.

The proportions given are for general practice. All fillets should be large.

## Bevel Gears.

Mr. Lewis gives the following method for determining the strength of bevel gears:

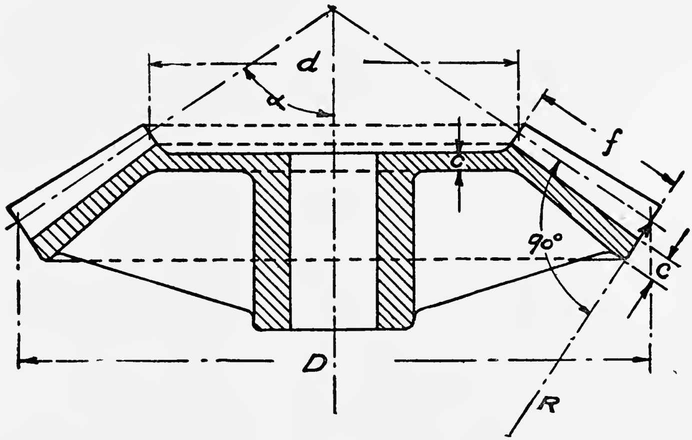



Fig. 8.

All dimensions in inches.

W = force acting on tooth face, assumed at mean diameter. Lbs.

D = outside pitch diameter.

$d$ = inside pitch diameter.

$p$ = circular pitch measured on the outside pitch-circle.

$n$ = actual number of teeth.

$f$ = width of the face.

N = formative number of teeth or $n$. secant $\alpha$; that is, the number of teeth on the circle whose radius is R.

$$y=\left(0.124-\frac{0.684}{N}\right) \text{ and } W=spfy\times\frac{D^3-d^3}{3D^2(D-d)}$$

In good practice $d/D \leqq 2/3$; under these conditions a close approximation to the above formula is given by, $(W=spfy)\frac{d}{D}$

If desired, bevel gears can be treated in a similar manner as spur gears; that is, the calculation can be made for the pitch of the smallest pinion having a required number of teeth that will transmit the given twisting moment.

The following additional nomenclature is required:

$D_m$ = mean diameter of bevel gear.

$d_m$ = mean diameter of bevel pinion.

$n$ = number of teeth in the pinion.

N = formative number of teeth. $N = n.$ secant $\alpha$

$\alpha$ = tangent $^{-1}\frac{d}{D}$, gear reduction

$f$ = width of tooth face measured on the pitch cone.

$p_m$ = circular pitch measured on the circle whose diameter is $d_m$.

$p$ = circular pitch measured on outside pitch circle.

$$p = p_m \times \frac{(d_m cosec\,\alpha) + f}{d_m\, cosec\,\alpha}$$

$c$ = face $\div p_m$, ranging from 2 to 3.

As in spur gears this may be exceeded in carefully made gears.

For gears in which the ratio between the inside diameter and the outside diameter is not less than ⅔ the center of the resultant pressure on the tooth can be assumed as acting on the circle whose diameter is $d_m$. Hence if the twisting moment on the pinion shaft is known, W the resultant force acting on the gear face is

$$\frac{2\,M_t}{d_m} = W.$$

The pitch can now be found from the formula,

$$W = sp_m f\left(0.124 - \frac{0.684}{N}\right)$$

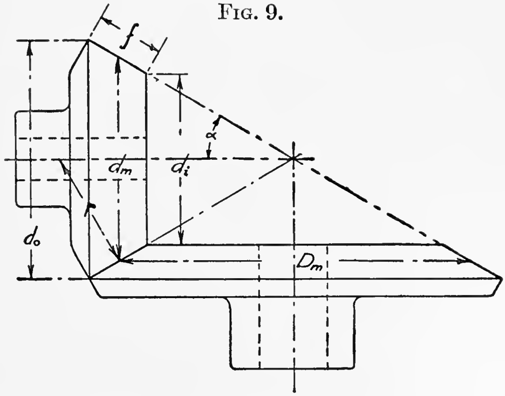
The teeth are designed as though developed on a circle whose radius is $r$ instead of $\frac{d_m}{2}$, so that $N = n.$ sec. $\alpha$.

The formula giving the pitch of the smallest pinion having a given number of teeth is,

$$p_m = \sqrt[3]{\frac{6.283\,M_t}{nsc\left(0.124 - \frac{0.684}{N}\right)}}$$

**Bevel Gears.**

Fig. 9.




$$p_c = p_m \frac{d_m \, cosec \, \alpha + f}{d_m \, cosec \, \alpha}$$

$$d_m = \frac{n \times p_m}{3.1416}$$

$$d_o = \frac{n \times p_c}{3.1416}$$

$M_t$ = twisting moment in inch pounds.

For mitre gears $\alpha = 45$ and Sec. $\alpha = cosec. \alpha = 1.414$.

All dimensions in inches.

Face $f = (2\frac{1}{2}$ to $3\frac{1}{2})\ p_c$

$$\text{Length of hub} = f + \frac{\text{wheel diameter}}{20} = H$$

$C = \frac{p_c}{2}$

$E = 1\frac{7}{8} \times$ bore.

$J = \frac{5}{4}\ p_c$

$L = J + \frac{3}{4}$ inch taper per foot.

$M = \frac{4}{10}\ p_c$

$P = \frac{1}{4}\ J$.

Bevel gears may be either cast or cut. Cast gears are frequently made from a cut pattern.

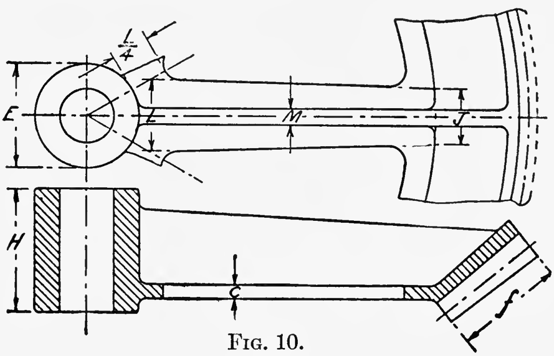



Fig. 10.

**Laying-Out Teeth for Bevel Gears.**

The teeth of bevel gears are laid out the same as teeth of spur gears, excepting that they are laid out on a pitch circle of radius R (see Fig. 8) and not on the actual pitch circle having a diameter D.

### Gear Efficiencies.

Spur gears (cast teeth) and bearings 93 per cent.
Spur gears (cut teeth) and bearings 96 per cent.
Bevel gears (cast teeth) and bearings 92 per cent.
Bevel gears (cut teeth) and bearings 95 per cent.

FIG. 11.

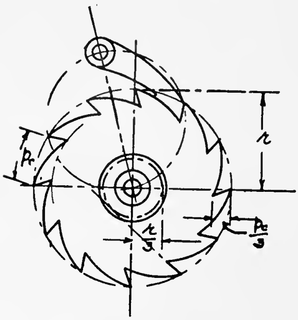


FIG. 12.

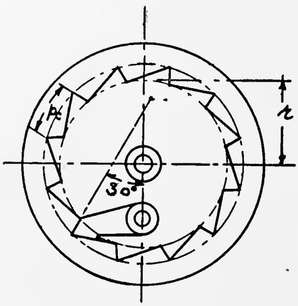
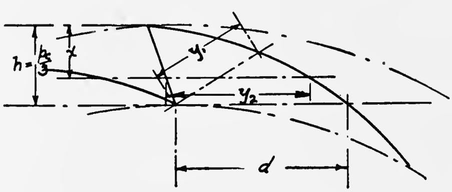


FIG. 13.




### Ratchet Wheels.

W = the load in pounds transmitted to the tooth by ratchet.
$r$ = radius of ratchet wheel.
M = W. r – Turning moment on ratchet-wheel shaft.
$s$ = allowable working fiber stress, pounds per square inch.
$f$ = tooth face in inches.
$p_c$ = circular pitch in inches.
$h$ = height of tooth in inches $= \frac{p_c}{3}$
$d$ = width at root of tooth, parallel to force acting on the tooth.

$d \geqq \frac{p_c}{2}$ depending upon number of teeth.

$y$ = depth of rectangle about which bending is assumed.
$s$ = 2,500 to 4,000 pounds. The latter value when not subjected to any shock.

The bending moment of any section is $M = W.x$

then $M = W.x = \frac{fy^2}{6} \times S$

from which $W = \frac{fs}{6} \times \frac{y^2}{x}$

from this the ratio of $y^2/x$ can be found and the minimum section $y_1$ or $y_2$ must exceed the above value found for $y$.

The section at root can be found from the following equation: When the number of teeth is small, 8 to 12, $y$ can be assumed at $p_c/2$ and

$$W = \frac{f.p_c.s}{8} = \frac{M}{r} \qquad (a)$$

since $$W=\frac{M}{r} \text{ and } r=\frac{p_c\times n}{2\pi}$$

from which $$p_c=\sqrt{\frac{16\,\pi\,M}{f.s.n}} \qquad (b)$$

where $n$ is the number of teeth not exceeding 12.

As the number of teeth increases the length $y$ approaches $p_c$ as a limit. If $y$ be assumed as $0.82\times p_c$, for 20 teeth and over, equations $a$ and $b$ become,

$$W=\frac{f.s.p_c}{3}=\frac{M}{r} \qquad (a_1)$$

$$p_c=\sqrt{\frac{6\pi\,M}{f.s.n}}\left(\begin{matrix}n \text{ not under}\\ 20\end{matrix}\right) \qquad (b_1)$$

For side and internal gears, at the root section, $y$ can be taken as equal to $p_c$, from which equations $a$ and $b$ become

$$W=\frac{f.s.p_c}{2} \quad \text{and} \qquad (a_2)$$

$$p_c=\sqrt{\frac{4\pi M}{f.s.n}} \qquad (b_2)$$

In the case of side teeth the length $p_c$ should be measured at the mean radius of the teeth.

It will be noted that the formulæ $b$ give the minimum ratchet wheel diameters. Having found $p_c$, the diameter of the wheel will be given by

$$D=\frac{p_c\times n}{\pi}.$$

If the diameter is fixed by other conditions of the problem, then W is found from W=M/r, and $p_c$ is found from one of several formulæ "*a*."

**Problem.**—A ratchet wheel and ratchet is called upon to retain a shaft subjected to a turning moment of 2,000 inch-pounds. Allowing a fiber stress of 2,500 pounds per square inch, what is the smallest ratchet wheel having 12 teeth that it would be advisable to use? Assume the face width as 1.5 inches.

$$p_c=\sqrt{\frac{16\pi M}{f.s.n}}=\sqrt{\frac{16\times 3.14\times 2000}{1.5\times 2500\times 12}}=1.49''\backsim 1.5''$$

$$D=\frac{1.50\times 12}{3.14}=5.75''$$

Had it been desired to have the diameter of the ratchet wheel 8 inches, then, assuming the face width as 1 inch,

$$W=\frac{2000}{4}=500 \text{ pounds} \qquad p_c=\frac{8W}{f.s.}=\frac{8\times 500}{1\times 2500}=1.6$$

The number of teeth will be

$$n=\frac{3.14\times 8}{1.6}\backsim 16, \text{ making } D=\frac{1.6\propto 16}{3.14}=8.15''$$

A couple of the sections should be checked as first shown, after the pitch has been found.

## Worm Gearing.

The data for the design of a worm and wheel are less satisfactory than those available for any other form of gearing. The following references cover the principal experiments made upon worm gearing:

Zeitschrift des Vereines deutscher Ingenieure, 1898, p. 1156 (Stribeck); 1900, p. 1473 (Ernst).

American Society of Mechanical Engineers, vol. vii, 1886 (Lewis). Zeitschrift des Vereines deutscher Ingenieure, 1903 (Bach and Roser).

Proceedings English Institution Mechanical Engineers, Jan., 1906.

Considerable of this data has been condensed by Fred. A. Halsey in "Worm and Spiral Gearing," Van Nostrand Science Series, No. 116.

In the usual worm and wheel the worm or screw is an involute rack tooth, having an angle of 29 degrees. A cutter similar to the worm is made, and the teeth on the worm wheel are milled with this cutter. When the cutter is replaced by the worm the worm and wheel should mesh accurately and the contact be line rather than point contact.

The worm may have but a single thread, like the ordinary screw or bolt, but more frequently for transmission is made with

2, 3, or even 4 threads. The pitch $p$ is the distance measured on the pitch circle between corresponding points on adjacent teeth of the worm wheel.

The lead, or advance of the worm wheel corresponding to one revolution of the worm, is $m.p$ where $m$ is the number of threads on the worm.

Common proportions for such a worm and wheel are:

Angle of tooth, 29 degrees.

Number of teeth on wheel no less than 30.

Height of tooth above pitch line at central section $\frac{3}{10}\,p$.

Depth of tooth below pitch line at central section $\frac{4}{10}\,p$.

Width of tooth on wheel, 1.5 to 2.5 times the pitch, $p$.

**Materials.**—For hand driving both worm and wheel may be made of cast-iron, or the worm may be turned from steel. For power driving at higher velocities, it is necessary to have both worm and wheel cut. The materials are preferably steel for the worm which may be hardened, and phosphor-bronze for the wheel. On account of the expense of phosphor-bronze it is customary to make only the rim containing the teeth of this material, and then key it to a cast-iron blank.

The teeth on the phosphor-bronze rim are cut by a hob as previously explained.

SECTION OF WORM TOOTH.

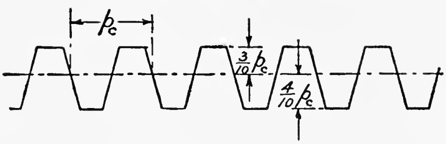



FIG. 14.

The following theory applies to worm gear reduction and efficiency.

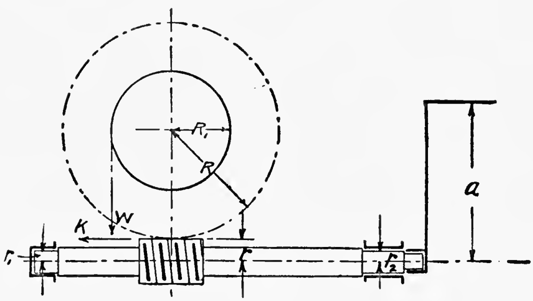



FIG. 15.

$F$ = force required to raise load W in pounds.
$F_o$ = force required to raise load W, if there was no friction.
$a$ = lever arm.
$W$ = load on drum whose pitch radius is R.
$R$ = pitch radius of worm wheel.
$K$ = axial pressure on worm, in pounds.
$p$ = circular pitch of worm wheel.
$n$ = number of teeth on wheel.
$f$ = width of tooth face on wheel measured on pitch circle of the worm.
$l$ = lead of worm, $m.p$.
$m$ = number of threads on worm.
$\eta$ = efficiency of drive.
$\rho$ = angle of lead. $= \text{tang}^{-1} \frac{m.p}{2\pi r}$
$r$ = pitch radius of worm.
$r_1$ = radius of thrust bearing.
$r^2$ = radius of crank bearing.
$\mu$ = coefficient of friction at bearings, say $1/10$.
$\Theta$ = angle of friction between worm and wheel.

All dimensions in inches.

Neglecting friction $F_o 2\pi r = K.l = \frac{W.R_1}{R} \times l.$

Since $2\pi R = m.p$ and $l = m.p$ hence $F_o = \frac{W.\ R_1}{a} \times \frac{m}{n}$

If the efficiency is $\eta$, then $F = \frac{F_o}{\eta} = \frac{W.R_1}{\eta.a} \times \frac{m}{n}.$

Equating moments, $M = F.a = K_1 r \tan(\rho + \Theta).$

Considering the bearing friction

$$F.a = Kr \tan(\rho+\Theta) + \mu K r_1 + \mu F r_2$$

from which

$$F = \frac{K[r \tan(\rho+\Theta) + \mu r_1]}{a - \mu r_2}$$

The friction loss in the journals will range from 2 to 10 per cent., hence

$$F = 1.02 \text{ to } 1.10 \frac{Kr \tan(\rho+\Theta)}{a}$$

The worm efficiency can be estimated as follows:

$$\eta_w = \frac{F_o}{F} = \frac{r \text{ tang } \rho\, (a - \mu r_2)}{a[r \text{ tang } (\rho + \Theta) + \mu r_1]}$$

$$\text{Approximate value } \eta = \frac{\tan \rho}{1.10 \tan (\rho + \phi)}$$

The efficiency of this entire machine will be the product of the efficiencies of the worm and worm shaft bearings, of the drum shaft bearings, and of the chain or rope running on the drum.

The following are a few conclusions drawn from the above theory and corroborated by the experiments previously referred to:

1. The angle of the screw or helix of the worm should be large, the lead being preferably about equal to the pitch diameter. This gives an angle of 17° 40′. Satisfactory angles are considered as lying between 15° and 20°.

2. Worms should be run in a bath of oil.

3. The satisfactory working of a worm depends upon the rise in temperature of the oil bath. This rise in temperature is due to the generation of heat in lost work or friction, and the ability of the mechanism to dissipate the heat thus generated. The conditions are complicated; the rise in temperature affects the viscosity of the oil and consequently the coefficient of friction between worm and wheel. Increased temperature and increased velocity increase the rate of heat loss by the mechanism.

Stribeck and Bach and Roser made experiments to determine these influences, and the tests of Bach and Roser were made upon worms of approximately the following dimensions:

| | |
|---|---|
| Pitch diameter | 3.06″ |
| Outside diameter | 3.69″ |
| Inside diameter | 2.21″ |
| Pitch | 1 ″ |
| Height of tooth | ⅓×pitch. |
| Lead | 3.00″ |
| Angle of lead | 17° 34′ |
| Length of worm | 5.83″ |

**Worm Wheel.**

| | |
|---|---|
| Number of teeth | 30 |
| Pitch diameter | 9.63″ |
| Width of tooth measured on arc | 3.87″ |
| Reduction | 10 to 1 |

In these experiments the condition under which the experiments were made ranged as follows:

| | |
|---|---|
| Axial pressure on worm, pounds | 244 to 2760 |
| Velocity of worm at pitch line, feet per minute | 1725 to 51 |
| Atmospheric temperature, degrees Fahrenheit | 47 to 57 |
| Rise in temperature, degrees Fahrenheit | 50 to 171 |
| Efficiency, per cent | 65 to 84 |
| Maximum temperature of oil bath, degrees Fahrenheit | 222 |

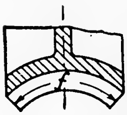
From these experiments Bach and Roser suggest the following formulæ to determine the allowable pressure upon a worm:

$$K = c.f.p \qquad C = 7.9a\,(t_1 - t_2) + 14.25b$$

$$a = \frac{13.12}{v} + 0.42 \qquad b = \frac{21500}{540 + v} - 25$$

K = axial pressure on worm, in pounds.

$v$ = velocity of worm at pitch line, feet per minute.



$f$ = width of tooth on worm wheel, measured on the arc of the worm pitch circle.

$p$ = pitch of worm wheel, measured from one tooth to the adjoining one.

These formulæ would seem applicable only to worms of approximately the same dimensions as those used in the tests.

Based upon the conclusions of Stribeck and Ernst Hütte gives the following formula:

$$K = c.f.p$$

$c$ = 250 to 400 for cast-iron where strength alone is considered.

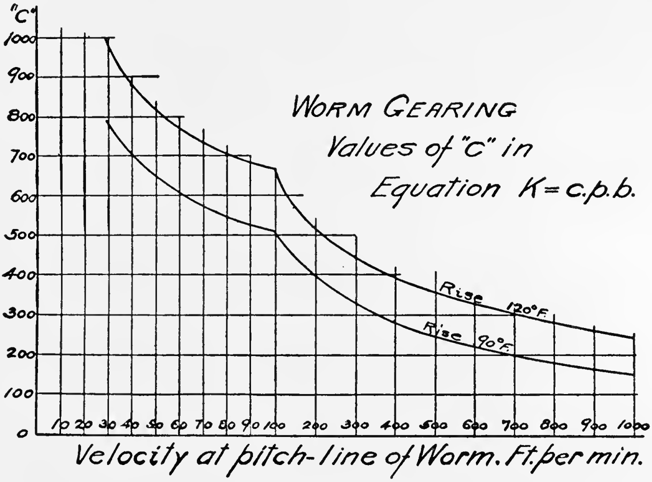
$c$ = 110 to 280, depending upon the revolutions of the worm and the efficiency of the oil bath.

According to Stribeck $c$ can be made 280 to velocities of 720 feet per minute.

$c$ = 450 to 715 for phosphor-bronze wheels on hardened steel worms.

In estimating the constant "$c$" for various velocities the temperature rise can be assumed at 90° F. for continuous service under ordinary conditions. For intermittent service where some opportunity is afforded the oil to cool this assumed temperature rise can be taken higher. For an illustration of worm design see the design of worm-driven hoist, p. 216.

The following curves give the value of C calculated by the given formulæ for 90° and 120° F. rise in temperature of the oil bath under continuous operation:




## Shafting.

Shafting may be either cold-rolled or hot-rolled. The former is finished in the process of its manufacture, while the latter can be finished all over or turned at the places at which bearings, couplings, pulleys, etc., are located. Both can be had in lengths up to 30 feet; however, 20 feet is a common shipping length for boxed shafting. Shafts of both kinds are to be had in the following sizes:

$1\frac{3}{16}$ to $3\frac{7}{16}$ inches increasing by $\frac{4}{16}$ inch.
$1\frac{1}{4}$ to $3\frac{1}{2}$ inches increasing by $\frac{1}{8}$ inch.
$3\frac{7}{16}$ to $4\frac{15}{16}$ inches increasing by $\frac{8}{16}$ inch.
$3\frac{1}{2}$ to 5 inches increasing by $\frac{1}{2}$ inch.

$5\frac{7}{16}$ inches, $5\frac{1}{2}$ inches, $5\frac{15}{16}$ inches, and 6 inches are also regularly turned. The larger forged sizes can be had in any dimensions.

## Strength of Shafting.

For shafting in which there is torsion only, by equating the external moment on the shaft with the resisting moment we have

$$M_t = \frac{pJ}{e}$$

Here $M_t$ = torsional moment on the shaft in inch-pounds.
$p$ = unit working shearing stress, pounds per square inch.
J = polar moment of inertia in inches.

Since for round shafts $J = \frac{\pi D^4}{32}$,

$d$ = diameter of the shaft in inches and $e = \frac{d}{2}$

We have from the first equation

$$d = \sqrt[3]{\frac{5.1 M_t}{p_s}}$$

Where the horse-power transmitted by a shaft is known the twisting moment is given by the following relation:

$$M_t = \frac{63{,}025 \times H.P.}{N}$$

here

H. P. = horse-power being transmitted by shaft.
N = revolutions of the shaft per minute.
$M_t$ = twisting moment, inch-pounds.

When the horse-power transmitted and the revolutions per minute of the shaft are known and the shaft is not subjected to bending of any consequence the following formula in which the shaft is designed for torsion only is convenient.

Here $p_s$ = is the maximum shearing fiber stress desired, in pounds per square inch.

$$d = \sqrt[3]{\frac{321{,}000 \times H.P.}{p_s \times N}} = 68.5 \sqrt[3]{\frac{H.P.}{p_s \times N}}$$

As a rule, shafting is subjected to combined bending and torsion and under these conditions the equivalent bending moment must be found and the shaft diameter determined for it.

$$M_{e.b.} = 0.35\, M_b + 0.65 \sqrt{M_b^2 + M_t^2}$$

and this is also sometimes written

$$M_{e.b.} = \tfrac{3}{8} M_b + \tfrac{5}{8} \sqrt{M_b^2 + M_t^2}$$

Here $M_{e.b.}$ is the equivalent bending moment in inch-pounds.

When the equivalent bending moment is used the diameter is given by

$$d = \sqrt[3]{\frac{10.2\, M_{e.b}}{p_b}}$$

$p_b$ = allowable working strength in bending.

### Torsional Deflection.

In many instances it is not only necessary that the shaft shall develop sufficient strength but also that it shall be amply stiff to resist twisting. This is especially the case in long shafts. Although practice varies, the following are rules sometimes used:

1. In mill practice to limit the deflection to 1 degree in a length of shaft equal to 20 times its diameter.

2. Ordinary service, no excessive fluctuations, to allow $1/10$ of a degree deflection per foot of shaft.

3. Fluctuating loads suddenly applied 0.075 degree deflection per foot of shaft.

4. Sudden reversals under full load, 0.05 degree deflection per foot of shaft.

The angle of twist of a shaft in degrees is given by

$$\triangle = 7000\,\frac{M_t \times L}{E_s \times d^4}$$

Since for steel the modulus of elasticity is 11,600,000 pounds $= E_s$, $L =$ length of shaft in feet, $M_t =$ twisting moment inch-pounds.

$$\triangle = \frac{M_t \times L}{1660 \times d^4}$$

To meet the requirement of stiffness given as No. 1 the diameter will be given by

$$d = \frac{1}{10}\sqrt[3]{M_t}$$

**Problem.**—A shaft is to transmit a twisting moment of 51,000 inch-pounds; what should its diameter be, if in addition to this twisting moment it is also subjected to a bending moment of 60,000 inch-pounds? Had the shaft been 100 feet long and it was deemed necessary to limit its angle of twisting to 1 degree in 20 diameters, what diameter should be used? Allow 7,000 pounds in shear and 9,500 pounds in flexure.

$$d = \sqrt[3]{\frac{5.1 \times 51000}{7000}} = 3\tfrac{3}{8}'' \qquad (a)$$

$$M_{e.b.} = 0.35\ M_b + 0.65\sqrt{M_b^2 + M_t^2} \qquad (b)$$

$$= (0.35 \times 60000) + 0.65\sqrt{(60000)^2 + (51000)^2}$$

$$= 72{,}200 \text{ pounds.}$$

$$d = \sqrt[3]{\frac{10.2 \times 72200}{9500}}\,4.27'', \text{ say } 4\tfrac{5}{16}''.$$

Designing for twisting of shaft, (c)

$$d = \tfrac{1}{10}\sqrt[3]{51000} = 3.715, \text{ say } 3\tfrac{3}{4}''.$$

## Combined Bending and Twisting.

The accompanying curves are intended to abridge the work of calculating the diameters of shafts subjected to combined bending and twisting. The diameters can be read for fiber stresses in flexure, varying from 7,000 to 10,000 pounds per square inch. The diagram is based upon the following formulæ:

$$M_{e.b.} = \tfrac{3}{8}M_b + \tfrac{5}{8}\sqrt{M_b^2 + M_t^2}$$

and $$d = \sqrt[3]{\frac{10\,M_{e.b.}}{p_b}}$$

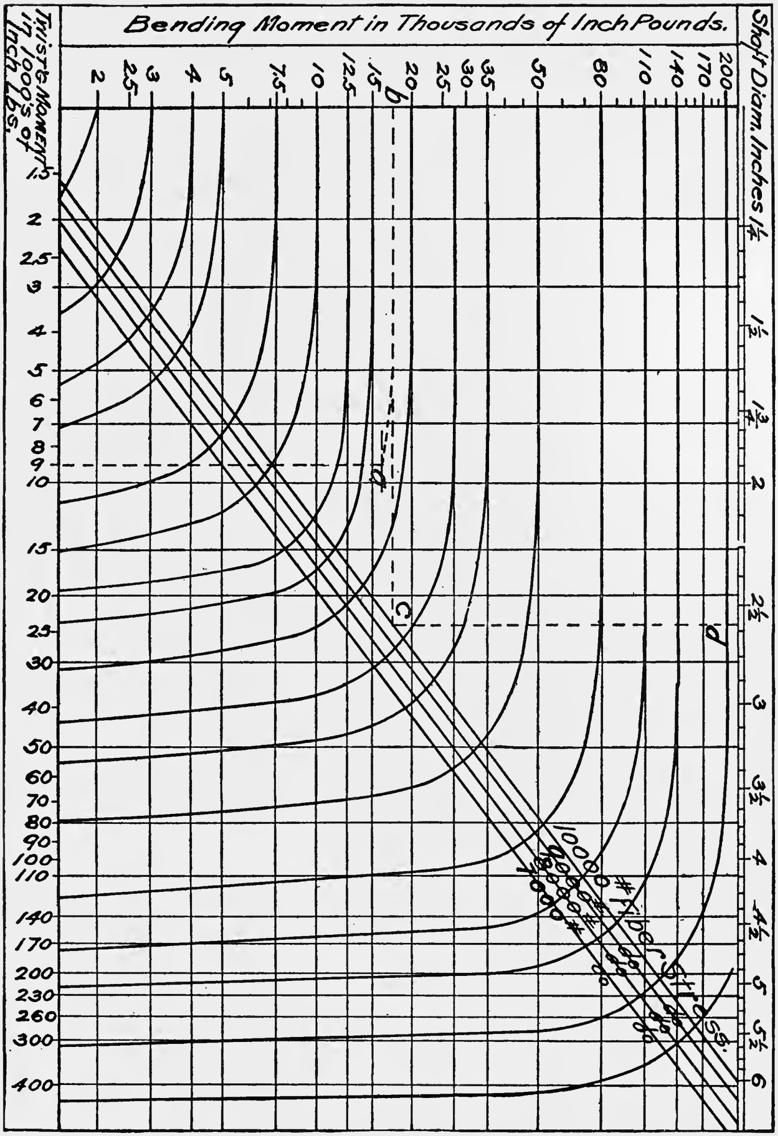
Both bending and twisting moments are plotted in thousands of inch-pounds. The curved lines represent constant equivalent bending moments, their numerical values are read upon the scale of bending moments. The following problem will illustrate the use of the diagram. A shaft is subjected to a twisting moment of 9000 inch-pounds and a bending moment of 16,000 inch-pounds; what diameter should be used if the extreme fiber stress in bending is limited to 10,000 pounds per square inch? From the point of intersection *a* of the ordinate at 9000 with the abscissa at 16,000 follow with the dotted line *ab* parallel to the curves of equivalent bending 15 and 20. From *b* draw the horizontal line *bc* until it intersects the line of desired fiber stress in *c*; from this point draw *cd* perpendicularly until it intersects the scale of shaft diameters in *d*. The shaft diameter is then read off as 2⅝ inches.

The diagram will also serve for shafts subjected to twisting only. The flexural stress is to be assumed as 1.3 of the desired shearing stress.

**Problem.**—A shaft is subjected to a twisting moment of 24,000 inch-pounds; what should its diameter be made, allowing a shearing stress of 6,400 pounds per square inch?

$$\text{Flexural stress} = 6400 \times 1.3 = 8320 \text{ pounds.}$$

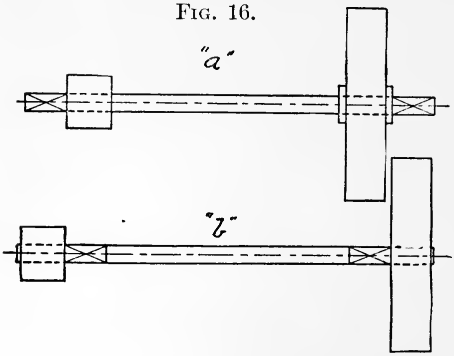
Now following around parallel to the curve nearest the 24,000 twisting moment abscissa, our line coincides with 15,000 pounds ordinate; a horizontal line through this point until it intersects the line of 8000 pounds fiber stress, and then this point of intersection projected upon the scale of diameters gives 2⅝ inches diameter.


Bending Moment in Thousands of Inch Pounds.
Twisting Moment in 1000's of Inch Lbs.
Shaft Diam. Inches
2 2.5 3 4 5 7.5 10 12.5 15 20 25 30 35 50 80 110 140 170 200
1.5 2 2.5 3 4 5 6 7 8 9 10 15 20 25 30 40 50 60 70 80 90 100 110 140 170 200 230 260 300 400
1¼ 1½ 1¾ 2 2¼ 3 3½ 4 4½ 5 5½ 6
a b c d
10000 # Fiber Stress.
9000 # " "
8000 # " "
7000 # " "

## Bending of Shafts.

In cases like these the forces at the pinion will usually be much larger than those at the gear, so that in general it will be sufficiently accurate to neglect the bending due to the gear force and design the shaft for the force acting at the pinion.

Fig. 16.




In all cases, where possible, both gear and pinion should be placed close to the bearings. The arrangement shown in case *a* will be stiffer than case *b*.

When the forces act as shown in the isometric sketch, Fig. 17, the approximation is more difficult, and where it is thought advisable to determine the bending moment with care it may be done as follows:

The method employed is a graphical one and uses the principles explained on page 76. The shaft is shown as acted upon by two parallel forces in a common plane, and each equal to 1000 pounds; a third force of 2000 pounds acts in a different plane. Fig. 18 represents the shaft drawn in a plane containing the axis of the shaft and the two 1000 pound forces. Fig. 19 is the end view, showing also the 2000 pound force and the angle B which it makes with the two other forces.

Fig. 17.

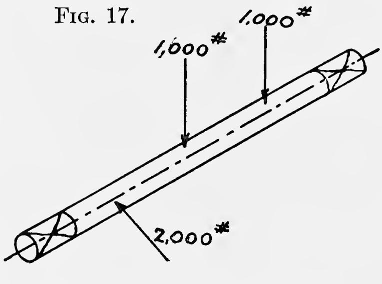



Fig. 20 is a force polygon; the forces BC and CD are both laid-off to a suitable scale and a pole O is taken, making the angle BOD about 90 degrees and measuring the pole distance OC in the scale that the forces BC and CD are laid-off to. Fig. 21 is an equilibrium polygon and, as previously explained, it can be interpreted as a bending-moment diagram. In Fig. 20 the three forces OB, OC and CB closing the triangle are in equilibrium and must meet in a point in the equilibrium polygon, hence through any point 2 in BC draw lines 21 and 23 parallel respectively to OB and OC. In the same manner CD, 23 and 34 must pass through the common point 3, which is located by the intersection of 23 with CD. 34 must be parallel to OD. Connect points 1 and 4, then 1-2-3-4-1 gives the equilibrium polygon or bending-moment diagram due to the two loads of 1000 pounds each. The magnitude

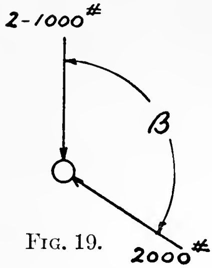



Fig. 19.

Fig. 18.

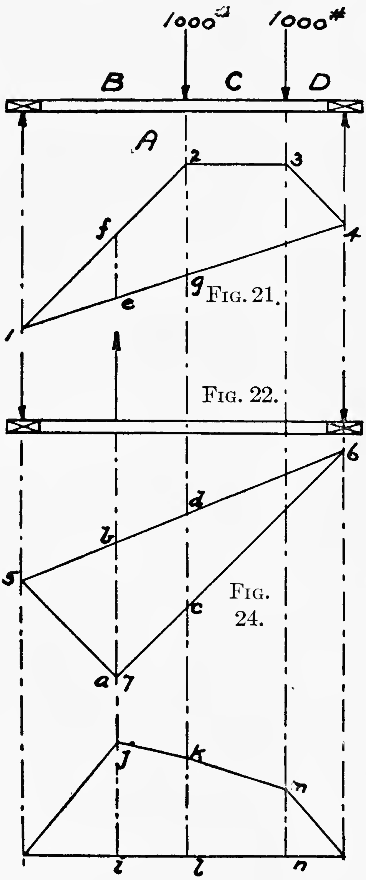



Fig. 21.

Fig. 22.

Fig. 24.

Fig. 28.

Fig. 20.

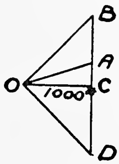



Fig. 23.

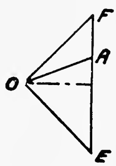
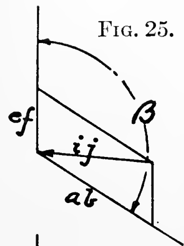



Fig. 25.



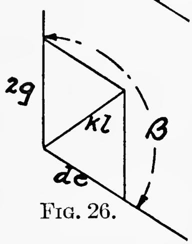




Fig. 26.

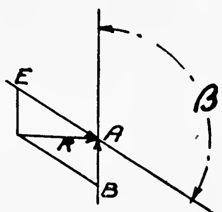



Fig. 27.

of this moment at any point, as at 2 under the local BC, is found by measuring the vertical intercept $2g$ in the equilibrium polygon by the same scale to which the beam span is laid-off, and multiplying this quantity by the pole distance OC in pounds.

Fig. 22 represents the shaft drawn in a plane containing the shaft axis and the force of 2000 pounds. Figs. 23 and 24 are the force and equilibrium polygons drawn as before.

It now remains to combine these diagrams, and this can be done in the same way that forces are combined to find a resultant. In Fig. 25 the directions of the forces are laid off, making the angle $\beta$. It is desired to find the resulting bending at the section in line with *ab*. *ef* is taken from Fig. 21 and laid-off on Fig. 25, and in the same way *ab* is taken from Fig. 24 and laid-off in its proper direction from the end of *ef*.

The parallelogram is completed and *ij* is found to be the resultant. The plane in which this bending occurs will contain the shaft axis and the line *ij* in Fig. 25.

In a similar manner the bending moments *kl* and *mn* can be determined and plotted in Fig. 28.

Fig. 28 shows the combined bending moments as though in one plane; this is immaterial, as only the magnitudes are required. The maximum is *ij*, and its magnitude is found by scaling *ij* in Fig. 28 by the scale used to lay-off the shaft span and multiplying this by the pole distance OC in pounds.

Should the resultants at the bearings be required they may be found by drawing AO parallel to 14 in Fig. 20 and AO parallel to 56 in Fig. 23. Now laying off the left-hand reactions AB and AE in Fig. 27 the resultant R is found in both magnitude and direction.

## Keys.

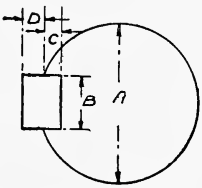
The two commonest types of keys are straight square keys and taper keys of rectangular cross-section. The former are commonly used for machine tools and the latter in general heavy work subjected to vibration. The taper keys are also used where it is desirable to have the key withdrawn to permit of the removal of the machine part keyed to the shaft.

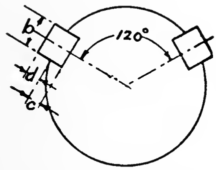
There is no generally accepted standard for key dimensions. The following are common dimensions for square keys.

| Diam. Shaft *Inches* | Key *Inches* | Diam. Shaft *Inches* | Key *Inches* |
|---|---|---|---|
| 1 to 1 3/16 | 3/16 | 3 15/16 to 5 1/2 | 13/16 |
| 1 1/4 to 1 7/16 | 1/4 | 5 15/16 to 6 1/2 | 15/16 |
| 1 1/2 to 1 5/8 | 5/16 | 6 15/16 to 7 1/2 | 1 1/16 |
| 1 11/16 to 2 1/8 | 7/16 | 7 15/16 to 8 1/2 | 1 3/16 |
| 2 3/16 to 2 5/8 | 9/16 | 8 15/16 to 9 1/2 | 1 5/16 |
| 2 11/16 to 3 1/2 | 11/16 | | |

Taper keys are usually specified to have width of not less than one-fourth the diameter of the shaft, and a minimum height of one-half the width. When two keys are used they are placed 120° apart and the minimum width is then one-sixth the shaft diameter, while the height is one-half of this width. The taper is commonly one-eighth inch per foot.

Keys.




| For 1 Key | | | | | For 2 Keys | | |
|---|---|---|---|---|---|---|---|
| Size of Shaft<br>A | Width<br>B | Depth in Shaft<br>C | Depth in Hub<br>D | Thick-ness at small end<br>E | Width<br>b | Depth in Shaft<br>c | Depth in Hub<br>d |
| *Inches* | *Inches* | *Inches* | *Inches* | *Inches* | *Inches* | *Inches* | *Inches* |
| 3/4 to 1 1/8 | 5/16 | 5/32 | 1/8 | 9/32 | 3/16 | 3/32 | 1/16 |
| 1 3/16 to 1 3/8 | 3/8 | 3/16 | 5/32 | 11/32 | 1/4 | 1/8 | 3/32 |
| 1 7/16 to 1 7/8 | 1/2 | 7/32 | 3/16 | 13/32 | 5/16 | 5/32 | 1/8 |
| 1 15/16 to 2 3/8 | 9/16 | 1/4 | 7/32 | 15/32 | 3/8 | 3/16 | 5/32 |
| 2 7/16 to 2 7/8 | 5/8 | 9/32 | 1/4 | 17/32 | 1/2 | 7/32 | 3/16 |
| 2 15/16 to 3 3/8 | 3/4 | 5/16 | 9/32 | 19/32 | 9/16 | 1/4 | 7/32 |
| 3 7/16 to 3 7/8 | 7/8 | 11/32 | 5/16 | 21/32 | 5/8 | 9/32 | 1/4 |
| 3 15/16 to 4 3/8 | 1 | 3/8 | 11/32 | 23/32 | 3/4 | 5/16 | 9/32 |
| 4 7/16 to 4 3/4 | 1 1/8 | 13/32 | 3/8 | 25/32 | 13/16 | 5/16 | 9/32 |
| 4 13/16 to 5 1/4 | 1 1/4 | 15/32 | 7/16 | 29/32 | 7/8 | 11/32 | 5/16 |
| 5 5/16 to 5 3/4 | 1 3/8 | 1/2 | 15/32 | 31/32 | 15/16 | 3/8 | 11/32 |
| 5 13/16 to 6 1/4 | 1 1/2 | 17/32 | 1/2 | 1 1/32 | 1 | 3/8 | 11/32 |
| 6 5/16 to 6 3/4 | 1 5/8 | 9/16 | 17/32 | 1 3/32 | 1 1/8 | 13/32 | 3/8 |
| 6 13/16 to 7 1/4 | 1 3/4 | 5/8 | 19/32 | 1 7/32 | 1 3/16 | 7/16 | 13/32 |
| 7 5/16 to 7 3/4 | 1 7/8 | 21/32 | 5/8 | 1 9/32 | 1 1/4 | 15/32 | 7/16 |
| 7 13/16 to 8 1/4 | 2 | 11/16 | 21/32 | 1 11/32 | 1 3/8 | 1/2 | 15/32 |

Depth of Keyway in Hub will be measured at small end of Keyway.
Taper of Keyway in Hub 1/8 inch to 1 foot.

**Feathers.**—When the piece secured to the shaft is prevented from rotating but is intended to slide along the shaft, the key preventing this rotation is called a feather. One manufacturer uses the following dimensions; ½ the depth of the key is in the shaft; these feathers are square.

| Diam. of Shaft. *Inches* | Side of Key *Inches* |
|---|---|
| 1 7/16 to 1 11/16 | 3/8 |
| 1 15/16 to 2 3/16 | 1/2 |
| 2 7/16 to 2 11/16 | 5/8 |
| 2 15/16 to 3 3/16 | 3/4 |
| 3 7/16 to 3 11/16 | 7/8 |
| 3 15/16 to 4 3/16 | 1 |
| 4 7/16 to 4 11/16 | 1 1/8 |
| 4 15/16 to 5 3/16 | 1 1/4 |
| 5 7/16 to 5 11/16 | 1 3/8 |
| 5 15/16 to 6 3/16 | 1 1/2 |
| 6 7/16 to 6 11/16 | 1 5/8 |

## Shaft Couplings.

There are a number of forms of shaft couplings; the most common are flange couplings, clamp couplings and cone couplings.

### Flange Couplings.

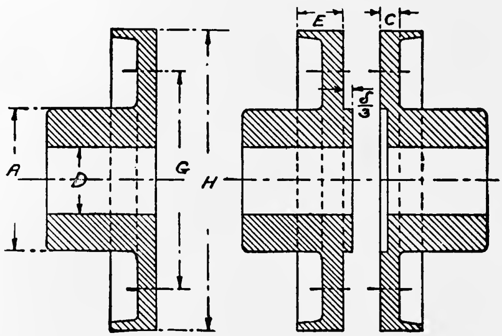



Fig. 29.

$$A = 1.8\,D + 0.8''.$$

Length of hub $\geqq 1\frac{1}{4}\,D$—for forced or shrunk fit.

Length of hub $\geqq 1\frac{1}{2}\,D$—for ordinary fit with key.

$$\delta = \frac{D}{8} + 0.4'' \text{ to } \frac{D}{8} + 0.6''.$$

When male and female couplings are used the recess is made 1/32 inch deeper than the extension on the male coupling. These couplings are used most frequently on shafts 2½ inches in diameter and over.

The following tables of coupling proportions are based upon Hütte, the driving assumed to be done by friction between the faces. Coefficient of friction 25 per cent.

FLANGE COUPLINGS.

| D | A | Bolts lbs. | S | B | C | E | G | H |
|---|---|---|---|---|---|---|---|---|
| 1½ | 3½ | 4 | ⅝ | 1⅞ | 7/16 | 1 1/16 | 5⅜ | 7¾ |
| 1¾ | 4 | 4 | ⅝ | 1⅞ | ½ | 1 ⅛ | 5⅞ | 8¼ |
| 2 | 4½ | 4 | ¾ | 2¼ | 9/16 | 1 5/16 | 6¾ | 9⅝ |
| 2¼ | 5 | 5 | ¾ | 2¼ | ⅝ | 1 ⅜ | 7¼ | 10⅛ |
| 2½ | 5⅜ | 5 | ⅞ | 2⅝ | 11/16 | 1 9/16 | 8 | 11¼ |
| 2¾ | 5¾ | 5 | ⅞ | 2⅝ | ¾ | 1 ⅝ | 8⅜ | 11⅝ |
| 3 | 6¼ | 5 | ⅞ | 2⅝ | 13/16 | 1 11/16 | 8⅞ | 12¼ |
| 3¼ | 6¾ | 6 | 1 | 3 | ⅞ | 1 ⅞ | 9¾ | 13½ |
| 3½ | 7⅛ | 6 | 1 | 3 | 15/16 | 1 15/16 | 10⅛ | 13⅞ |
| 3¾ | 7⅝ | 6 | 1 | 3 | 1 | 2 | 10⅝ | 14⅛ |
| 4 | 8 | 6 | 1 | 3 | 1 ⅛ | 2 ⅛ | 11 | 14⅞ |
| 4½ | 8⅞ | 7 | 1⅛ | 3⅜ | 1 ¼ | 2 ⅜ | 12¼ | 16½ |
| 5 | 9⅞ | 7 | 1¼ | 3¾ | 1 ⅜ | 2 ⅝ | 13⅝ | 17⅞ |
| 5½ | 10¾ | 8 | 1¼ | 3¾ | 1 ⅜ | 2 ⅝ | 14½ | 19⅛ |
| 6 | 11⅝ | 8 | 1⅜ | 4⅛ | 1 ½ | 2 ⅞ | 15¾ | 20⅞ |
| 6½ | 12½ | 9 | 1⅜ | 4⅛ | 1 ⅝ | 3 | 16⅝ | 21¾ |
| 7 | 13⅜ | 9 | 1½ | 4½ | 1 ¾ | 3 ¼ | 17⅞ | 23½ |
| 7½ | 14⅜ | 10 | 1½ | 4½ | 1 ⅞ | 3 ⅜ | 18⅞ | 24½ |
| 8 | 15¼ | 10 | 1⅝ | 4⅞ | 2 | 3 ⅝ | 20⅛ | 26¼ |

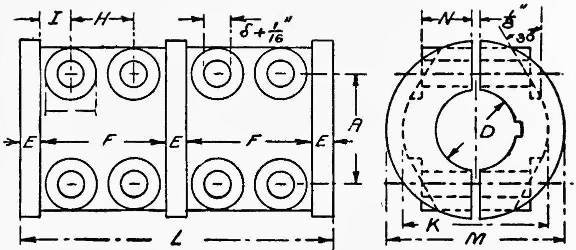



FIG. 30.

SHAFT COUPLINGS.

| D | A | δ | l | E | F | G | H | I | L | C | K | M | N | Key Sq. |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1¼ | 2⅛ | ½ | .. | ¼ | 2⅞ | 1⅛ | 1⅜ | ¾ | 6½ | 9/16 | 2¼ | 3 | 15/16 | ⅜ |
| 1½ | 2⅜ | ½ | .. | ¼ | 2⅞ | 1⅛ | 1⅜ | ¾ | 6½ | 9/16 | 2⅝ | 3⅜ | 1 1/16 | ⅜ |
| 1¾ | 2¾ | ⅝ | .. | ⅜ | 3⅜ | 1⅜ | 1⅝ | ⅞ | 7⅞ | 11/16 | 3 | 4 | 1 ¼ | ⅜ |
| 2 | 3 | ⅝ | .. | ⅜ | 3⅜ | 1⅜ | 1⅝ | ⅞ | 7⅞ | 11/16 | 3½ | 4½ | 1 ⅜ | ½ |
| 2¼ | 3⅜ | ¾ | .. | ½ | 4 | 1⅝ | 2 | 1 | 9½ | 13/16 | 4 | 5¼ | 1 ½ | ½ |
| 2½ | 3⅝ | ¾ | .. | ⅝ | 4 | 1⅝ | 2 | 1 | 9⅞ | 13/16 | 4⅜ | 5⅞ | 1 ¾ | ⅝ |
| 2¾ | 4 | ⅞ | .. | ⅝ | 4⅝ | 1⅞ | 2⅜ | 1⅛ | 11⅛ | 15/16 | 4¾ | 6¼ | 1 ⅞ | ⅝ |
| 3 | 4¼ | ⅞ | .. | ¾ | 4⅝ | 1 7 | 2⅜ | 1⅛ | 11½ | 15/16 | 5¼ | 6½ | 2 | ¾ |
| 3¼ | 4½ | ⅞ | .. | ¾ | 4⅞ | 1⅞ | 2⅝ | 1⅛ | 12 | 15/16 | 5⅝ | 7⅛ | 2 ½ | ¾ |
| 3½ | 4¾ | ⅞ | .. | ⅞ | 4⅞ | 1⅞ | 2⅝ | 1⅛ | 12⅜ | 15/16 | 6⅛ | 7¼ | 2 ⅝ | ⅞ |
| 3¾ | 5⅛ | 1 | .. | ⅞ | 5¼ | 2⅛ | 2¾ | 1¼ | 13½ | 1 1/16 | 6½ | 8 | 2 11/16 | ⅞ |
| 4 | 5⅜ | 1 | .. | ⅞ | 5¼ | 2⅛ | 2¾ | 1¼ | 13½ | 1 1/16 | 7 | 8½ | 2 ⅞ | 1 |

N. B.—$\delta$ = diameter of bolt. C = diameter of bolt holes. G = diameter spot face of bolt holes. All dimensions in inches.

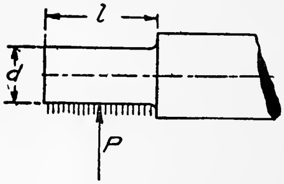
The table is based approximately upon the following proportions:

$$A = D + \delta + \tfrac{3}{8}'', \quad \delta = \frac{D}{5} + (\tfrac{1}{8}'' \text{ to } \tfrac{1}{4}''), \quad K = 1\tfrac{3}{4}\,D.$$

$$M = 2D + \tfrac{3}{8}'', \quad H = 2\tfrac{1}{2}\delta + \tfrac{1}{8}'', \quad G = 2\delta + \tfrac{1}{8}'' \text{ and } I = \delta + \tfrac{1}{4}''.$$

## Journals and Bearings.

FIG. 31.




For bending $\frac{Pl}{2} = \frac{pd^3}{10}$ (1)

$p$ = allowable working fiber stress in bending.

$a$ = unit pressure on projected area of bearing.

$P = a.d.l$ (2)

Equating 1 and 2, $\frac{a.dl^2}{2} = \frac{pd^3}{10} \therefore \frac{l}{d} = \sqrt{\frac{p}{5a}}$

For the limiting values of $a$ see table No. I.

$p$ = for steel 7000 to 9000 pounds per square inch.

There will be no torsional moment of any consequence unless the journal is in a line continued on both sides of it.

1. Journals must be designed to resist any bending and torsional strain that may come upon them.

2. The maximum allowable pressure per square inch of projected area of the journal, *i.e.* $l \times d$, depends upon the materials in sliding contact and should generally have the following limiting values:

TABLE I.

| Materials | Lbs. per sq. in. |
|---|---|
| Hardened tool steel on hard tool steel | 2150 |
| Hardened tool steel on bronze | 1280 |
| Unhardened tool steel on bronze | 860 |
| Soft steel or wrought iron on thick bronze | 560 |
| Cast-iron on bronze | 420 |
| Wrought-iron on cast-iron | 360 |

These pressures may be exceeded in cases where the actions of the pressures reverse or are intermittent. In the case of sheave wheel bearings, according to Hütte these pressures may be doubled or trebled.

3. The length of the journal must be chosen so that the bearing will not heat unduly.

Hütte gives the following formulæ:

$$l \geqq \frac{PN}{w} \quad \text{or} \quad N \leqq \frac{wl}{P}$$

$l$ = length of journal in inches.

P = total load in pounds on the journal.

N = revolutions per minute of the journal.

$w$ = constant depending upon the frictional work and the coefficient of friction.

The heating will depend upon the coefficient of friction of the journal running on the bearing with the particular lubrication existing.

Ring-oiled bearings and those with forced lubrication can be designed with $w$ taken very high, 500,000.

The following values may be assumed for $w$:

Ordinary bearings, air cooled, babbitt, 210,000.
Babbitted bearings, best lubrication, 500,000.
Fly-wheel and crank shaft bearings, bronze, 85,000–170,000.
Fly-wheel and crank shaft bearings, babbitt, 170,000–250,000.

## Step Bearings.

If P, as before, is the total load on the bearing in pounds, then

$$P = 0.785\, d^2.s$$

Here $d$ = diameter of the bearing in inches.
$s$ = allowable bearing pressure as previously given.

For hardened tool steel on bronze $P = 1000d^2$.

In the case of hardened tool steel on bronze as used for draw-bridges and turntables, $s$ can be taken at 3000 pounds, as in these cases the velocity is very low.

Considering the heating of the bearing, we have $d \geqq \frac{PN}{w}$

in which the following values are to be used for $w$:

Ordinary step bearing . . . . . . . . . . . . $w \leqq 220{,}000$

Turbine step bearing . . . . . . . . . . . . $w \leqq 700{,}000$

## Bearings.

The diameter and length of a bearing are found as previously shown; the other dimensions of the bearing will depend more upon what can be conveniently made and that has been found satisfactory in practice than upon any calculations for strength. The following empirical proportions are therefore given. There is no standard practice, the proportions used by each firm deviating slightly from those used by others.

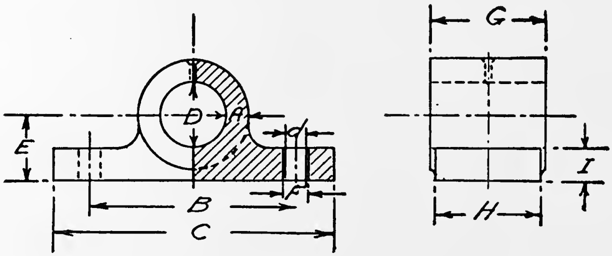



FIG. 32.

| | |
|---|---|
| Unit = D + ½ inch = Δ | E = 0.8 Δ |
| A = 0.28 Δ | F = 0.3 Δ |
| B = 2.5 Δ | G = 1.4 Δ |
| C = 3.4 Δ | H = 1.3 Δ |
| $d$ = 0.25 Δ | I = 0.4 Δ |

$d$ = diameter of bolts. Two required.

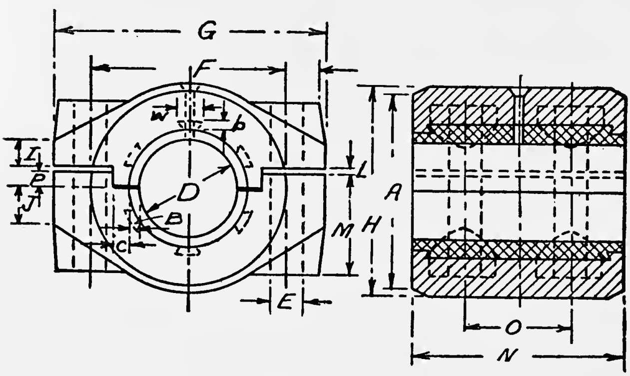



FIG. 33.

### Dimensions of Bearings (Fig. 33).

All dimensions in inches.

| Diam. of Shaft "D" | Thickness of Babbitt | C | Diam. of Bolt "d" | Depth of Babbitt Recess b | A | L |
|---|---|---|---|---|---|---|
| 1½ | ¼ | 3/16 | ⅝ | 3/16 | 3¼ | ⅛ |
| 1¾ | ¼ | ¼ | ⅝ | 3/16 | 3¾ | ⅛ |
| 2 | ¼ | ¼ | ⅝ | 3/16 | 4¼ | ⅛ |
| 2¼ | ¼ | 5/16 | ¾ | 3/16 | 4½ | ⅛ |
| 2½ | 5/16 | 5/16 | ¾ | 3/16 | 5 | ⅛ |
| 2¾ | 5/16 | ⅜ | ⅞ | 3/16 | 5½ | ⅛ |
| 3 | 5/16 | 7/16 | ⅞ | ¼ | 6 | ⅛ |
| 3¼ | 5/16 | ½ | 1 | ¼ | 6½ | 3/16 |
| 3½ | ⅜ | ½ | 1 | ¼ | 6¾ | 3/16 |
| 3¾ | ⅜ | 9/16 | 1⅛ | ¼ | 7¼ | 3/16 |
| 4 | ⅜ | ⅝ | 1⅛ | ¼ | 7¾ | 3/16 |
| 4¼ | 7/16 | ⅝ | 1¼ | ¼ | 8¼ | 3/16 |
| 4½ | 7/16 | 11/16 | 1¼ | ¼ | 8¾ | ¼ |
| 5 | 7/16 | 13/16 | 1¼ | 5/16 | 9¾ | ¼ |
| 5½ | ½ | ⅞ | 1¼ | 5/16 | 10¾ | ¼ |
| 6 | ½ | 1 | 1½ | 5/16 | 11½ | ¼ |
| 6½ | ½ | 1⅛ | 1½ | 5/16 | 12½ | ¼ |
| 7 | ½ | 1¼ | 1½ | 5/16 | 13½ | ¼ |

Core bolt holes $-d+\frac{1}{8}$ inch.

Width of babbitt groove, $w=d-\frac{1}{8}$ inch.

$F=2D$ $\qquad I=\frac{D}{4}$ $\qquad P=\frac{D}{8}+\frac{1}{8}$ inch.

$G=F+2d+\frac{1}{2}$ inch. $\qquad J=\frac{D}{2}-\frac{1}{4}$ inch.

$H=2D+\frac{d}{2}$ $\qquad M=D\times\frac{7}{8}$

### Proportions of Bearing Box (Fig. 34).

| D | d | δ | J | K | L | M | N | F | m | a | e | h | i |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 1½ | ½ | ½ | 1/16 | 3/16 | .. | ¼ | 2¼ | 1/16 | .. | 1 | 3/16 | 4½ | .. |
| 1¾ | ⅝ | ⅝ | 1/16 | 3/16 | .. | ¼ | 2½ | 1/16 | .. | 1⅜ | 3/16 | 5½ | .. |
| 2 | ⅝ | ⅝ | 1/16 | ¼ | .. | 5/16 | 3 | 1/16 | .. | 1⅜ | ¼ | 6 | .. |
| 2¼ | ¾ | ¾ | 1/16 | ¼ | .. | 5/16 | 3¼ | 1/16 | .. | 1½ | ¼ | 7 | .. |
| 2½ | ¾ | ¾ | ⅛ | ¼ | .. | 5/16 | 3¾ | ⅛ | .. | 1½ | ¼ | 7½ | .. |
| 2¾ | ¾ | ⅞ | ⅛ | ¼ | .. | 5/16 | 4 | ⅛ | .. | 1½ | ¼ | 8½ | .. |
| 3 | ¾ | ⅞ | ⅛ | ¼ | ¾ | ⅜ | 4¼ | ⅛ | ¾ | 1½ | ¼ | 9 | ¾ |
| 3¼ | ⅞ | 1 | ⅛ | ¼ | ¾ | ⅜ | 4¾ | ⅛ | ¾ | 1¾ | ⅜ | 10 | ¾ |
| 3½ | ⅞ | 1 | ⅛ | ¼ | ¾ | ⅜ | 5 | ⅛ | ¾ | 1¾ | ⅜ | 10½ | ⅞ |
| 3¾ | 1 | 1⅛ | ⅛ | ¼ | ¾ | ⅜ | 5¼ | 3/16 | ¾ | 1⅞ | ⅜ | 11 | ⅞ |
| 4 | 1 | 1⅛ | 3/16 | 5/16 | 1 | 7/16 | 5¾ | 3/16 | 1 | 1⅞ | ⅜ | 11½ | 1 |
| 4¼ | 1⅛ | 1¼ | 3/16 | 5/16 | 1 | 7/16 | 6 | 3/16 | 1 | 2⅛ | ⅜ | 12 | 1 |
| 4½ | 1⅛ | 1¼ | 3/16 | 5/16 | 1 | 7/16 | 6½ | 3/16 | 1 | 2⅛ | ⅜ | 12 | 1⅛ |
| 5 | 1¼ | 1½ | 3/16 | ⅜ | 1¼ | ½ | 7 | 3/16 | 1¼ | 2¼ | ⅜ | 12½ | 1¼ |
| 5½ | 1¼ | 1½ | 3/16 | ⅜ | 1¼ | ½ | 7¾ | ¼ | 1¼ | 2¼ | ½ | 13 | 1⅜ |
| 6 | 1¼ | 1½ | ¼ | 7/16 | 1½ | 9/16 | 8½ | ¼ | 1½ | 2¼ | ½ | 13½ | 1½ |
| 6½ | 1½ | 1¾ | ¼ | 7/16 | 1½ | 9/16 | 9 | ¼ | 1½ | 2⅝ | ½ | 14 | 1½ |
| 7 | 1½ | 1¾ | ¼ | 7/16 | 1¾ | 9/16 | 9¾ | ¼ | 1¾ | 2⅝ | ½ | 15 | 1⅝ |

The proportions for the dimensions not given in the table are noted below.

$d$ = diameter of the bolts securing cap on bearing.

$\delta$ = diameter of the bolts in the base.

$A = 1.5\delta$

$B = \frac{A}{5}$

$C = \frac{5}{4}D$

$E = \frac{D}{4}$

$G = 1.5\ F$

$H = 1.5d$

$I = B$

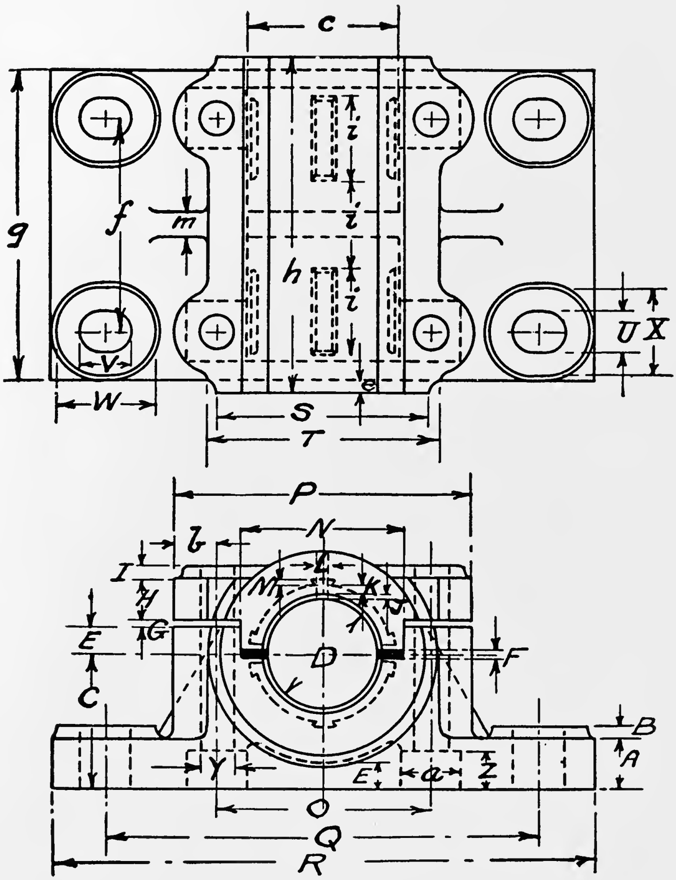



Fig. 34.

$O = N + 1.75d$

$P = O + 2.75d$

$Q = P + 4\delta$

$R = Q + 3.5\delta$

$S = 1.8D$

$T = S + 2e$

$U = \begin{cases} \delta + \frac{1}{16}'' \text{ for } 1\frac{1}{2}'',\ 1\frac{3}{4} \text{ and } 2'' \text{ diameter.} \\ \delta + \frac{1}{8}'' \text{ for the others.} \end{cases}$

$V = 1.5\delta$

$W = U + V + X$

$X = 1\frac{1}{2}\delta$

$Y = d + \frac{1}{16}$ inch.

$Z = \frac{5}{4}\delta$

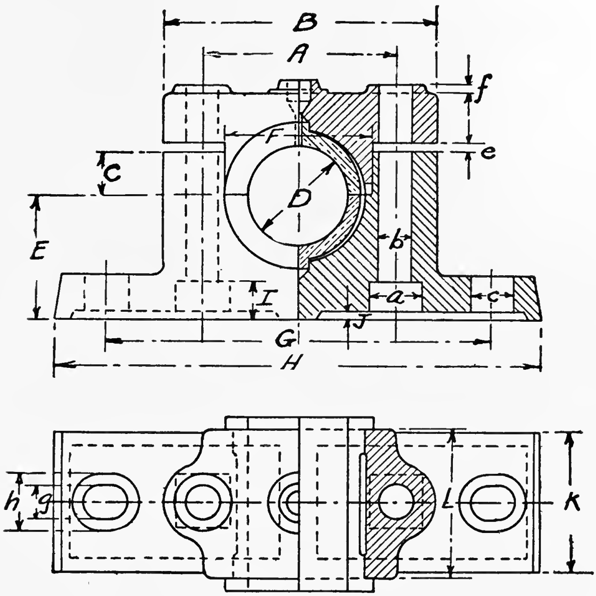
$b = 1\frac{1}{8}d$

$c = O - a$

$f = g - 3.5\delta$

$g = h - 2e$

Fig. 35.



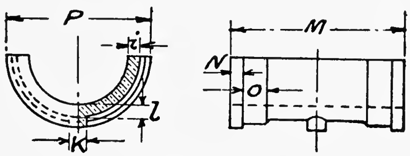




Fig. 36.

## Bearings.

Fig. 35—Unit $\Delta = D + \frac{1}{2}$ inch.

$A = 1.7\Delta$ $B = A + 2.75d$ $C = 0.38\Delta$
$E = 1.07\Delta$ $F = 1.25\Delta$ $G = 3.3\Delta$
$H = G + 3.5\delta$ $I = \frac{5}{4}\delta$ $J = 0.07\Delta$
$K = 1.21\Delta$ $L = 1.11\Delta$
$a = 1\frac{1}{2}d + \frac{3}{8}$ inch $b = 0.28\Delta$ $c = 1.5\delta$
$d = \delta = 0.25\Delta$ $e = \frac{D}{16}$ $f = \frac{d}{8}$
$g = 0.28\delta$ $h = 1.5\delta$
$\delta$ = diameter of bolts in cap.
$d$ = diameter of bolts in base.

Fig. 36.—Brasses—Unit for brasses $\propto = 0.09\,D + 0.15$ inch.
$i = 0.75 \propto$ $N = l = \propto$
$K = 1.3 \propto$ $O = 1.8 \propto$

The length of the brasses will vary with the bearing, but is here taken as $M = 1.5\,D$; $P = D + 3.6 \propto$

## Brackets.

Some idea of a bracket can be had from Fig. 37—Unit $\Delta = D + \frac{1}{2}$ inch.

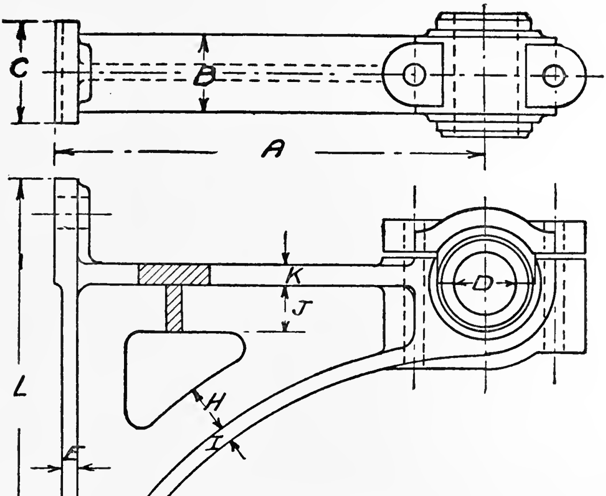
$A$ = about $5.5\Delta$ $H = 0.6\Delta$
$B = 0.9\Delta$ $I = 0.2\Delta$
$C = 1.3\Delta$ $J = 0.6\Delta$
$E = 0.2\Delta$ $K = 0.25\Delta$
$F = 0.3\Delta$ $L = 6.5\Delta$
$G = \frac{F}{3}$

## Step Bearings.

Fig. 38.—Unit D
$A = 1.2\,D$ $G = 0.25\,D$
$B = 0.4\,D$ $H = I = K = 1\frac{3}{4}\delta$
$C = 0.35\,D$ $L = 0.35\,D$
$E = 0.3\,D$ $M = 0.3\,D$
$F = D$

No. bolts = 4 and diameter of bolts $= \delta = \frac{D}{4}$

Fig. 37.



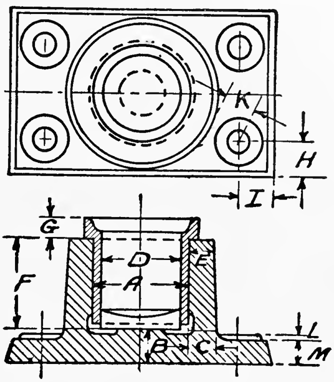




Fig. 38.

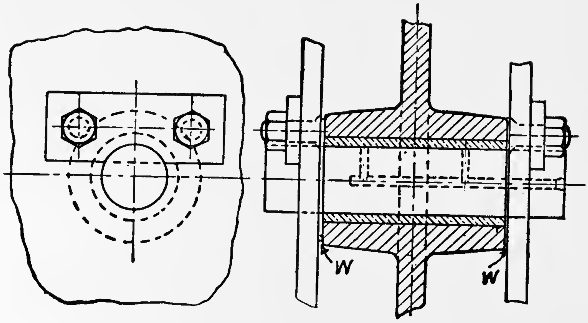



FIG. 39.

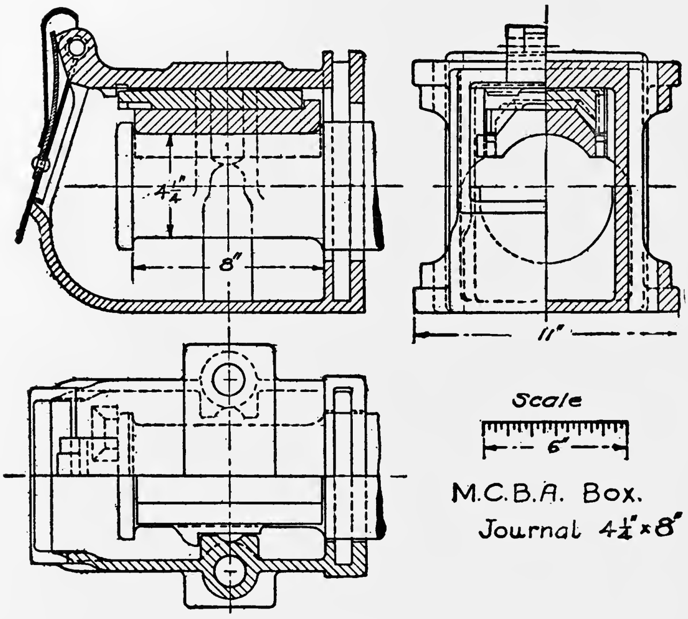



FIG. 40.

The bearing shown in Fig. 39 is a very common type in crane practice. The pin is held by a key or keys as shown and the wheel, in whose hub is secured a brass bushing, runs on the pin. Brass

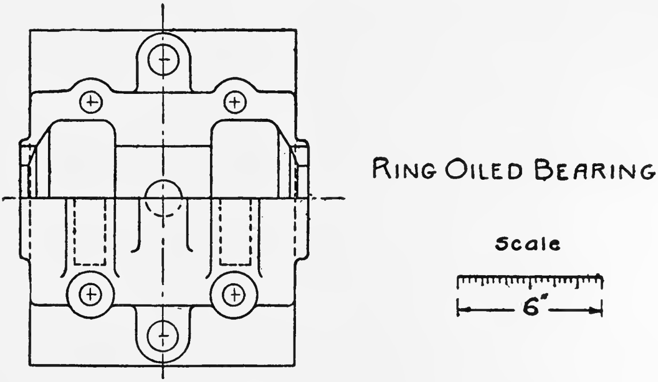
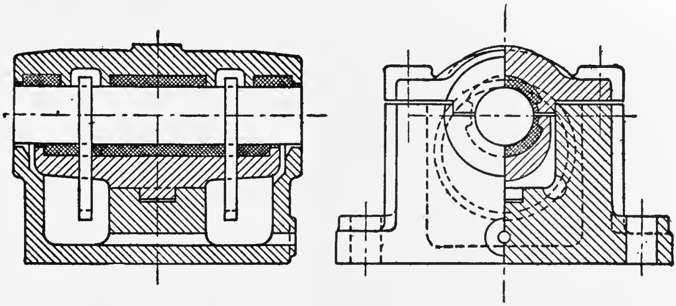



Fig. 41.



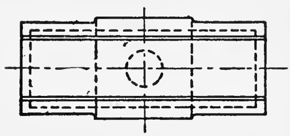


Fig. 42.

washers W--W at the ends of the hub take the wear. The bearing is oiled through a hole in the shaft.

An oil-cup is fastened to the shaft by a short length of pipe placing the cup above the top of the shaft.

The thickness of the bushing can be made

$$t = 0.08D + 0.10 \text{ inch.}$$

Ring-oiled bearing—Figures 41 and 42 illustrate a ring-oiled bearing suitable for high speeds where continuous lubrication is desired.

## Dimensions of Bolts and Nuts.

Franklin Institute Standard.

| Bolts and Threads | | | | | | Rough Nuts and Heads | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| Diameter of Bolt | Threads per Inch | Diameter at Root of Thread | Width of Flat | Area of Bolt Body | Area of Bolt at Root of Thread | Short Diameter of Square and Hexagon | Long Diameter of Square | Long Diameter of Hexagon | Thickness of Nuts | Thickness of Heads |
| *Ins.* | *No.* | *Ins.* | *Ins.* | *Sq. Ins.* | *Sq. Ins.* | *Sq. Ins.* | *Ins.* | *Ins.* | *Ins.* | *Ins.* |
| 1/4 | 20 | .185 | .0062 | .049 | .027 | 1/2 | .707 | .577 | 1/4 | 1/4 |
| 5/16 | 18 | .240 | .0070 | .077 | .045 | 19/32 | .840 | .686 | 5/16 | 19/64 |
| 3/8 | 16 | .294 | .0078 | .110 | .068 | 11/16 | .972 | .794 | 3/8 | 11/32 |
| 7/16 | 14 | .344 | .0089 | .150 | .093 | 25/32 | 1.105 | .902 | 7/16 | 25/64 |
| 1/2 | 13 | .400 | .0096 | .196 | .126 | 7/8 | 1.238 | 1.010 | 1/2 | 7/16 |
| 9/16 | 12 | .454 | .0104 | .249 | .162 | 31/32 | 1.370 | 1.119 | 9/16 | 31/64 |
| 5/8 | 11 | .507 | .0113 | .307 | .202 | 1 1/16 | 1.503 | 1.227 | 5/8 | 17/32 |
| 3/4 | 10 | .620 | .0125 | .442 | .302 | 1 1/4 | 1.768 | 1.443 | 3/4 | 5/8 |
| 7/8 | 9 | .731 | .0140 | .601 | .420 | 1 7/16 | 2.033 | 1.660 | 7/8 | 23/32 |
| 1 | 8 | .837 | .0156 | .785 | .550 | 1 5/8 | 2.298 | 1.876 | 1 | 13/16 |
| 1 1/8 | 7 | .940 | .0180 | .994 | .694 | 1 13/16 | 2.563 | 2.093 | 1 1/8 | 29/32 |
| 1 1/4 | 7 | 1.065 | .0180 | 1.227 | .893 | 2 | 2.829 | 2.309 | 1 1/4 | 1 |
| 1 3/8 | 6 | 1.160 | .0210 | 1.485 | 1.057 | 2 3/16 | 3.094 | 2.526 | 1 3/8 | 1 3/32 |
| 1 1/2 | 6 | 1.284 | .0210 | 1.767 | 1.295 | 2 3/8 | 3.359 | 2.742 | 1 1/2 | 1 3/16 |
| 1 5/8 | 5 1/2 | 1.389 | .0227 | 2.074 | 1.515 | 2 9/16 | 3.624 | 2.959 | 1 5/8 | 1 9/32 |
| 1 3/4 | 5 | 1.490 | .0250 | 2.405 | 1.744 | 2 3/4 | 3.889 | 3.175 | 1 3/4 | 1 3/8 |
| 1 7/8 | 5 | 1.615 | .0250 | 2.761 | 2.048 | 2 15/16 | 4.154 | 3.392 | 1 7/8 | 1 15/32 |
| 2 | 4 1/2 | 1.712 | .0280 | 3.142 | 2.302 | 3 1/8 | 4.420 | 3.608 | 2 | 1 9/16 |
| 2 1/4 | 4 1/2 | 1.962 | .0280 | 3.976 | 3.023 | 3 1/2 | 4.950 | 4.042 | 2 1/4 | 1 3/4 |
| 2 1/2 | 4 | 2.175 | .0310 | 4.909 | 3.715 | 3 7/8 | 5.480 | 4.475 | 2 1/2 | 1 15/16 |
| 2 3/4 | 4 | 2.425 | .0310 | 5.940 | 4.619 | 4 1/4 | 6.011 | 4.908 | 2 3/4 | 2 1/8 |
| 3 | 3 1/2 | 2.629 | .0357 | 7.069 | 5.428 | 4 5/8 | 6.541 | 5.341 | 3 | 2 5/16 |
| 3 1/4 | 3 1/2 | 2.879 | .0357 | 8.296 | 6.510 | 5 | 7.071 | 5.774 | 3 1/4 | 2 1/2 |
| 3 1/2 | 3 1/4 | 3.100 | .0384 | 9.621 | 7.548 | 5 3/8 | 7.602 | 6.207 | 3 1/2 | 2 11/16 |
| 3 3/4 | 3 | 3.317 | .0410 | 11.045 | 8.641 | 5 3/4 | 8.132 | 6.640 | 3 3/4 | 2 7/8 |
| 4 | 3 | 3.567 | .0410 | 12.566 | 9.993 | 6 1/8 | 8.662 | 7.073 | 4 | 3 1/16 |
| 4 1/4 | 2 7/8 | 3.798 | .0435 | 14.186 | 11.329 | 6 1/2 | 9.193 | 7.506 | 4 1/4 | 3 1/4 |
| 4 1/2 | 2 3/4 | 4.028 | .0460 | 15.904 | 12.743 | 6 7/8 | 9.723 | 7.939 | 4 1/2 | 3 7/16 |
| 4 3/4 | 2 5/8 | 4.255 | .0480 | 17.721 | 14.220 | 7 1/4 | 10.253 | 8.372 | 4 3/4 | 3 5/8 |
| 5 | 2 1/2 | 4.480 | .0500 | 19.635 | 15.763 | 7 5/8 | 10.784 | 8.805 | 5 | 3 13/16 |
| 5 1/4 | 2 1/2 | 4.730 | .0500 | 21.648 | 17.572 | 8 | 11.314 | 9.238 | 5 1/4 | 4 |
| 5 1/2 | 2 3/8 | 4.953 | .0526 | 23.758 | 19.267 | 8 3/8 | 11.844 | 9.671 | 5 1/2 | 4 3/16 |
| 5 3/4 | 2 3/8 | 5.203 | .0526 | 25.967 | 21.262 | 8 3/4 | 12.375 | 10.104 | 5 3/4 | 4 3/8 |
| 6 | 2 1/4 | 5.423 | .0555 | 28.274 | 23.098 | 9 1/8 | 12.905 | 10.537 | 6 | 4 9/16 |

## Wire Rope for Cranes and Hoists.

As usually constructed for this purpose the ropes are made as follows:

| | Strands | No. Wires per strand | No. Wires in Rope | $\frac{d}{\delta}$ |
|---|---|---|---|---|
| A | 6 | 19 | 114 | 15 |
| B | 6 | 37 | 222 | 21 |
| C | 8 | 19 | 152 | 18 |

$d$ = diameter of rope, in inches. $\delta$ = diameter of wire, in inches.

The ultimate strengths will vary with the material from which the wires are drawn.

| Material | Pounds per sq. inch. Ultimate Strength. |
|---|---|
| Swedish Iron | 80,000 |
| Crucible Steel | 180,000 |
| Plow Steel | 220,000 |

If a wire is drawn around a circumference whose diameter is D the extreme fiber stress produced by this bending is

$$p_b = \frac{\delta \times E}{D}$$

where $\delta$ = diameter of the wire in inches.
E = modulus of elasticity.
D = diameter of the drum or sheave in inches.

In a wire rope as actually made the stress will be $p_b = \propto \frac{\delta \times E}{D}$ where according to Bach $\propto$ = approximates $\frac{3}{8}$. This is due to the rope construction, the individual wires being wound in spirals to form the strands, and these strands in turn wound as spirals into the rope. The strands are usually wound around a flexible hemp core. The wires are thus allowed to move somewhat upon each other.

In the case of a rope wound around a drum and carrying a load the maximum fiber stress will be the sum of the extreme stress due to bending and the direct stress due to the load.

Total fiber stress $= p_t = p_b + p_t$

$$p_T = \left( \frac{S}{i \times \frac{\pi\,\delta^2}{4}} \right) + \left( \frac{3}{8} \times \frac{E \times \delta}{D} \right)$$

Here S = load on rope in pounds.
$i$ = total number of wires in rope.
$\delta$ = diameter of a single wire in inches.

When wire ropes are to be used in places sufficiently hot to char the hemp core, this is replaced by a soft iron core. The iron core stiffens the rope materially, necessitating larger drums and sheaves.

When both the direct and the bending stresses are carefully considered the allowable working stress may be the ultimate strength divided by from 3.5 to 4.0.

## Standard Hoisting Rope.

Standard Strengths, adopted May 1, 1910. Six strands—nineteen wires to the strand—one hemp core.

| Swedes Iron | | | | | | Crucible Cast Steel | | | Extra Strong Crucible Cast Steel | | | Plow Steel | | | Monitor Plow Steel | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Diameter in inches | Circumference in inches | Approx. Weight per foot | Approx. Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in ft. advised | Approx. Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in ft. advised. | Approx. Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in ft. advised | Approx. Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in ft. advised. | Approx. Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in ft. advised |
| 2¾ | 8⅝ | 11.95 | 111 | 22.2 | 17 | 211 | 42.2 | 11 | 243 | 48.6 | 11 | 275 | 55 | 11 | 315 | 63 | 11 |
| 2½ | 7⅞ | 9.85 | 92 | 18.4 | 15 | 170 | 34 | 10 | 200 | 40 | 10 | 229 | 46 | 10 | 263 | 53 | 10 |
| 2¼ | 7⅛ | 8. | 72 | 14.4 | 14 | 133 | 26.6 | 9 | 160 | 32 | 9 | 186 | 37 | 9 | 210 | 42 | 9 |
| 2 | 6¼ | 6.30 | 55 | 11 | 12 | 106 | 21.2 | 8 | 123 | 24.6 | 8 | 140 | 28 | 8 | 166 | 33 | 8 |
| 1⅞ | 5¾ | 5.55 | 50 | 10 | 12 | 96 | 19 | 8 | 112 | 22 | 8 | 127 | 25 | 8 | 150 | 30 | 8 |
| 1¾ | 5½ | 4.85 | 44 | 8.8 | 11 | 85 | 17 | 7 | 99 | 19.8 | 7 | 112 | 22 | 7 | 133 | 27 | 7 |
| 1⅝ | 5 | 4.15 | 38 | 7.5 | 10 | 72 | 14.4 | 6.5 | 83 | 16.6 | 6.5 | 94 | 19 | 6.5 | 110 | 22 | 6½ |
| 1½ | 4¾ | 3.55 | 33 | 6.5 | 9 | 64 | 12.8 | 6 | 73 | 14.6 | 6 | 82 | 16 | 6 | 98 | 20 | 6 |
| 1⅜ | 4¼ | 3. | 28 | 5.5 | 8.5 | 56 | 11.6 | 5.5 | 64 | 13 | 5.5 | 72 | 14 | 5.5 | 84 | 17 | 5½ |
| 1¼ | 4 | 2.45 | 22.8 | 4.56 | 7.5 | 47 | 9.4 | 5 | 53 | 10.6 | 5 | 58 | 12 | 5 | 69 | 14 | 5 |
| 1⅛ | 3½ | 2. | 18.6 | 3.72 | 7 | 38 | 7.6 | 4.5 | 43 | 8.6 | 4.5 | 47 | 9.5 | 4.5 | 56 | 11 | 4½ |
| 1 | 3 | 1.58 | 14.5 | 2.90 | 6 | 30 | 6 | 4 | 34 | 6.80 | 4 | 38 | 7.6 | 4 | 45 | 9 | 4 |
| ⅞ | 2¾ | 1.20 | 11.8 | 2.36 | 5.5 | 23 | 4.6 | 3.5 | 26 | 5.20 | 3.5 | 29 | 5.8 | 3.5 | 35 | 7 | 3½ |
| ¾ | 2¼ | .89 | 8.5 | 1.70 | 4.5 | 17.5 | 3.5 | 3 | 20.2 | 4.04 | 3 | 23 | 4.6 | 3 | 26.3 | 5.3 | 3 |
| ⅝ | 2 | .62 | 6. | 1.20 | 4 | 12.5 | 2.5 | 2.5 | 14 | 2.80 | 2.5 | 15.5 | 3.1 | 2.5 | 19 | 3.8 | 2½ |
| 9/16 | 1¾ | .50 | 4.7 | .94 | 3.5 | 10 | 2 | 2.25 | 11.2 | 2.24 | 2.25 | 12.3 | 2.4 | 2.25 | 14.5 | 2.9 | 2¼ |
| ½ | 1½ | .39 | 3.9 | .78 | 3 | 8.4 | 1.68 | 2 | 9.2 | 1.84 | 2 | 10 | 2 | 2 | 12.1 | 2.4 | 2 |
| 7/16 | 1¼ | .30 | 2.9 | .58 | 2.75 | 6.5 | 1.30 | 1.75 | 7.25 | 1.45 | 1.75 | 8 | 1.6 | 1.75 | 9.4 | 1.9 | 1¾ |
| ⅜ | 1⅛ | .22 | 2.4 | .48 | 2.25 | 4.8 | .96 | 1.50 | 5.30 | 1.06 | 1.50 | 5.75 | 1.15 | 1.50 | 6.75 | 1.35 | 1½ |
| 5/16 | 1 | .15 | 1.5 | .30 | 2 | 3.1 | .62 | 1.25 | 3.50 | .70 | 1.25 | 3.8 | .76 | 1.25 | 4.50 | .9 | 1¼ |
| ¼ | ¾ | .10 | 1.1 | .22 | 1.50 | 2.2 | .44 | 1 | 2.43 | .49 | 1 | 2.65 | .53 | 1 | 3.15 | .63 | 1 |

## Special Flexible Steel Hoisting Rope.

Made of 6 strands, 37 wires to the strand, hemp center. This rope is designed for cranes, dredges, and other uses, where the full measure of strength, combined with extreme flexibility, is essential. The wires are of a necessity finer than those in a 19-wire strand, and should not be subject to much abrasive wear.

| Plow Steel. | | | | | Monitor Plow Steel. | | |
|---|---|---|---|---|---|---|---|
| Diameter in inches | Circumference in inches | Approximate Weight per foot | Approximate Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Approximate Strength in tons of 2000 lbs. | Proper working load in tons of 2000 lbs. | Diameter of drum or sheave in feet advised |
| 2¾ | 8⅝ | 11.95 | 265 | 53 | 278 | 55 | .... |
| 2½ | 7⅞ | 9.85 | 214 | 43 | 225 | 45 | .... |
| 2¼ | 7⅛ | 8 | 175 | 35 | 184 | 37 | .... |
| 2 | 6¼ | 6.30 | 130 | 26 | 137 | 27 | .... |
| 1¾ | 5½ | 4.85 | 108 | 22 | 113 | 23 | .... |
| 1⅝ | 5 | 4.15 | 90 | 18 | 95 | 19 | .... |
| 1½ | 4¾ | 3.55 | 80 | 16 | 84 | 17 | 3.75 |
| 1⅜ | 4¼ | 3 | 68 | 14 | 71 | 14 | 3.50 |
| 1¼ | 4 | 2.45 | 55 | 11 | 58 | 11 | 3.20 |
| 1⅛ | 3½ | 2 | 44 | 9 | 46 | 9.2 | 2.83 |
| 1 | 3 | 1.58 | 35 | 7 | 37 | 7.4 | 2.50 |
| ⅞ | 2¾ | 1.20 | 27 | 5 | 29 | 5.8 | 2.16 |
| ¾ | 2¼ | .89 | 21 | 4 | 23 | 4.6 | 1.83 |
| ⅝ | 2 | .62 | 14 | 3 | 16 | 3.2 | 1.75 |
| 9/16 | 1¾ | .50 | 11.5 | 2.3 | 12½ | 2.5 | 1.50 |
| ½ | 1½ | .39 | 9.25 | 1.85 | 9.75 | 1.9 | 1.33 |
| 7/16 | 1¼ | .30 | 7.2 | 1.4 | 7.50 | 1.5 | 1.15 |
| ⅜ | 1⅛ | .22 | 5.1 | 1 | 5.30 | 1.06 | 1 |

### Extra Flexible Plow Steel Hoisting Rope

Nineteen Wires to the Strand, Eight Strands—One Hemp Center

| Diameter in Inches | Approximate Circumference in Inches | Weight per Foot in Pounds | Approximate Breaking Stress in Tons of 2000 Pounds | Proper Working Load in Tons of 2000 Pounds | Minimum Diameter of Drum or Sheave in Feet |
|---|---|---|---|---|---|
| 1½ | 4¾ | 3.48 | 86 | 17.2 | 3.5 |
| 1⅜ | 4¼ | 2.51 | 74 | 14.8 | 3.0 |
| 1¼ | 4 | 2.13 | 60 | 12.0 | 2.75 |
| 1⅛ | 3½ | 1.82 | 50 | 10.0 | 2.50 |
| 1 | 3 | 1.32 | 35 | 7.0 | 2.25 |
| ⅞ | 2¾ | 1.05 | 27 | 5.4 | 2.00 |
| ¾ | 2¼ | .89 | 21 | 4.2 | 1.75 |
| ⅝ | 2 | .53 | 15 | 3.0 | 1.63 |
| 9/16 | 1¾ | .43 | 12.3 | 2.46 | 1.38 |
| ½ | 1½ | .31 | 8.55 | 1.71 | 1.25 |
| 7/16 | 1¼ | .27 | 7.95 | 1.59 | 1.13 |
| ⅜ | 1⅛ | .18 | 4.25 | .85 | 1.00 |
| 5/16 | 1 | .12 | 2.92 | .58 | .88 |
| ¼ | ¾ | .066 | 1.95 | .39 | .75 |

### General Uses.

Swedes Iron, for hoisting and counterweight service on elevators. Frequently used for transmission of power.

Crucible cast steel, for coal hoists, ore hoists, derricks, elevator hoisting, ballast unloaders.

Extra strong crucible steel, for the same purposes as Crucible Cast Steel Rope.

Plow Steel and Monitor Grade, for heavy dredging, logging, stump pulling, derricks, coal and ore hoisting service.

Figures 43 to 48 illustrate the usual fittings for wire rope. Fig. 43.—Rope sheave. Figs. 44 and 45.—Clamps. Fig. 47.—Thimble. Fig. 48.—Blocks.

**Problems.**—A ¾ inch plow steel rope, 6 strands, 19 wires each is wound around a drum 22 inches in diameter, and loaded with 10,000 pounds. Assume its ultimate strength at 210,000 pounds per square inch, what is the probable factor of safety when both direct stress and bending stresses are considered?

**Solution.**—Number of wires in rope $6 \times 19 = 114$.

Diameter of each rope $= \frac{.75}{15} = 0.05$ inch.

Area of steel in cross-section of rope $= \frac{114 \times 3.14 \times .05^2}{4} = 0.224$ sq. in.

Direct tensile stress $\frac{10,000}{0.224} = 44.700$ pounds.

Bending stresses $= \frac{3}{8} \times \frac{E \times \delta}{D} = \frac{3}{8} \times \frac{30,000,000 \times .05}{22} = 25,600$ pounds.

Combined stress $44,700 + 25,600 = 70,300$ pounds.

Factor $\frac{210,000}{70,000} = 3$

Fig. 43.

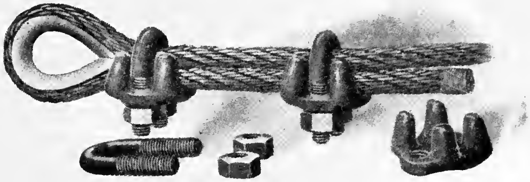


Fig. 44.

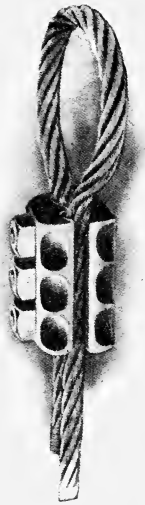


Fig. 45.

Fig. 46.

Fig. 47.

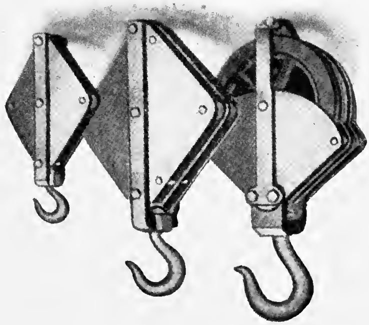


Fig. 48.

## Chains.

The average strength of chains as given by manufacturers can be expressed by the following formula:

$$L \text{ (ultimate)} = 60{,}000 \times d^2$$

This corresponds to an ultimate strength per square inch of the chain material of about 38,000 pounds, the ultimate strength of the original material being approximately 50,000 pounds per square inch.

The ordinary loads recommended by manufacturers for these chains correspond to ⅓ of these ultimate loads

$$L \text{ (working load)} = 20{,}000 \times d^2$$

*d* being the diameter of the stock in inches.
L being the load in pounds.

Although this loading is sometimes used in crane work, it seems unduly high. The chain does not usually conform to the drum or sheaves and as a consequence bending stresses result in the links, thus increasing the stress. Bach recommends the following values:

$$L \text{ (working load)} = 14{,}000 \times d^2 \text{ (infrequent loads).}$$

$$L \text{ (working load)} = 11{,}200 \times d^2 \text{ (general work).}$$

Bulletin No. 18, "The Strength of Chain Links," issued by the Engineering Experiment Station of the University of Illinois, recommends the following formulæ:

$P = 0.4\ d^2S$ (Open links).
$P = 0.5\ d^2S$ (Stud links).
d = stock diameter in inches.
P = safe load in pounds.
S = maximum allowed tensile stress, pounds per square inch.

**Problem.**—A load of 40,000 pounds is to be carried by 4 chains; what size chain should be used?

**Solution.**—Using Bach's formula we have

$$L = \frac{40000}{4} = 11200 \times d^2$$

$$d = \sqrt{\frac{10000}{11200}} = 0.943 \text{ inch, say } 15/16 \text{ inch.}$$

## CHAINS

| Size of Chain | Dist. from cen. of one link to cen. of next | Weight per foot in lbs. Approximately | Outside Width | D. B. G. Special Crane | | | Crane | | |
|---|---|---|---|---|---|---|---|---|---|
| | | | | Proof Test Lbs. | Average Breaking Strain Lbs. | Ordinary safe load General Use Lbs. | Proof Test Lbs. | Average Breaking Strain Lbs. | Ordinary safe load General Use. Lbs. |
| 1/4 | 25/32 | 3/4 | 15/16 | 1,932 | 3,864 | 1,288 | 1,680 | 3,360 | 1,120 |
| 5/16 | 27/32 | 1 | 1 1/8 | 2,898 | 5,796 | 1,932 | 2,520 | 5,040 | 1,680 |
| 3/8 | 31/32 | 1 1/2 | 1 5/16 | 4,186 | 8,372 | 2,790 | 3,640 | 7,280 | 2,427 |
| 7/16 | 1 5/32 | 2 | 1 1/2 | 5,796 | 11,592 | 3,864 | 5,040 | 10,080 | 3,360 |
| 1/2 | 1 11/32 | 2 1/2 | 1 13/16 | 7,728 | 15,456 | 5,152 | 6,720 | 13,440 | 4,480 |
| 9/16 | 1 15/32 | 3 3/10 | 2 | 9,660 | 19,320 | 6,440 | 8,400 | 16,800 | 5,600 |
| 5/8 | 1 23/32 | 4 1/10 | 2 3/16 | 11,914 | 23,828 | 7,942 | 10,360 | 20,720 | 6,907 |
| 11/16 | 1 13/16 | 5 | 2 3/8 | 14,490 | 28,980 | 9,660 | 12,600 | 25,200 | 8,400 |
| 3/4 | 1 15/16 | 6 2/10 | 2 9/16 | 17,388 | 34,776 | 11,592 | 15,120 | 30,240 | 10,080 |
| 13/16 | 2 1/16 | 6 7/10 | 2 3/4 | 20,286 | 40,572 | 13,524 | 17,640 | 35,280 | 11,760 |
| 7/8 | 2 3/16 | 8 3/8 | 2 15/16 | 22,484 | 44,968 | 14,989 | 20,440 | 40,880 | 13,627 |
| 15/16 | 2 7/16 | 9 | 3 3/16 | 25,872 | 51,744 | 17,248 | 23,520 | 47,040 | 15,680 |
| 1 | 2 1/2 | 10 1/2 | 3 3/8 | 29,568 | 59,136 | 19,712 | 26,880 | 53,760 | 17,920 |
| 1 1/16 | 2 5/8 | 12 | 3 9/16 | 33,264 | 66,538 | 22,176 | 30,240 | 60,480 | 20,160 |
| 1 1/8 | 2 3/4 | 13 5/8 | 3 13/16 | 37,576 | 75,152 | 25,050 | 34,160 | 68,320 | 22,773 |
| 1 3/16 | 3 1/16 | 13 7/10 | 4 | 41,888 | 83,776 | 27,925 | 38,080 | 76,160 | 25,387 |
| 1 1/4 | 3 1/8 | 16 | 4 3/16 | 46,200 | 92,400 | 30,800 | 42,000 | 84,000 | 28,000 |
| 1 5/16 | 3 3/8 | 16 1/2 | 4 3/8 | 50,512 | 101,024 | 33,674 | 45,920 | 91,840 | 30,613 |
| 1 3/8 | 3 9/16 | 19 1/4 | 4 9/16 | 55,748 | 111,496 | 37,165 | 50,680 | 101,360 | 33,787 |
| 1 7/16 | 3 11/16 | 19 7/10 | 4 3/4 | 60,368 | 120,736 | 40,245 | 54,880 | 109,760 | 36,587 |
| 1 1/2 | 3 7/8 | 23 | 5 1/8 | 66,528 | 133,056 | 44,352 | 60,480 | 120,960 | 40,320 |
| 1 9/16 | 4 | 25 | 5 5/16 | 70,762 | 141,524 | 47,174 | 65,520 | 131,140 | 43,180 |
| 1 5/8 | 4 1/4 | 28 | 5 1/2 | 74,382 | 148,764 | 49,588 | .... | .... | .... |
| 1 11/16 | 4 1/2 | 30 | 5 11/16 | 78,733 | 157,466 | 52,488 | .... | .... | .... |
| 1 3/4 | 4 3/4 | 31 | 5 7/8 | 82,320 | 164,640 | 54,880 | .... | .... | .... |
| 1 13/16 | 5 | 33 | 6 1/16 | 88,256 | 176,512 | 55,504 | .... | .... | .... |
| 1 7/8 | 5 1/4 | 35 | 6 3/8 | 94,360 | 188,720 | 62,906 | .... | .... | .... |
| 1 15/16 | 5 1/2 | 38 | 6 9/16 | 100,800 | 201,600 | 67,200 | .... | .... | .... |
| 2 | 5 3/4 | 40 | 6 3/4 | 107,520 | 215,040 | 71,680 | .... | .... | .... |
| 2 1/16 | 6 | 43 | 6 15/16 | 114,240 | 228,480 | 76,160 | .... | .... | .... |
| 2 1/8 | 6 1/4 | 46 1/2 | 7 1/8 | 121,240 | 242,480 | 80,823 | .... | .... | .... |
| 2 3/16 | 6 1/2 | 49 1/2 | 7 5/16 | 128,576 | 257,152 | 85,750 | .... | .... | .... |
| 2 1/4 | 6 3/4 | 52 3/4 | 7 5/8 | 136,080 | 272,160 | 90,720 | .... | .... | .... |
| 2 3/8 | 6 7/8 | 58 1/4 | 8 | 151,580 | 303,160 | 101,053 | .... | .... | .... |
| 2 1/2 | 7 | 64 1/2 | 8 3/8 | 168,000 | 336,000 | 112,000 | .... | .... | .... |
| 2 5/8 | 7 1/8 | 70 | 8 3/4 | 180,544 | 361,088 | 120,362 | .... | .... | .... |
| 2 3/4 | 7 1/4 | 73 | 9 1/8 | 193,088 | 386,176 | 128,725 | .... | .... | .... |
| 2 7/8 | 7 1/2 | 76 | 9 1/2 | 205,408 | 410,816 | 136,938 | .... | .... | .... |
| 3 | 7 3/4 | 83 | 9 7/8 | 217,728 | 435,456 | 145,152 | .... | .... | .... |

NOTE.—The pitch is approximate. For sprocket wheels the pitch should be exact, *i.e.*, the chain calibrated. Manufacturers recommend enlarging the pitch of chains for sprocket wheel service by 1/16 inch to and including 5/8 inch chain and by 1/8 inch for larger chain.

DIMENSIONS OF CHAIN LINKS.

| Size of Chain | D. B. G. & Crane | | Stud. | | Coil | |
|---|---|---|---|---|---|---|
| | Outside Length | Outside Width | Outside Length | Outside Width | Outside Length | Outside Width |
| | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* | *Inches.* |
| 3/16 | 15/16 | 7/8 | ... | .. | .. | .. |
| 1/4 | 1 5/16 | 15/16 | ... | .. | .. | .. |
| 5/16 | 1 1/2 | 1 1/8 | ... | .. | .. | .. |
| 3/8 | 1 3/4 | 1 5/16 | ... | .. | .. | .. |
| 7/16 | 2 1/16 | 1 1/2 | ... | .. | .. | .. |
| 1/2 | 2 3/8 | 1 13/16 | ... | .. | 2 3/8 | 1 3/4 |
| 9/16 | 2 5/8 | 2 | ... | .. | 2 3/4 | 1 7/8 |
| 5/8 | 3 | 2 3/16 | ... | .. | 3 | 2 1/8 |
| 11/16 | 3 1/4 | 2 3/8 | ... | .. | 3 1/4 | 2 3/8 |
| 3/4 | 3 1/2 | 2 9/16 | 4 3/8 | 2 3/4 | 3 1/2 | 2 1/2 |
| 13/16 | 3 3/4 | 2 3/4 | 4 3/4 | 3 | 3 7/8 | 2 3/4 |
| 7/8 | 4 | 2 15/16 | 5 | 3 1/4 | 4 1/8 | 3 |
| 15/16 | 4 3/8 | 3 3/16 | 5 3/8 | 3 1/2 | 4 3/8 | 3 1/4 |
| 1 | 4 5/8 | 3 3/8 | 5 7/8 | 3 3/4 | 4 5/8 | 3 1/2 |
| 1 1/16 | 4 7/8 | 3 9/16 | 6 1/4 | 3 7/8 | 5 | 3 5/8 |
| 1 1/8 | 5 1/8 | 3 13/16 | 6 1/2 | 4 1/8 | 5 3/8 | 3 7/8 |
| 1 3/16 | 5 9/16 | 4 | 6 3/4 | 4 1/4 | 5 1/2 | 4 1/8 |
| 1 1/4 | 5 3/4 | 4 3/16 | 7 1/8 | 4 1/2 | 5 3/4 | 4 1/4 |
| 1 5/16 | 6 1/8 | 4 3/8 | 7 3/8 | 4 5/8 | 6 | 4 1/2 |
| 1 3/8 | 6 7/16 | 4 9/16 | 7 3/4 | 4 7/8 | 6 1/4 | 4 3/4 |
| 1 7/16 | 6 11/16 | 4 3/4 | 8 1/8 | 5 1/8 | 6 5/8 | 5 |
| 1 1/2 | 7 | 5 1/8 | 8 1/2 | 5 3/8 | 6 7/8 | 5 1/4 |
| 1 9/16 | 7 3/8 | 5 5/16 | 8 7/8 | 5 5/8 | 7 1/4 | 5 1/2 |
| 1 5/8 | 7 3/4 | 5 1/2 | 9 1/4 | 5 7/8 | 7 1/2 | 5 3/4 |
| 1 11/16 | 8 1/8 | 5 11/16 | 9 5/8 | 6 | .. | .. |
| 1 3/4 | 8 1/2 | 5 7/8 | 10 | 6 1/4 | .. | .. |
| 1 13/16 | 8 7/8 | 6 1/16 | 10 1/4 | 6 1/2 | .. | .. |
| 1 7/8 | 9 1/4 | 6 3/8 | 10 1/2 | 6 3/4 | .. | .. |
| 1 15/16 | 9 5/8 | 6 9/16 | 10 3/4 | 7 | .. | .. |
| 2 | 10 | 6 3/4 | 11 1/8 | 7 1/4 | .. | .. |

Chain should be frequently annealed and lubricated. The load should be removed, the chain slackened and the grease worked between the links with a stiff brush.

Should any portion of the chain show excessive wear such section should be removed and the chain pieced.

Sheave wheels should have a diameter at least 20 to 30 times the diameter of the chain stock.

## Drums.

Wire rope drums should have a diameter of from 400 to 500 times the diameter of the individual wires of which the rope is made. Expressed in terms of the rope diameter we have:

| Description of Wire. | Hand driven. | | Mach. driven. |
|---|---|---|---|
| 6 strands, 19 wires each | 27 | to | 33 |
| 8 strands, 19 wires each | 22 | to | 28 |
| 6 strands, 37 wires each | 19 | to | 24 |

For hemp ropes:
Drum diameter (hand driven) 7 to 10 times the diameter.
Drum diameter (power driven) 30 to 50 times the diameter.
Sheave proportions, Fig. 49.

$d$ = diameter of rope.

$$R=\frac{d}{2}+\frac{1}{32}''$$

$$a=1.5 \text{ to } 2d$$

$$b=\frac{d}{3}+\frac{3}{16}''$$

$$c=\frac{d}{6}+\frac{3}{16}''$$

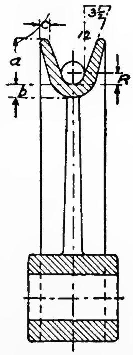


Fig. 49.

The load on the journal carrying the sheave should not exceed 800 to 1000 pounds per square inch of projected area of the journal.

The drum thickness under the grooves is approximately:

$$t=\frac{D}{18}+\frac{3}{8}''$$

where t = drum thickness in inches.
D = pitch diameter of drum in inches.

## Wire Rope Drums.

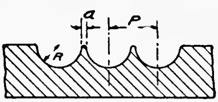


Fig. 50.

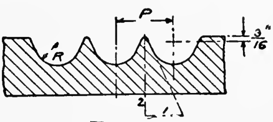


Fig. 51.

$$R=\text{radius of rope}+\frac{1}{32}''$$

$$a \geqq \frac{3}{16}''$$

$$P=2R+a$$

$$P=1\frac{1}{8}\text{ diameter of rope}+\frac{5}{16}''$$

## Chain Drums.

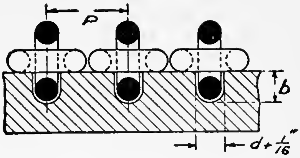



Fig. 52.

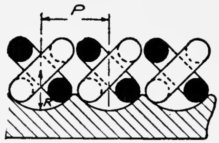



Fig. 53.

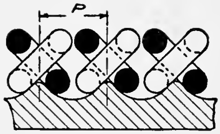



Fig. 54.

Pitch circumference.

Fig. 52: $P=3\frac{1}{2}d+\frac{1}{8}$ inch. $b=1\frac{1}{4}d+\frac{1}{16}$ inch.

Fig. 53: $P=2.8d+\frac{1}{8}$ inch. $R=1\frac{3}{4}d$

Fig. 54: $P=2.8d+\frac{1}{8}$ inch.

If L = length of chain to be rolled on the drum.
$n$ = number of turns of chain on drum.
W = width required upon the drum by the chain.
D = pitch diameter of drum, *i.e.*, the diameter measured at the chain center.

For chain drum Fig 52 $L=n\pi D$

$$W=\frac{L}{\pi D}(3.5d+\tfrac{1}{8})$$

For chain drums Figs. 53 and 54

$$W=\frac{L}{\pi D}(2.8d+\tfrac{1}{8})$$

Chain drums should have the following diameters:

Hand driven drums $D \geqq 20 \times d$

Power driven drums $D \geqq 30 \times d$

Where $d$ is the diameter of the chain stock, and D is the pitch diameter of the drum.

In order that the rope or chain may run properly on the drum, the rope's travel on each side of the middle of the drum should not exceed 1/50 the distance from the drum to the sheave.

## Wheels.

The width and diameter of a wheel depend upon the load. Wheels are made of cast-iron, preferably with chilled tread ground to suit the rail section, or they may be made of hard steel castings. Hard material is required to reduce the wear to a minimum.

The allowable load per lineal inch of roller or wheel is

$$w = \tfrac{2}{3} D \sqrt{\frac{2p^3}{E}} = C \times D$$

$w$ = load in pounds per one inch of width of wheel tread.
D = diameter of roller or wheel in inches.
$p$ = maximum desired fiber stress in pounds per square inch.
E = modulus of elasticity.

Assuming for rail steel $p$ = 20,000 to 30,000 pounds and E = 30,000,000, $c$ becomes 500 − 900.

Similarly taking $p$ = 8,000 and E = 15,000,000 for cast-iron, $c$ is 220.

From these the load upon a wheel becomes

$$W = c.b.D$$

W = total load on wheel in pounds.

$$c = \tfrac{2}{3} \sqrt{\frac{2p^3}{E}}$$

Hence for cast-iron W = 220.$b$.D,

for steel W = 500 to 900.$b$.D

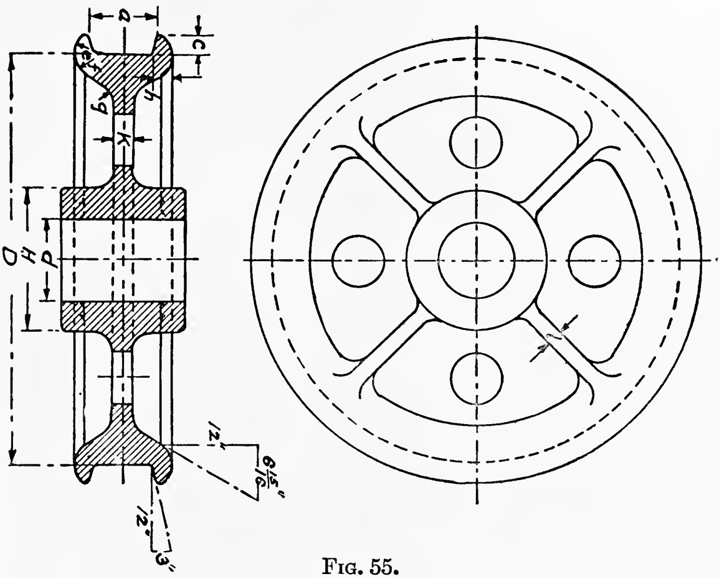



FIG. 55.

It is of the greatest importance that the trolley and especially the truck wheels for the bridge should be turned, or where necessary ground, accurately to the proper diameters, otherwise the crane or trolley will bind on the track, greatly increasing the power required to move it.

The pressure per square inch of projected area of the axle bearing should be limited to from 800 to 1,000 pounds.

The proportions of Fig. 55 are,

$a = 1.25b$ $\qquad c = \frac{D}{50} + \frac{3}{8}$ inch.

$d =$ diameter of the shaft, in inches.

$H = 1.5\,d + \frac{3}{4}$ inch. $\qquad e = c + \frac{3}{16}$ inch.

$D =$ diameter of the wheel, in inches.

$f = c + \frac{1}{4}$ inch. $\qquad g = c$

$h = c$ $\qquad k = c$

$l = c - \frac{1}{8}$ inch.

The number of ribs can be taken as $N = \frac{D}{8} + 2$.

## Ball Bearings.

Ball bearings have found some little application in crane design, a common instance being the bearing that permits the hook to swivel.

The authoritative investigation of ball bearings is that of Stribeck, and a *résumé*, to some extent, of Stribeck's work was presented to the American Society of Mechanical Engineers by Mr. Henry Hess in 1906 and 1907.

The following are some of the conclusions taken from those papers:

1. Ball bearings do not wear. Should the design be such that wear occurs, the bearing is useless.

2. Balls may be subjected to loads increasing as the shape of the supporting surface becomes more nearly complemental to that of the ball.

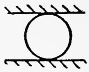


Fig. 56.

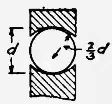



Fig. 57.

Fig. 57 will carry a greater load than Fig 56, but in Fig. 57 the radius $r$ must always exceed the ball radius.

3. The frictional resistance of a ball bearing is lower the fewer the number of balls. Bearings can usually be designed with from 10 to 20 balls.

4. Speed of rotation so long as it is uniform does not affect the carrying capacity of balls in radial bearings.

This does not apply to thrust bearings of the collar type; here the load decreases with increased speed.

5. Both load and speed variations materially reduce the carrying capacity of ball bearings. The effect of these can only be estimated after experience with similar conditions.

6. Balls must be accurately sized, otherwise they will not share the load properly.

7. Balls and races must have a high finish and be made of a material of uniform quality, hardness and structure throughout.

8. Balls and races should generally be of the best steels and most carefully tempered.

9. Thrust bearings.

Ball thrust bearings are materially affected by high speeds, so that their utility is greatly reduced at speeds above 1500 revolutions per minute.

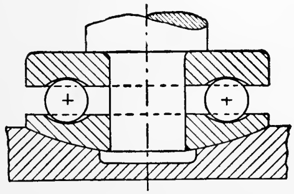


Fig. 58.

The following conditions are set down as relating to the satisfactory design and operation of ball bearings:

$a$ = proper selection of size for the given load and conditions.
$b$ = bearings must be lubricated.
$c$ = bearings must be kept free of grit, moisture, and acid.
$d$ = the inner race must be firmly secured to the shaft.
$e$ = the outer race must be a slip fit in its seat.
$f$ = thrust should always be taken up, whether in one or opposite directions, by the same bearings.
$g$ = bearings should never be dismembered, or at least only one at a time, thus avoiding the danger of mixing the balls.

The following deductions from the experiments of Stribeck apply to radial bearings:

1. The sum of the normal pressures acting on the balls, when from 10 to 20 balls are used in the bearing, is approximately 1.2 times the bearing pressure (P).

2. The ratio of the total bearing pressure (P) to the maximum

pressure on a ball (P) approximates the number of balls (Z) divided by 4.37, or, allowing for unavoidable play between balls,

etc., $\frac{Z}{5.}$ $\quad \frac{P}{P_o} = \frac{Z}{5}$.

3. In the formula $P_o = Kd^2$ when of type Fig. 57, r being equal to $\frac{2}{3} \times d$, both balls and races being the best quality tool steel, $k = 1400$.

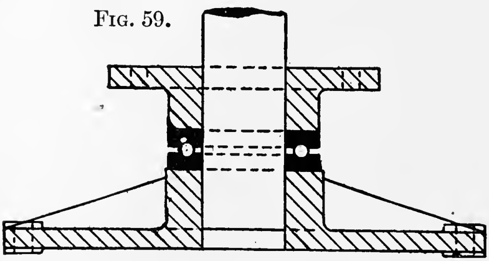
4. In the same formula when the track surfaces are flat, conical or cylindrical, other than shown in Fig. 57, $k = 420$ to 700. The higher values to be used when the motion of the ball relative to the race approaches pure rolling.

The following data credited to Bach and Stribeck are taken from Hütte, 1905:

$P = kd^2$ $\quad$ P = load in pounds. $\quad$ $d$ = ball diameter in inches.

For cast-iron balls between two straight faces $k = 35.5$.
Steel balls on straight, conical or cylindrical races
$k = 1400$ for intermittent service.
$k = 700$ to 1060 for continuous service.
In races as shown in Fig. 57 with $r = \frac{2}{3} \times d$.
$k = 2850$ for intermittent driving.
$k = 1400$ to 2100 for continuous driving.

FIG. 59.



Ball step bearing at mast.

Ball step bearings are frequently used at the base of the masts. Fig. 59 illustrates such a bearing for a 5 ton (10,000 pounds) jib crane. The reaction at the bearing is 16,000 pounds. Trying 23—¾ inch diameter balls the load per ball is $\frac{16000}{23} \sim 700$ pounds.

The load per ball is given by $P = Kd^2$, from which

$$K = \frac{P}{d^2} = \frac{700}{.75^2} = 1245 \text{ pounds},$$

which is well below the allowable 1400 pounds.

* In ball bearings for crane hooks designs have been noted with K = 4000—5000 pounds.

In this particular case the number of balls is really influenced largely by the diameter of the pin taking the horizontal thrust, thus making the diameter of the race large.

## Roller Bearings.

For roller bearings, see Proceedings of the Engineers' Club of Philadelphia, October, 1907.

The following is from Hütte and is credited to Stribeck:

P = load on bearing in pounds.
$i$ = number of rollers.
$d$ = diameter of rollers in inches. Mean diameter of cones.
$l$ = working length in inches.

The rollers must be designed so that the load P will be properly distributed over the length l and number i of the roller.

$$P = c.i.l.d$$

$c$ has the following values:

Rollers and tracks hard cast-iron $c = 350$.
Rollers and tracks hard steel $c = 850$.

Should $l > 5d$, $c$ should be taken correspondingly smaller. In the case of accurately made, hardened steel rollers on hardened steel bushings $c$ may reach 2100.

## Crane Hooks. *

Crane hooks are usually forgings of soft steel. The design is carried out by Unwin, Bethmann and others by analyzing the stresses at the section BB of maximum bending. The fiber stress is estimated to be the algebraic sum of the tension due to the direct load and the fiber stress on the section due to bending. As this method is an approximation the fiber stress should be limited to from 8000 to 10,000 pounds per square inch at the outer fibers.

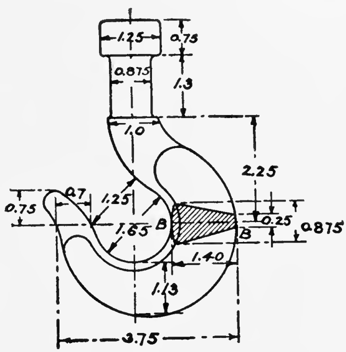


Fig. 60.

The hook proportions in Fig. 60 are given by Unwin. The following points should be observed:

1. The point of the hook should be carried well up to prevent the sling from slipping off the hook.

* The theory of curved beams as applied to hooks is tedious and has not been adopted in their design.

2. For rope slings $a = 0.75d$ to $d$ ($d$ is the rope diameter).
   For chain slings $a = d$ to $1.5d$ ($d$ is chain diameter).

$a$ is the radius of the inside circle of the hook.

The following method is given by Bach, or Hütte, Edition 1905, Vol. 1, p. 696.

The section at BB is generally a trapezoid, but may also be either an ellipse or a rectangle.

CC is the axis through the center of gravity.

$h$ varies from $2a$ to $3a$.

In the trapezoidal section $\frac{b_1}{b_2} = \frac{h}{a} + 1$

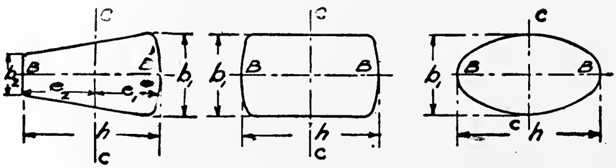



Fig. 61.

The formulæ reduce to

$$P = p\frac{a}{e_1} \times F \qquad (1)$$

where P = load in pounds on the hook.
  $p$ = maximum fiber stress in pounds per square inch.
  $a$ = radius of inside circle of hook in inches.
  F = area of section at BB in square inches.
  $x$ = a constant depending upon the area and form of the section, together with the radius of curvature of the line through the center of gravity of the section.

From formula 1 we have:

$$b_1 = \frac{P}{p.a} \times C \text{ where } C = \frac{b_1 e_1}{xF}$$

The following values of C are taken from Hütte, 1905:

| $\frac{h}{a}$ | 1.0 | 1.5 | 1.8 | 1.9 | 2.0 | 2.1 | 2.2 | 2.3 | 2.4 | 2.5 | 2.75 | 3.0 | 3.5 | 4.0 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| Rectangular Section | 12.59 | 7.25 | 5.76 | 5.39 | 5.07 | 4.79 | 4.53 | 4.31 | 4.10 | 3.92 | 3.52 | 3.22 | 2.75 | 2.41 |
| Trapezoidal Section | 15.0 | 8.96 | 7.25 | 6.85 | 6.42 | 6.05 | 5.77 | 5.48 | 5.25 | 5.06 | 4.55 | 4.18 | 3.59 | 3.28 |
| Elliptical Section | 21.51 | 12.58 | 10.07 | 9.41 | 8.89 | 8.47 | 7.96 | 7.58 | 7.23 | 6.92 | 6.24 | 5.73 | | |

When the stress is carefully determined, the working fiber stress can be taken higher, say 12,000 to 16,000 pounds per square inch in the larger hooks.

**Example.**—Design a hook to carry 20,000 pounds. Allow 8000 pounds fiber stress per square inch on the section at the root of the screw thread, and 12,000 pounds per square inch on the extreme fibers as a result of combined tension and flexure.

Finding the diameter at $d_1$. Area at root of thread is:

$$\frac{P}{p} = \frac{20.000}{8.000} = 2.5 \text{ square inches.}$$

Referring to table of standard screw threads, page 56, a bolt 2¼ in. in diameter will be required.

$$d_2 = 1.25 d_1 = 1.25 \times 2.25$$
$$= 2.81, \text{ say } 2\tfrac{3}{4} \text{ inches.}$$

Now considering the section of maximum bending

$a = 2\tfrac{1}{2}''$, taking $h = 2.5 \times a = 6.25''$

$$\frac{b_1}{b_2} = \frac{h}{a} + 1 = 2.5 + 1 = 3.5.$$

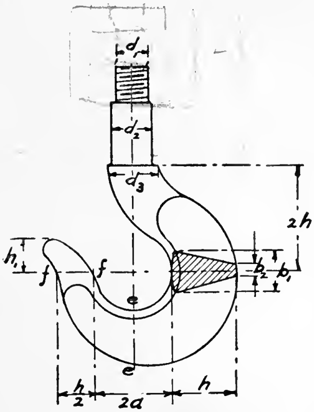


FIG. 62.

C by tables for trapezoidal section is 5.06, hence:

$$b_1 = \frac{P}{p \times a} \times C = \frac{20,000 \times 5.06}{12,000 \times 2.5} = 3.37 \text{ inches}$$

$$b_2 = \frac{3.37}{3.5} = 0.963 \sim 1 \text{ inch.}$$

The other sections can be proportioned as given in Fig. 62, section *ee* being 0.8 and section *ff* being 0.5 the length of the main section BB. The section at *ee* will be trapezoidal, that at *ff* circular.

DOUBLE HOOK.

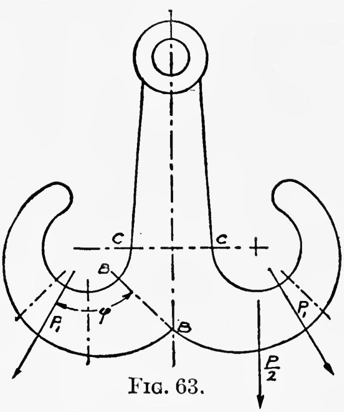


FIG. 63.

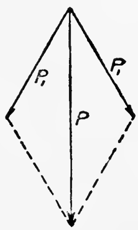


FIG. 64.

The design of double hooks follows similarly to that of the single hook. If $\varphi$ is the angle that the pull P makes with the section being designed, say BB, then by substitution in the previously given formula we have

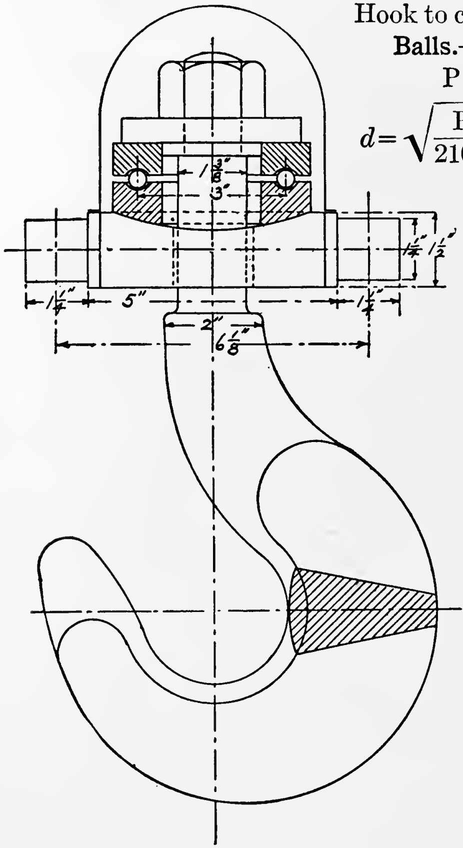
$$b_1 = \frac{P_1 sin\varphi}{p \times a} \times C$$

C is found in the tables just given for the required ratio of $\frac{h}{a}$. Since one side of the hook could raise ½P, the section CD should be designed for this condition and here

$$b_1 = \frac{P \times C}{2 \times p \times a}$$

Double hooks are sometimes called Twin or Sister Hooks.

Swivelling Crane-Hook with Trunnion Suspension.




Hook to carry a load of 5000 pounds.

**Balls.**—$P = kd^2 \qquad k = 2100$

$P = 5000 \div 20 = 250.$

$$d = \sqrt{\frac{P}{2100}} = \sqrt{\frac{250}{2100}} = 0.34 \backsim \tfrac{3}{8} \text{ in.}$$

This indicates a requirement of 20 balls at least ⅜ inch in diameter. The design will be influenced by the fact that the races must surround the bolt at the top of the hook. In heavier hooks $k$ can be taken higher, say 4200 pounds. The area at the root of the thread will be:

$$\text{Area} \times \text{fiber stress} = 5000$$

$$\text{Area } 5000 \div 8000 = 0.625 \text{ square inches.}$$

This corresponds to a bolt 1⅛ inches diameter. See table of bolt dimensions, page 56.

Bolt diameter through trunnion, $1.25 \times 1.125 = 1.408$ inches, approximately $1\frac{3}{8}$ inches.

**Trunnion.**—The height required for the trunnion, assuming the load as a central one and the width $4\frac{1}{2}$ inches $-1.63$ inches $=$ 2.87 inches.

$$M=\frac{pI}{e}$$

$$M=\frac{5000\times6.13}{4}=\frac{8000\times bh^2}{6}$$

Since for a rectangle $\frac{I}{e}=\frac{bh^2}{6}$

$$h=\sqrt{\frac{5000\times6.13\times6}{4\times8000\times2.87}}=1.42''.$$

$h=1.42$ inches, approximately 1.5 inches.

It should be borne in mind that the spherical cut to permit the hook to adjust itself and thus bring pressure uniformly upon all the balls weakens the trunnion. This, however, is allowed for here in the fact that the balls make the load more nearly a uniform one on the trunnion than a central one, as was assumed.

**Trunnion Bearings.**—Owing to the fact that there is likely to be very little swinging of the hook in the trunnion bearings under load, and that the loads are probably infrequent and then less than full load, the journal pressure may be taken high. On the assumption of 1500 pounds per square inch the projected area should be

$$\frac{2500}{1500}=1.67 \text{ square inches.}$$

If the journal is now assumed as having a width equal to its diameter, the diameter will be determined for strength as a cantilever as follows:

$$M=\frac{pI}{e}$$

$$M=\frac{2500\times d}{2}$$

For a circle $\frac{I}{e}=\frac{\pi d^3}{32}$, approximately $\frac{d^3}{10}$

Taking $p=8000$ pounds per square inch,

$$M=\frac{2500\times d}{2}=\frac{8000\times d^3}{10}$$

$$d=\sqrt{\frac{2500\times 10}{2\times 8000}}=1.25 \text{ inches.}$$

The area $1.25\times 1.25=1.56$ square inches. This is sufficiently near the 1.67 square inches estimated as desirable.

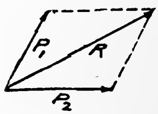
## Elements of Graphic Statics.

**Equilibrium.**—Graphic statics treats of forces in equilibrium, *i.e.*, neither translation nor rotation of the body upon which the forces act must take place. For forces acting in the same plane this condition will be fulfilled when

$\Sigma$ horizontal components of forces $=0$
$\Sigma$ vertical components of forces $=0$
$\Sigma$ moment of forces about any point $=0$

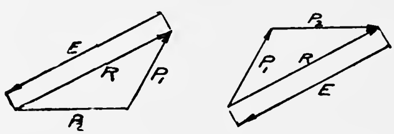
**Representation of a Force.**—A force may be represented by a line, its length denoting the force's magnitude, the line's position the force's line of action; an arrow placed upon the line may indicate the force's direction.

**Force Parallelogram.**—The resultant of two forces acting in a plane is the diagonal of the parallelogram constructed upon these forces.




A force equal and opposite to R is the equilibrant, as it would hold the two forces $P_1$ and $P_2$ in equilibrium.

**Force Triangle.**—Given the two forces $P_1$ and $P_2$ the resultant can be determined by laying $P_1$ and $P_2$ off consecutively so that the sense of the forces shall be the same, *i.e.*, either clock-wise or counter clock-wise, and then joining their extremities.




**Force Polygon.**—Any number of forces acting in a plane and through a point can be treated similarly, *i.e.*, a resultant can be found for any two forces; this resultant can then be combined with another of the forces and so on until the last force is thus combined. The final resultant will be the resultant of all the forces, or a force equal and opposite to it will be the equilibrant of the system. In the force triangle and force polygon it will be noticed that the equilibrant follows around in the same sense as the forces,

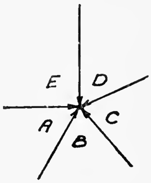



FIG. 65.

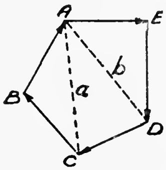



FIG. 66.

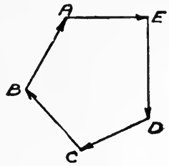



FIG. 67.

the resultant in the opposite sense. The actual drawing of the several resultants can be dispensed with. When the system is in equilibrium, the figure will close and form a force polygon.

Fig. 65.—The forces.

Fig. 66.—*a* resultant of AB and BC.
*b* resultant of *a* and CD.
*c* resultant of *b* and DE=AE.

Fig. 67.—Force polygon. AE shown as the equilibrant of the other forces.

## Equilibrium Polygon.

A number of coplanar forces may act upon a body and the entire system be in equilibrium when the forces do not pass through a common point.

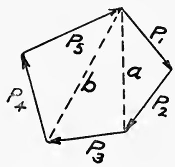



FIG. 68.

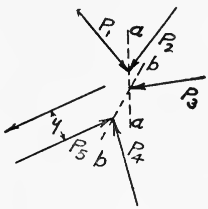



FIG. 69.

Fig. 68 is a force polygon and Fig. 69, which gives the actual location of the forces, is an equilibrium polygon. The resultant of $P_1$ and $P_2$ must pass through their intersection and be parallel to $a$ in the force polygon.

The line locating the force $a$ is $a-a$. The resultant of $a$ and $P_3$ must pass through the intersection of $a-a$ and $P_3$ and be parallel to $b$ in the force polygon. The resultant of $b$ and $P_4$ must pass through the intersection of $b-b$ and $P_4$ and be parallel to $P_5$. When the force polygon closes the system is in equilibrium so far as translation is concerned, but if R and $P_5$ do not coincide in the equilibrium polygon there will be a couple whose moment is

$$+P_5 \times y = +R \times y$$

This will require a couple having an equal but opposite moment acting upon the system to produce equilibrium. When $y$ is zero the force $P_5$ and R coincide and the system is in equilibrium, *i.e.*, the equilibrium polygon closes. A system of coplanar non-concurrent forces are in equilibrium when both the force and the equilibrium polygons close.

### Uses of Equilibrium Polygons.

The equilibrium polygon may be used to determine reactions.

In Fig. 71 a beam is loaded with vertical loads $P_1$, $P_2$ and $P_3$; the lines of the reactions are known but not their magnitudes. Lay off the forces $P_1$, $P_2$ and $P_3$ consecutively so that they form

Fig. 70.

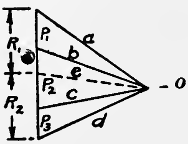
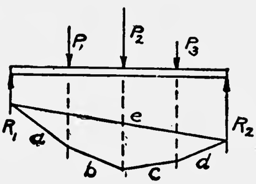



Fig. 71.




the force polygon. Take any point O and connect the extremities of $P_1$, $P_2$ and $P_3$, by the rays $a$, $b$, $c$, and $d$, with this pole O.

In the force polygon Fig. 70 we have several force triangles, so that if $P_1$ is the equilibrant of $a$ and $b$, $a$, $b$, and $P_1$ must act through a common point, and they are so drawn in Fig. 71. $P_2$ is the equilibrant of $b$ and $c$, which are also drawn through a common point; so also with $P_3$ and $c$ and $d$. Since the lines $a$, $b$, $c$, and $d$ in Fig. 70 represent the actual location of the forces $a$, $b$, $c$, and $d$ of the force polygon Fig. 70, Fig. 71 is the equilibrium polygon and the system being in equilibrium the side $e$ must close the polygon, and is drawn through the intersection of $a$ with $R_1$ and $d$ with $R_2$. Now through the pole O draw $e$ parallel to $e$ in the equilibrium polygon. It will cut the line $P_1$, $P_2$ and $P_3$ into two parts respectively equal to $R_1$ and $R_2$, because $R_1$, $a$ and $e$ pass through a common point in

Fig. 71 and close a force triangle in Fig. 70. So also in the case of the force triangle $R_2$ *e*, *d*.

The equilibrium polygon *a*, *b*, *c*, *d*, *e*, in Fig. 71 represents a bending-moment diagram. The bending moment at any section is the product of the vertical intercept between the sides of the equilibrium polygon at that section and the perpendicular distance of the pole O from the line of forces $P_1$, $P_2$, and $P_3$. The use of this bending-moment diagram can be simplified by taking a pole O a definite number of pounds from the forces, *i.e.*, 1000, 10,000, etc., and using the same scale as was used in laying out the forces. The scale of bending moment will then be the product of the scale of vertical intercepts in feet by the pole distance in pounds. Illustration—Assume the scale of the forces as 1 inch = 5000 pounds and the pole distance as 10,000 pounds and that 1 inch = 5 feet. Then the scale for the bending-moment diagram is 1 inch = 10,000 × 5 = 50,000 foot-pounds.

The general method of applying graphics to the analysis of crane stresses is illustrated in the following example. A common

Fig. 72.

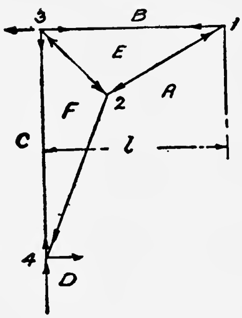
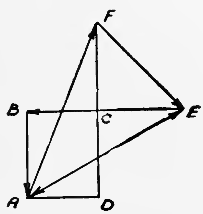



Fig. 73.




type of crane frame is shown in the accompanying Fig. 72. As the stresses in the frame are the result of the external forces acting upon it, these forces must first be located. The analysis is simplified by placing a letter between each pair of forces, and also a letter in each triangle as shown. The load AB must be held up by an equal and opposite force DC so that the sum of the vertical forces shall be zero. Since AB and DC are *l* feet apart they form a couple acting upon the frame; this necessitates a couple with an equal moment but acting in the opposite sense. Such a couple is furnished by DA and CB. This couple holds the crane frame back to a wall or column. Now the forces BE, EA, and AB acting at the point 1 are in equilibrium, hence, laying off these forces consecutively, we have, since the forces must act in the same sense, BA acting down, AE acting to the right and EB acting to the left. Placing the arrows at the point 1 as shown, they indicate the action of the members upon the pin that is assumed as being at the apex 1. The forces acting at point 2 can now be considered. Since the force acting in EA pushes on the pin 1, to do this it must be in compression and it must also push upon the pin at 2, which can be indicated by the arrow at 2.

Laying off the forces acting at point 2, we have EA acting down; draw AF and FE to close the force triangle. Since the forces must act in the same sense as EA acts, AF acts up and FE acts down. Finally taking up the forces at points 3 and 4 and placing the arrows indicating the direction of the forces properly at these points, we have the completed stress diagram.

Since on the frame <———> indicates compression >———< must indicate tension, and both the magnitudes and characters of the stresses are known.

## Graphics of Mechanisms.

Graphics may also be applied to the determination of forces acting in machine members, and the influence of frictional resistances may be included in the analysis. For the purpose of graph-

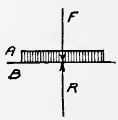



Fig. 74.

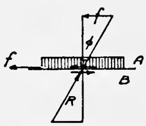



Fig. 75.

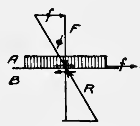



Fig. 76.

ical analysis the machine is taken at various positions in its cycle of motions and for each position the forces acting upon it are assumed in equilibrium. The work may be abridged when the position producing maximum stress in a member or a number of members can be seen by inspection.

We will consider here the influence of sliding friction, journal friction, rolling friction and tooth friction. For a more complete treatment consult any book on mechanics or graphic statics of mechanisms.

**Sliding Friction.**—In the above illustrations the block A is sliding upon the face B in Fig. 74; there is no frictional resistance offered between the faces A and B, so that in moving A upon B no force is required to overcome any friction, hence the resultant pressure R exerted by B upon A is equal and opposite to F.

Figs. 75 and 76.—Here there is frictional resistance between A and B and a force $f$ is required to move A to the left over B; the resultant R is the hypothenuse of the right-angled triangle having F and $f$ for its other sides. All these forces F, $f$ and R must pass through the common point C, the center of pressure. Both F and R must generally be considered as the resultants of distributed pressures on both sides of these resultant forces.

The coefficient of friction is $\mu = \text{tang}\ \phi = \frac{f}{F}$. The small parallel arrows, one on A and one on B, show the direction of relative motion of A to B, and it should be noticed that the resultant R slants towards the tails of these arrows.

**Journal Friction.**—This is simply a special case of sliding friction. With straight faces the radius is infinite, while with circles the radius is finite. The illustration Fig. 77 is analogous to Fig. 76. The small circle is called the friction circle, its radius $\rho = \mu r$, and the resultant R will be tangent to it, passing through the center of pressure between A and B and sloping towards the tails of the small arrows indicating the relative motions of A to B.

**Rolling Friction.**—The influence of rolling friction is to move the center of pressure from C to $C^1$ in the direction of the rolling.

Fig. 77.

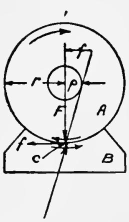



Fig. 78.

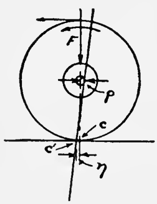
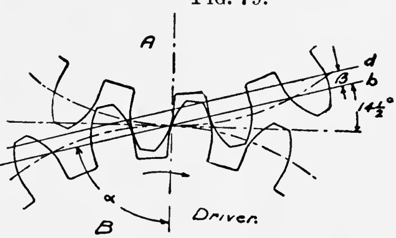



Fig. 79.




The resultant R will then pass through $C^1$ and be tangent to the friction circle of the journal. The effect of rolling friction is so small that it may generally be neglected where journal and other frictional losses are considered. (See Fig. 78.)

**Tooth Friction.**—Fig. 79.—Tooth friction includes both rolling and sliding. The influence of friction is to move the resultant line of action *cd* a distance $\beta$ from the constructive line of action *ab* or the line of action when friction is neglected; thus *cd* is parallel to *ab* and farther from the center of the driver B than *ba*.

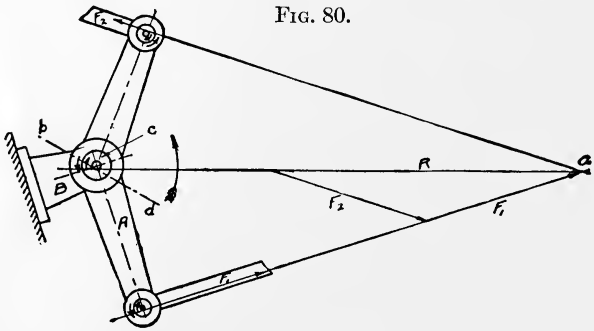
$$B = \frac{\mu p_c}{2} \sin \alpha$$

$\alpha$ = angle of involute tooth, commonly 75° 30 minutes.
$\mu$ = coefficient sliding friction.
$p_c$ = circular pitch.

A couple of simple illustrations will show the methods of using graphics on mechanisms.

The lever A Fig. 80 rotates on the pin B; it is assumed that the directions of $F_1$ and $F_2$ are known and are tangent to their respec-

Fig. 80.




tive friction circles. The piece A will then be acted upon by the three forces $F_1$, $F_2$, and R. These forces, being in equilibrium, must intersect in a common point *a*. Produce $F_1$ and $F_2$ until they intersect at *a*, draw R tangent to its friction circle as indicated by the arrows drawn at the center of pressure between A and B and the line B *b d*. $F_1$ being known in magnitude, $F_2$ and R are found by completing the force triangle, as shown.

Fig. 81.

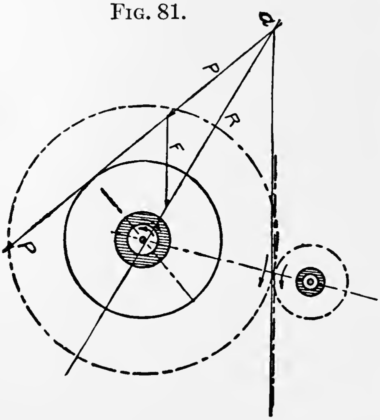



Fig. 81 represents a drum with gear attached turning upon a shaft A, the gear meshes with the pinion C. The drum and gear being bolted together make one piece, which is acted upon by 3 forces—P, the load carried by the drum; F, a force acting upon the gear teeth, and R, the resultant acting through the shaft. The center of pressure between drum and shaft is located at C; the resultant R must be drawn through *a* and tangent to the friction circle as indicated by the small arrows. P being known, R and F are found by completing the triangle.

### Rolling Friction and Power for Crane Travel.

$$\frac{F}{W}=\frac{f+\mu r}{R}$$

$$F=\frac{(f+\mu r)\ W}{R}$$

FIG. 82.

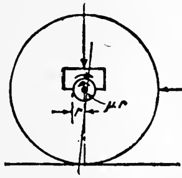
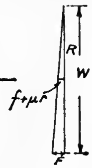



FIG. 83.




The influence of rolling friction is to move the point through which the resultant pressure acts on the track in the direction of motion of the axle. The amount depends upon the materials of the track and wheel and is generally assumed as 0.002 to 0.003 inch for steel or iron.

Taking a wheel 24 inches in diameter with 4 inch diameter axle and $\mu=0.08$, the force F per ton of load W would be

$$F=\frac{[.003+(0.08\times 2)]}{12}W=\frac{0.163}{12}\times 2000=27.2 \text{ pounds.}$$

The calculations for trolley and bridge travel in overhead electric cranes are complicated by the tendency of the wheels to bind sideways against the rails, due to the twisting of the trolley or bridge. This tendency, of course, is more marked the greater the ratio of gauge of track to wheel base, and hence is worse for bridges than for trolleys.

An allowance must also be made for friction in the driving mechanism. In the case of trolleys there will generally be 3 gear reductions; allowing an efficiency of 92 per cent. for each,

$$\text{Efficiency}=0.92^3=78 \text{ per cent.}$$

So that the power to drive the trolley can be put at

$$\text{H.P.}=1\tfrac{1}{8} \text{ to } 1\tfrac{1}{4}\left[\frac{(W_t+L)\ (f+\mu r)\times S}{R\times 33000}\right]$$

$W_t$ = weight of trolley, blocks, tackle, etc.
L = weight of load.
$f$ = 0.002 inch to 0.003 inch.
$\mu$ = 0.08 to 0.10.
$r$ = radius of axle journal in inches.
R = radius of wheel in inches.

**Problem.**—What horse-power will be required for the cross travel of the trolley of a 25 ton crane (50,000 pounds)? Assume the trolley weight as 15,000 pounds, wheels 12 inches in diameter, axles 4 inches in diameter. Travel 100 feet per minute.

$$H.\ P. = 1.25 \frac{[(15000+50000)[.003+(0.08\times2)]\times100}{6\times33,000} = 6.68$$

Such a crane was equipped with an 8 horse-power motor and when moving full load about 80 feet per minute used 6½ horse-power.

In the case of bridge travel the factor must be increased to 1¼ or 1½ on account of binding of the wheels upon the rails, and the weight of the bridge must also be included; the formula becomes

$$H.\ P. = 1\tfrac{1}{4} \text{ to } 1\tfrac{1}{2} \frac{(W_b+W_t+L)\ (f+\mu r)\times S}{R\times33000}$$

$W_b$ = weight of bridge in pounds.

**Problem.**—Taking the 25 ton crane just cited, assume the bridge weight as 30,000 pounds, the bridge wheels 24 inches in diameter, and the axle journals 4 inches in diameter. The bridge travel is to be 250 feet per minute.

$$H.\ P. = 1\tfrac{1}{2}\times\frac{(30000+15000+50000)\ [.003+(0.08\times2)]\times250}{12\times33000}$$

$$H.\ P. = 14.7$$

The above crane used 16 horse-power when carrying full load at about 220 feet per minute.

Designers usually abridge the work of estimating the force to drive a crane by assuming that under ordinary conditions a force of 30 to 40 pounds per ton (2,000 pounds) will be required with a ratio of wheel to axle diameter of 6 to 1. This covers all friction from the track to the motor. It is further assumed that the force varies inversely with this ratio of wheel to axle. Thus a ratio of 3 to 1 would require $\frac{6}{3}\times30=60$ pounds, while a ratio of 7 to 1 demands $\frac{6}{7}\times30=25.7$ pounds.

**Example.**—The power for bridge travel in the last problem would be, $H.\ P. = \frac{95000}{2000}\times\frac{40\times250}{33000} = 14.4.$

Ordinarily no consideration need be given the subject of acceleration in either hoisting or in crab or bridge travel.

The usual arrangement of controller and resistances permits ample force for acceleration to meet ordinary requirements. Should the magnitude of the force required for acceleration be desired it can be determined as follows:

**Hoisting.**—Assuming a 25 ton load to start from rest and by uniform acceleration to attain a velocity of 12 feet per minute in 5 seconds.

The acceleration being assumed as uniform it will equal the velocity at the end of the first second.

$$\text{Acceleration}=\frac{12}{60\times 5}=0.04 \text{ feet per second.}$$

$$\text{Force}=\text{Mass}\times\text{acceleration.}$$

$$F=\left(\frac{25\times 2000}{32.2}\right)\times 0.04=62 \text{ pounds.}$$

This means that to attain the desired velocity at the end of 5 seconds the crane must exert a force of 50,062 pounds upon the 50,000 pound load.

The force required for acceleration in bridge travel is illustrated by the problem given of a 25 ton crane. Bridge weight 30,000 pounds, trolley weight 15,000 pounds. Speed 250 feet per minute. Full speed to be attained in 7 seconds.

$$\text{Acceleration}=\frac{250}{60\times 7}=0.595 \text{ foot per second.}$$

$$\text{Force}=\text{Mass}\times\text{acceleration.}$$

$$F=\frac{(50000+30000+15000)}{32.2}\times 0.595=1750 \text{ pounds.}$$

The force required to overcome friction is

$$F=\frac{1.5\times(50000+30000+15000)}{12}\times[.003+(0.08\times 2)]=1940 \text{ pounds}$$

In this case the force required for acceleration is 90 per cent. of that demanded by friction.

The series motors most commonly used for crane service meet the usual acceleration requirements. In the case of belt or clutch driving or of operating by a steam engine the accelerating forces are generally required to be known for travelling and slewing. See locomotive-crane motions.

## Blocks.

P = pull in rope or chain running on drum or sheave.
$P_1$ = pull in rope or chain running off drum or sheave.
W = load being raised or lowered.
$P_o$ = pull on rope or chain neglecting friction.
B = force on journal.
$r$ = radius of journal.
R = radius of sheave.
$d$ = diameter of chain or rope.
$\mu_1$ = coefficient of journal friction.
$\alpha$ = angle of contact of rope with sheave.
$s$ = the distance the rope leaving the last sheave passes through.
$x$ = the ratio of the force running off a sheave to that running on the same sheave.
$h$ = the height the load moves.
$\eta$ = efficiency.
$\varsigma'$ = coefficient of chain friction.

Fig. 84

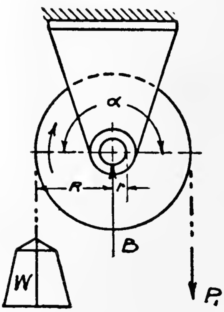



Fig. 85

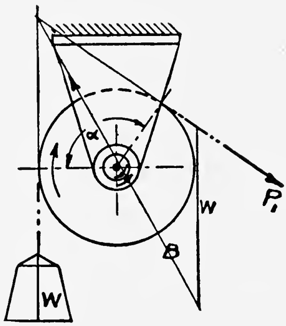



Fig. 84 is the simplest case and here $P_o = W = P$.

The working force is $P_1 = \dfrac{W}{\eta} = xW$.

The distance passed over is $s = h$.

Now considering the determination of the efficiency. If no frictional resistances occurred either in the rope or sheave journal, then equating moments about the journal $PR = P_1R$.

The effect of friction in the rope or chain is to increase the moment arm on the load side and decrease it on the force side.

If this change of length of arm is called $e_1$ for ropes, and $e_2$ for chains, then

$$2e_1 = 0.457\frac{d^2}{R}$$

$$2e_2 = \varphi\frac{d}{R}$$

In the general case in Fig. 85, $B = 2P\sin\frac{\alpha}{2}$.

Now equating the moments,

$$\text{Rope,} \quad P(R - e_1) = P_1(R + e_1) + \mu Br \qquad (1)$$

$$\text{Chain,} \quad P(R - e_2) = P_1(R + e_2) + \mu Br \qquad (2)$$

$$\text{Rope,} \quad P = P_1\left(1 + \frac{2e_1}{R} + \frac{2\mu\, sin\frac{\alpha}{2}}{Rr}\right) \qquad (3)$$

$$\text{Chain,} \quad P = P_1\left(1 + \frac{2e_2}{R} + \frac{2\mu\, sin\frac{\alpha}{2}}{Rr}\right) \qquad (4)$$

It follows from these that

$$\text{For rope,} \quad x = \frac{1}{\eta} = \left(1 + \frac{2e_1}{R} + \frac{2\mu\, sin\frac{\alpha}{2}}{Rr}\right)$$

$$\text{For chain,} \quad x = \frac{1}{\eta} = \left(1 + \frac{2e_2}{R} + \frac{2\mu\, sin\frac{\alpha}{2}}{Rr}\right)$$

It usually happens that $\alpha = 180$ degrees. For rope and chain $R = 10.d$, $r = 1.5d$, $\mu = 0.08$, then depending upon whether the chain is dry or greased, for a fixed sheave $\eta = 0.94$ to $0.96$, corresponding to $x = 1.06$ and $x = 1.04$, while for a floating sheave, Fig. 86, $\eta = .97$ and $x = 1.03$.

In the case of hemp rope sheaves, if $\alpha = 180$ degrees $R = 4.d$, $r = 0.4 \times d$, and $\mu = 0.08$.

Hemp Rope and Sheaves.

| Diam. rope, inches | $\frac{5}{8}$ | 1.0 | $1\frac{3}{8}$ | $1\frac{3}{4}$ | 2 |
|---|---|---|---|---|---|
| Fixed Sheave..... | 0.94–0.96 | 0.91–0.95 | 0.89–0.93 | 0.87–0.92 | 0.85–0.91 |
| Loose Sheave..... | 0.97 | 0.96 | 0.95 | 0.94 | 0.93 |

Fig. 86.

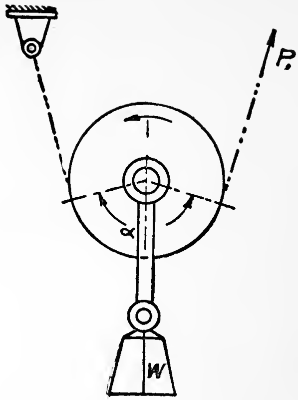
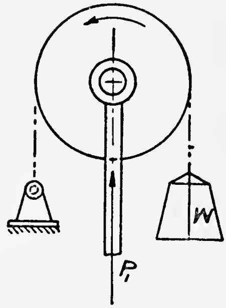



Fig. 87.




$$N = \sqrt{P^2 + P_1^2 - 2\,PP_1 \cos\alpha}$$

$$\text{or } P_1 = W\frac{x}{\sqrt{1+x^2-2x\cos\alpha}}$$

$$\text{For } \alpha = 180° \quad P_1 = W\frac{x}{1+x} \text{ and } \eta = \frac{1+x}{2x}.$$

In Fig. 87 the load is held by the rope and the driving force is applied to the sheave.

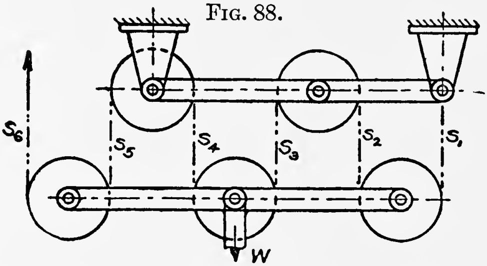
$$P_o = 2W, \quad P_1 = W(1+x)$$

$$\frac{P_o}{P_1} = \frac{2}{1+x}$$

**Efficiency of Blocks Containing Several Sheaves.**

First considering the case where the last sheave over which the rope passes is a floating sheave, Fig. 88.

Fig. 88.




As generally designed the upper sheaves would be on one axle and the lower sheaves would then be on another axle, and this latter would carry the hook. The sheaves are shown spread out for clearness, the theory not being affected by this arrangement.

$P_o$ = pull in $S_6$ without friction.

$n$ = the number of sheaves.

$$P_o = \frac{W}{n+1}$$

Since $S_2 = S_1 x$, $S_3 = S_2 x = S_1 x^2$, $P = S_{n+1} = S_1 x^n$ and

$$W = S_1 + S_2 + S_3 + \ldots\ldots\ldots\ldots S_{n+1}$$

$$W = S_1(1 + x + x^2 + \ldots\ldots\ldots\ldots x^n)$$

The latter being a geometrical series, the sum of n terms is

$$S = \frac{a(r^n - 1)}{r - 1} \text{ and we have } W = \frac{S_1(x^{n+1} - 1)}{x - 1}$$

$$\text{And } \frac{P}{W} = \frac{S_1 x^n (x-1)}{S_1 (x^{n+1} - 1)} = \frac{x^n(x-1)}{x^{n+1} - 1}$$

$$\text{From which the efficiency is } \eta = \frac{P_o}{P} = \frac{1}{n+1} \times \frac{x^{n+1} - 1}{x^n(x-1)}$$

The amount of rope or chain that will have to be drawn-in to raise the load W a distance $h$ will be, $S = (n+1)h$.

## Efficiencies of Blocks for Chain and Wire Rope.

For wire rope and greased chain $x = 1.04$.

For dry chain $x = 1.06$.

| Number of Sheaves | $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Chain (dry) ......... | 1.06 | .94 | .92 | .89 | .86 | .84 | .82 | .79 | .77 | .76 |
| Chain (greased), Wire rope .... | 1.04 | .96 | .94 | .93 | .92 | .90 | .88 | .86 | .84 | .83 |

In a similar way it may be shown that when the last sheave over which the rope or chain runs is a fixed sheave, that

$$\frac{P}{W} = \frac{x^n(x-1)}{x^n - 1}, \qquad P_o = \frac{W}{n}$$

$$\eta = \frac{1}{n} \times \frac{x^n - 1}{x^n(x-1)} \text{ and } S = h.n.$$

## Efficiency of Blocks for Chain and Wire Rope.

(Last sheave fixed.)

| Number of Sheaves | $x$ | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
|---|---|---|---|---|---|---|---|---|---|---|
| Chain (dry).......... | 1.06 | .91 | .89 | .87 | .84 | .82 | .80 | .78 | .76 | .73 |
| Chain (greased) } Wire rope..... } .... | 1.04 | .95 | .93 | .91 | .89 | .88 | .86 | .84 | .83 | .81 |

The few examples of types of blocks will illustrate the use of the formulæ.

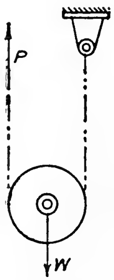



Fig. 89.

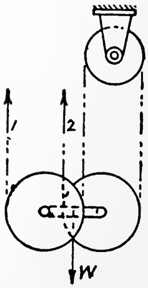



Fig. 90.

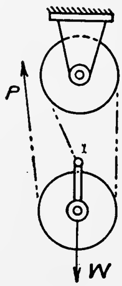



Fig. 91.

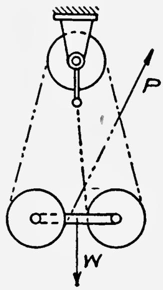



Fig. 92.

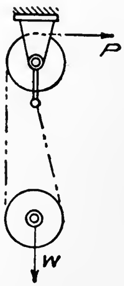



Fig. 93.

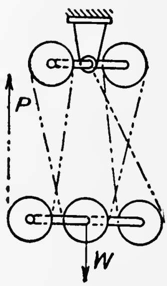



Fig. 94.

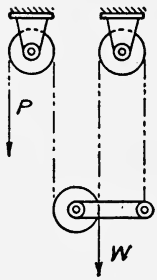



Fig. 95.

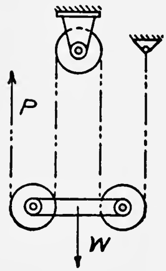



Fig. 96.

Fig. 89.—This is a simple floating sheave.

For wire rope $\eta = 96$ per cent. $s = 2.h$.

Fig. 90.—Sometimes 89 is made double; the end which in 89 is fastened, in 90 is run over an equalizing sheave. This block is identical with 89. The rope now has two ends, 1 and 2, which are carried around sheaves and pulled together. The only function

of the upper sheave is to equalize the loads on the two ends of the rope. As in Fig. 89, $\eta=96$ per cent., $s=2h$.

As there are four ropes carrying the load, the load on each rope neglecting friction is $P_o=\frac{W}{4}$.

Fig. 91.—In this case the last sheave the rope passes-off is a floating sheave. There are two sheaves.

$$P_o=\frac{W}{3}, \quad s=3h \text{ and } \eta=96 \text{ per cent.}$$

In this case, as in the former one, the rope may be taken around double, in which case an equalizing sheave is placed at 1, and the two ends of the rope are then carried around the double sheaves as in the former instance. The efficiencies are the same as those just given. Since there are 6 ropes holding the load, the pull without friction is

$$P_{o1}=\frac{W}{6}.$$

Fig. 92.—Here the last sheave that the rope passes from is a floating sheave. $P_o=\frac{W}{4}$, $\eta=93$ per cent., $s=4h$. This may also be made with an equalizing sheave, and a double rope run over double sheaves as before. The efficiency remains the same, but the load on the rope becomes $P_{o1}=\frac{W}{8}$.

Fig. 93.—The last sheave in this case is fixed.

$$P_o=\frac{W}{2}, \quad s=2h \text{ and } \eta=95 \text{ per cent.}$$

As in the previous cases, double rope can be used if the rope is run over an equalizing sheave. In this case $P_{o1}=\frac{W}{4}$.

Fig. 94.—Here the last sheave is free $n=5$.

$$P_o=\frac{W}{6} \qquad s=6.h \qquad \eta=92 \text{ per cent.}$$

In hydraulic cranes and elevators, blocks and tackle are frequently used where the desire is to increase the velocity of the load and the distance it passes over relative to that passed over by the working force. Figs. 95 and 96 illustrate such cases.

Fig. 95.—The last sheave is fixed.

$$\frac{P}{W}=\frac{x(x^{n}-1)}{x-1}, \qquad \eta=\frac{n}{x}\times\frac{x-1}{x^{n}-1}.$$

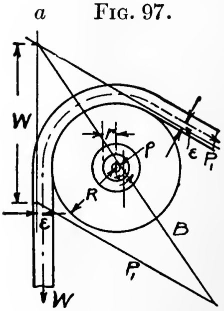
Fig. 96.—The last sheave in this case is free.

$$\frac{P}{W}=\frac{x^{n+1}-1}{x-1} \text{ and } \eta=(n+1)\,\frac{x-1}{x^{n+1}-1}.$$

## Graphical Determination of Stresses.

The accompanying figure shows the graphical method of determining the force $P_1$ and the journal pressure B. The effect of friction is to increase the arm R of the load W by the amount $\varepsilon$, and decrease the effective arm on the side of the force $P_1$ by the same amount. The journal friction will bring the resultant journal pressure through the point $a$ and tangent to the friction circle whose radius is $\rho$, on the side towards the force $P_1$ as shown. Since W is known, the other two forces are found by completing the triangle.

FIG. 97.




$$\rho=\varphi' r$$

$\varepsilon_1=0.23\,\delta^2$ for wire ropes.

$\varepsilon_2=\varphi'\,\dfrac{\delta}{2}$ for chains.

where $\rho$ = radius of friction circle.

$\varphi'$ = coefficient of friction.

$\delta$ = diameter of rope or diameter of chain iron.

$r$ = radius of journal.

## Blocks.

Figs. 98 to 101 are rope blocks. Figs. 102 and 103 illustrate lower blocks for cranes as built by one company. Their hooks are generally steel castings, and are suspended on either balls or rollers running on hardened and ground steel plates. The frame holding the sheaves is also a steel casting.

A lower block of somewhat different type is shown in Figs. 104 and 105. The hook is a forging, suspended on a swivelling pin through steel balls running on hardened and ground plates.

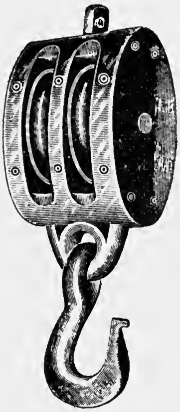


Fig. 98.

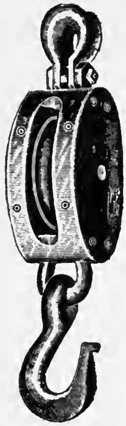


Fig. 99.

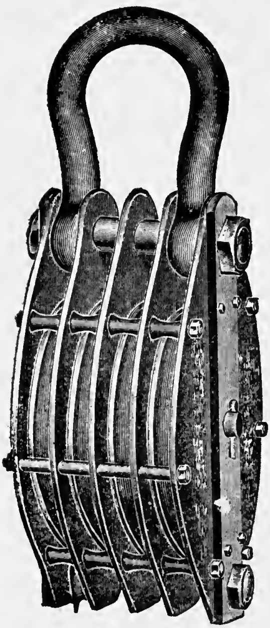


Fig. 100.

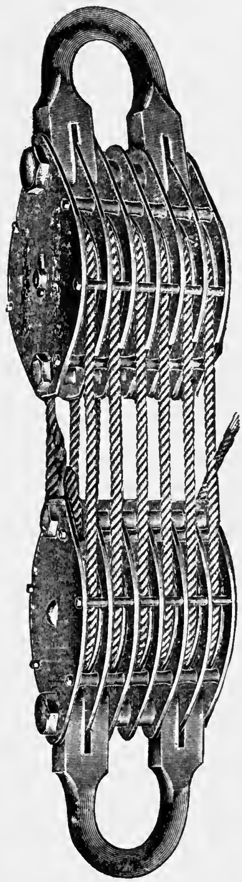


Fig. 101.

Fig. 102.

Fig. 103.

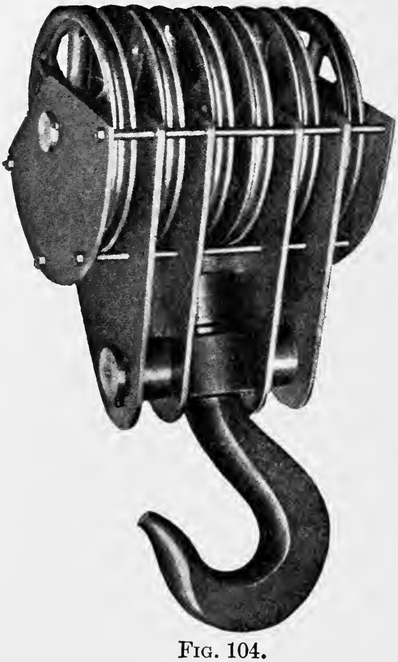


Fig. 104.

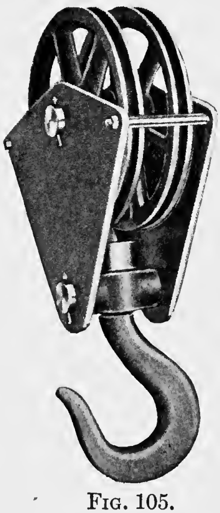


Fig. 105.

## Trolley Movement.

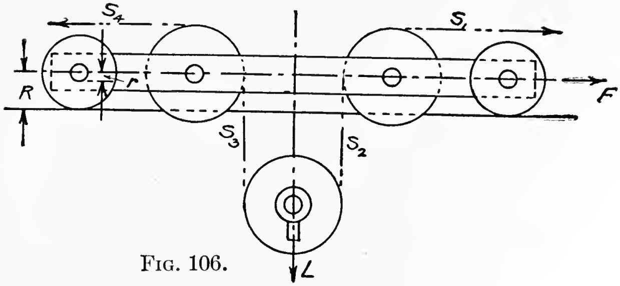



FIG. 106.

$S_1$ is the pull in a rope or chain running on a pulley, $S_2$ is the pull in the same leaving the pulley, then for wire rope or greased chain,

When the contact with the pulley is 90° $S_2 = 1.03\ S_1$.

When the contact with the pulley is 180° $S_2 = 1.04\ S_1$.

In the above trolley we then have:

$$S_2 = 1.03\ S_1.$$
$$S_3 = 1.04\ S_2.$$
$$S_4 = 1.03\ S_3.$$
$$S_2 + S_3 = L = S_2 + 1.04\ S_2.$$

from which $L = 2.04 S_2 \therefore S_2 = \frac{L}{2.04}$.

from this $S_1 = 0.477 \times L.$

$$S_3 = 1.04\ S_2 = 0.51\ L.$$
$$S_4 = 1.03 \times S_3 = 0.526\ L.$$
$$S_4 - S_1 = 0.05\ L.$$

The wheel friction is: $F_2 = \left(\frac{fD + \mu r L}{R}\right) 1.5$

When the distance the trolley travels is considerable and the hoisting chain or rope heavy, the pull due to the rope may be considerable, and can be calculated as follows:

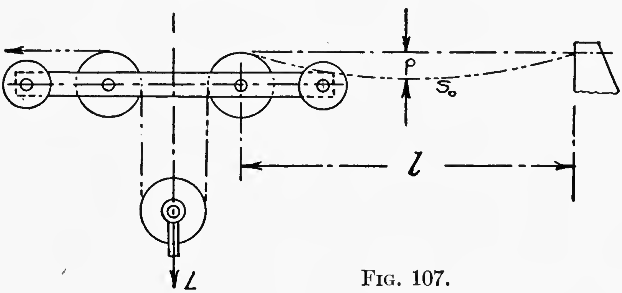



FIG. 107.

$l$ = the maximum length of the rope from trolley to sheave.
$S_o$ = pull in pounds due to span of rope or chain.
$w$ = weight per foot of the rope or chain.
$\rho$ = deflection in feet.

Then, $S_o = \dfrac{wl^2}{8\rho}$.

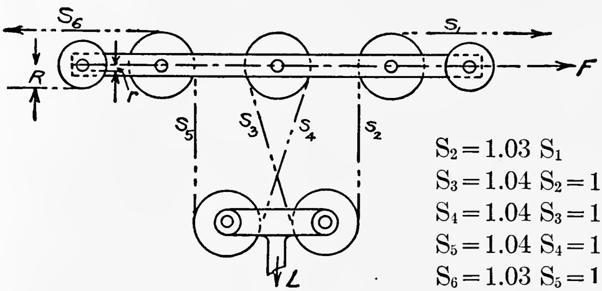
The total force is the sum of these resistances.

$$F = (S_4 - S_1) + \left[\left(\frac{fD + \mu rL}{R}\right)1.5\right] + S_o.$$

$$= 0.05\ L + \left[\left(\frac{fD + \mu rL}{R}\right)1.5\right] + S_o.$$

Here $F$ = force required to move trolley, the force being applied at the trolley.
$f$ = coefficient of rolling friction. (0.002 to 0.003.)
$D$ = live load + weight of trolley, block and tackle.
$\mu$ = coefficient of sliding friction.
$r$ = radius of trolley axle, inches.
$L$ = live load, pounds.
$R$ = trolley wheel radius, in inches.

The analysis would be similar if the load was carried upon any greater number of ropes.




$S_2 = 1.03\ S_1$
$S_3 = 1.04\ S_2 = 1.07\ S_1$
$S_4 = 1.04\ S_3 = 1.11\ S_1$
$S_5 = 1.04\ S_4 = 1.16\ S_1$
$S_6 = 1.03\ S_5 = 1.19\ S_1$

FIG. 108.

Now as before, $S_2 + S_3 + S_4 + S_5 = L$ and $L = 4.37\ S_1$, from which $S_1 = 0.23\ L$ and $S_6 - S_1 = (1.19\ S_1 - S_1) = 0.19\ S_1 = (0.19 \times 0.23)L = 0.044L$.

In this case the total force to move the trolley is:

$$F = 0.044L + \left(\frac{fD + \mu rL}{R} \times 1.5\right) + S_o.$$

## Structural Material.

The principal rolled sections used in cranes are I beams, channels, angles, plates, flats and rounds. Manufacturers' handbooks afford the best sources of information regarding these sections, and in actual design they should be freely consulted. The following data are abridged from these books.

SHEARED PLATES.

| Width in Inches | Thickness in Inches — Length in Inches | | | | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 | 15/16 | 1 | 1 1/8 | 1 1/4 | 1 1/2 |
| 15 | 240 | 320 | 400 | 500 | 500 | 550 | 500 | 475 | 475 | 475 | 425 | 400 | 375 | 360 | 300 | 280 | 280 |
| 16 | 240 | 320 | 400 | 500 | 500 | 550 | 500 | 475 | 475 | 475 | 425 | 400 | 375 | 360 | 300 | 280 | 280 |
| 17 | 240 | 320 | 400 | 500 | 500 | 550 | 500 | 475 | 475 | 475 | 425 | 400 | 375 | 360 | 300 | 280 | 280 |
| 18 | 240 | 360 | 400 | 500 | 500 | 500 | 500 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 19 | 240 | 360 | 400 | 500 | 500 | 500 | 500 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 20 | 240 | 360 | 400 | 500 | 500 | 500 | 500 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 21 | 216 | 360 | 400 | 500 | 525 | 525 | 525 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 22 | 216 | 360 | 400 | 500 | 525 | 525 | 525 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 23 | 204 | 360 | 400 | 500 | 525 | 525 | 525 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 24 | 204 | 360 | 400 | 500 | 525 | 550 | 550 | 550 | 550 | 550 | 500 | 475 | 425 | 400 | 350 | 350 | 325 |
| 25 | 204 | 360 | 400 | 500 | 525 | 550 | 550 | 550 | 550 | 550 | 500 | 475 | 425 | 400 | 350 | 350 | 325 |
| 26 | 180 | 360 | 400 | 500 | 525 | 550 | 550 | 550 | 550 | 550 | 500 | 475 | 425 | 400 | 350 | 350 | 325 |
| 27 | 168 | 340 | 400 | 500 | 500 | 550 | 550 | 500 | 500 | 500 | 450 | 450 | 400 | 380 | 330 | 300 | 300 |
| 28 | 168 | 340 | 400 | 500 | 500 | 550 | 550 | 500 | 500 | 500 | 450 | 450 | 400 | 380 | 330 | 300 | 300 |
| 29 | 156 | 340 | 400 | 500 | 500 | 550 | 550 | 500 | 500 | 500 | 450 | 450 | 400 | 380 | 330 | 300 | 300 |
| 30 to 35 | ... | 320 | 400 | 500 | 500 | 550 | 500 | 475 | 475 | 475 | 425 | 400 | 375 | 360 | 300 | 280 | 280 |
| 36 to 41 | ... | 360 | 400 | 500 | 500 | 500 | 500 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 42 to 47 | ... | 360 | 400 | 500 | 525 | 525 | 525 | 550 | 550 | 550 | 500 | 500 | 450 | 400 | 400 | 400 | 350 |
| 48 to 53 | ... | 360 | 400 | 500 | 525 | 550 | 550 | 550 | 550 | 550 | 500 | 475 | 425 | 400 | 350 | 350 | 325 |
| 54 to 59 | ... | 340 | 400 | 500 | 500 | 550 | 550 | 500 | 500 | 500 | 450 | 450 | 400 | 380 | 330 | 300 | 300 |
| 60 to 65 | ... | 320 | 400 | 500 | 500 | 550 | 500 | 475 | 475 | 475 | 425 | 400 | 375 | 360 | 300 | 280 | 280 |
| 66 to 71 | ... | 300 | 350 | 430 | 450 | 475 | 425 | 425 | 425 | 410 | 375 | 340 | 330 | 320 | 280 | 260 | 260 |
| 72 to 77 | ... | 260 | 300 | 400 | 425 | 450 | 400 | 400 | 400 | 390 | 350 | 320 | 320 | 300 | 260 | 240 | 240 |
| 78 to 83 | ... | 240 | 275 | 380 | 400 | 420 | 375 | 375 | 375 | 370 | 325 | 300 | 300 | 300 | 240 | 220 | 220 |
| 84 to 89 | ... | 200 | 250 | 350 | 375 | 385 | 350 | 350 | 350 | 350 | 300 | 280 | 275 | 275 | 230 | 210 | 210 |
| 90 to 95 | ... | 180 | 230 | 330 | 340 | 350 | 350 | 325 | 325 | 325 | 275 | 260 | 260 | 260 | 220 | 200 | 200 |
| 96 to 101 | ... | 120 | 175 | 240 | 250 | 275 | 275 | 275 | 275 | 275 | 240 | 240 | 220 | 220 | 200 | 180 | 180 |
| 102 to 107 | ... | ... | 150 | 200 | 230 | 230 | 250 | 250 | 250 | 250 | 230 | 230 | 210 | 210 | 190 | 170 | 170 |
| 108 to 113 | ... | ... | ... | 180 | 180 | 200 | 220 | 225 | 225 | 225 | 220 | 220 | 200 | 200 | 180 | 160 | 160 |
| 114 to 119 | ... | ... | ... | ... | ... | 180 | 200 | 210 | 210 | 210 | 200 | 200 | 180 | 180 | 170 | 150 | 150 |
| 120 to 125 | ... | ... | ... | ... | ... | 120 | 150 | 150 | 180 | 180 | 175 | 175 | 160 | 160 | 160 | 144 | 144 |

In the tables of angles the areas are given; the weights in pounds per foot of section can be obtained by multiplying the areas by 3.4.

The moments of inertia and location of the centers of gravity of the angles are the most useful data for these sections, and they are given on page 99.

All data are based upon dimensions in inches and weights in pounds.

### Edged Plates.

| Width in Inches | Thickness in Inches | | | | | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 | 15/16 | 1 |
| | Length in Feet | | | | | | | | | | | | | |
| 4 | 50 | 50 | 50 | 50 | 50 | 50 | 40 | 40 | 30 | 30 | 30 | 28 | 28 | 28 |
| 5 | 30 | 42 | 42 | 42 | 42 | 40 | 30 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| 6 | 30 | 42 | 42 | 42 | 42 | 40 | 35 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| 7 | 25 | 42 | 42 | 42 | 42 | 40 | 35 | 30 | 30 | 30 | 30 | 30 | 30 | 30 |
| 8 | 25 | 42 | 42 | 42 | 42 | 42 | 38 | 36 | 32 | 30 | 29 | 28 | 26 | 25 |
| 9 | 25 | 42 | 42 | 42 | 42 | 42 | 38 | 34 | 32 | 30 | 29 | 28 | 26 | 25 |
| 10 | 25 | 42 | 42 | 42 | 42 | 42 | 38 | 33 | 32 | 30 | 29 | 28 | 26 | 25 |
| 11 | 25 | 42 | 42 | 42 | 42 | 42 | 38 | 33 | 31 | 29 | 28 | 27 | 25 | 24 |
| 12 | 25 | 42 | 42 | 42 | 42 | 42 | 37 | 32 | 30 | 28 | 27 | 26 | 24 | 23 |
| 13 | | 42 | 42 | 42 | 42 | 42 | 37 | 32 | 30 | 27 | 25 | 24 | 22 | 20 |
| 14 | | 42 | 42 | 42 | 42 | 40 | 35 | 30 | 28 | 26 | 25 | 23 | 22 | 20 |
| 14½ | | 42 | 42 | 42 | 42 | 36 | 33 | 30 | 28 | 25 | | | | |

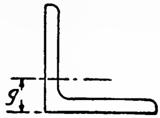
### Dimensions and Areas of Angles.

| Uneven Legs | Even Legs | 1/8 | 3/16 | 1/4 | 5/16 | 3/8 | 7/16 | 1/2 | 9/16 | 5/8 | 11/16 | 3/4 | 13/16 | 7/8 | 15/16 | 1 |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | 8 ×8 | | | | | | | 7.75 | 8.68 | 9.61 | 10.5 | 11.4 | 12.3 | 13.2 | 14.1 | 15.0 |
| | 6 ×6 | | | | | 4.36 | 5.06 | 5.75 | 6.43 | 7.11 | 7.78 | 8.44 | 9.09 | 9.74 | 10.4 | 11.0 |
| *7 ×3½ | | | | | | | 4.40 | 5.00 | 5.59 | 6.17 | 6.75 | 7.31 | 7.87 | 8.42 | 8.97 | 9.50 |
| 6 ×4 | *5 ×5 | | | | | 3.61 | 4.18 | 4.75 | 5.31 | 5.86 | 6.41 | 6.94 | 7.47 | 7.99 | 8.50 | 9.00 |
| 6 ×3½ | | | | | | 3.42 | 3.97 | 4.50 | 5.03 | 5.55 | 6.06 | 6.56 | 7.06 | 7.55 | 8.03 | 8.50 |
| 5 ×4 | | | | | | 3.23 | 3.75 | 4.25 | 4.75 | 5.23 | 5.72 | 6.19 | 6.65 | 7.11 | | |
| 5 ×3½ | | | | | 2.56 | 3.05 | 3.53 | 4.00 | 4.47 | 4.92 | 5.37 | 5.81 | 6.25 | 6.67 | | |
| 5 ×3 | 4 ×4 | | | | 2.40 | 2.86 | 3.31 | 3.75 | 4.18 | 4.61 | 5.03 | 5.44 | 5.84 | | | |
| *4 ×3 | | | | | 2.25 | 2.67 | 3.09 | 3.50 | 3.90 | 4.30 | 4.68 | 5.06 | 5.43 | | | |
| 4 ×3 | 3½ ×3½ | | | | 2.09 | 2.48 | 2.87 | 3.25 | 3.62 | 3.98 | 4.34 | 4.69 | 5.03 | | | |
| 3½ ×3 | | | | | 1.93 | 2.30 | 2.65 | 3.00 | 3.34 | 3.67 | 4.00 | 4.31 | 4.62 | | | |
| 3½ ×2½ | 3 ×3 | | | 1.44 | 1.78 | 2.11 | 2.43 | 2.75 | 3.06 | 3.36 | 3.65 | | | | | |
| 3 ×2½ | *2¾ ×2¾ | | | 1.31 | 1.62 | 1.92 | 2.22 | 2.50 | 2.78 | | | | | | | |
| 3 ×2 | 2½ ×2½ | | | 1.19 | 1.47 | 1.73 | 2.00 | 2.25 | | | | | | | | |
| 2½ ×2 | *2¼ ×2¼ | | 0.81 | 1.06 | 1.31 | 1.55 | 1.78 | 2.00 | | | | | | | | |
| | 2 ×2 | | 0.72 | 0.94 | 1.15 | 1.36 | 1.56 | | | | | | | | | |
| | 1¾ ×1¾ | | 0.62 | 0.81 | 1.00 | 1.17 | 1.30 | | | | | | | | | |
| | 1½ ×1½ | 0.36 | 0.53 | 0.69 | 0.84 | 0.99 | | | | | | | | | | |

*Special sections.

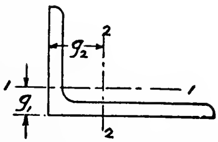
DIMENSIONS AND AREAS OF ANGLES—Continued.

Moments of inertia and distances between centers of gravity and backs of angles.



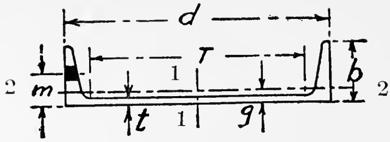

| Thickness Angles | ¼ | | ⅜ | | ½ | | ⅝ | | ¾ | | ⅞ | | 1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | I | g | I | g | I | g | I | g | I | g | I | g | I | g |
| 8 ×8 | | | | | 48.6 | 2.2 | 59.4 | 2.2 | 69.7 | 2.3 | 79.6 | 2.3 | 89.0 | 2.4 |
| 6 ×6 | | | 15.4 | 1.6 | 19.9 | 1.7 | 24.2 | 1.7 | 28.2 | 1.8 | 31.9 | 1.8 | 35.5 | 1.9 |
| *5 ×5 | | | 8.7 | 1.4 | 11.3 | 1.4 | 13.6 | 1.5 | | | | | | |
| 4 ×4 | | | 4.4 | 1.1 | 5.6 | 1.2 | 6.7 | 1.2 | 7.7 | 1.3 | 8.6 | 1.3 | | |
| 3½×3½ | | | 2.9 | 1.0 | 3.6 | 1.1 | 4.3 | 1.1 | 5.0 | 1.2 | 5.5 | 1.2 | | |
| 3 ×3 | 1.2 | .84 | 1.8 | .89 | 2.2 | .93 | 2.6 | .98 | | | | | | |
| *2¾×2¾ | .95 | .78 | 1.3 | .82 | 1.7 | .87 | | | | | | | | |
| 2½×2½ | .70 | .72 | .98 | .76 | 1.2 | .81 | | | | | | | | |
| 2¼×2¼ | .50 | .65 | .70 | .70 | | | | | | | | | | |
| 2 ×2 | .35 | .59 | .48 | .64 | .59 | .68 | | | | | | | | |

The properties of angles of intermediate thickness can be interpolated from the tables with sufficient accuracy for most purposes.



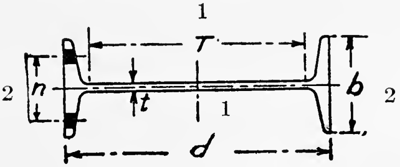

| Thickness Angles | Axis | ¼ | | ⅜ | | ½ | | ⅝ | | ¾ | | ⅞ | | 1 | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | I | g | I | g | I | g | I | g | I | g | I | g | I | g |
| *7 ×3½ | 1 | | | | | 4.4 | .78 | 5.3 | .82 | 6.1 | .87 | 6.8 | .91 | 7.5 | .96 |
| | 2 | | | | | 25.4 | 2.5 | 30.9 | 2.6 | 36.0 | 2.6 | 40.8 | 2.7 | 45.4 | 2.7 |
| 6 ×4 | 1 | | | 4.9 | .94 | 6.3 | .99 | 7.5 | 1.0 | 8.7 | 1.1 | 9.8 | 1.1 | 10.7 | 1.2 |
| | 2 | | | 13.5 | 1.94 | 17.4 | 1.99 | 21.1 | 2.03 | 24.5 | 2.1 | 27.7 | 2.1 | 30.8 | 2.2 |
| 6 ×3½ | 1 | | | 3.3 | .79 | 4.3 | .83 | 5.1 | .88 | 5.8 | .93 | 6.6 | .97 | 7.2 | 1.0 |
| | 2 | | | 12.9 | 2.0 | 16.6 | 2.1 | 20.1 | 2.1 | 23.3 | 2.2 | 26.4 | 2.2 | 29.2 | 2.3 |
| *5 ×4 | 1 | | | 4.7 | 1.0 | 6.0 | 1.1 | 7.1 | 1.1 | | | | | | |
| | 2 | | | 8.1 | 1.5 | 10.5 | 1.6 | 12.6 | 1.6 | | | | | | |
| 5 ×3½ | 1 | | | 3.2 | .86 | 4.1 | .91 | 4.8 | .95 | 5.6 | 1.0 | 6.2 | 1.0 | | |
| | 2 | | | 7.8 | 1.6 | 10.0 | 1.7 | 12.0 | 1.7 | 13.9 | 1.7 | 15.7 | 1.8 | | |
| 5 ×3 | 1 | | | 2.0 | .70 | 2.6 | .75 | 3.1 | .80 | 3.5 | .84 | 3.9 | .88 | | |
| | 2 | | | 7.4 | 1.7 | 9.5 | 1.8 | 11.4 | 1.8 | 13.2 | 1.8 | 14.8 | 1.9 | | |
| *4 ×3½ | 1 | | | 3.0 | .96 | 3.8 | 1.00 | 4.5 | 1.00 | | | | | | |
| | 2 | | | 4.2 | 1.2 | 5.3 | 1.2 | 6.4 | 1.3 | | | | | | |
| 4 ×3 | 1 | | | 1.9 | .78 | 2.4 | .83 | 2.9 | .87 | 3.3 | .92 | 3.7 | .96 | | |
| | 2 | | | 4.0 | 1.3 | 5.1 | 1.3 | 6.0 | 1.4 | 6.9 | 1.4 | 7.8 | 1.5 | | |
| 3½×3 | 1 | | | 1.9 | .83 | 2.3 | .88 | 2.8 | .92 | 3.2 | .96 | 3.5 | 1.0 | | |
| | 2 | | | 2.7 | 1.1 | 3.5 | 1.1 | 4.1 | 1.2 | 4.9 | 1.2 | 5.2 | 1.3 | | |
| 3½×2½ | 1 | .78 | .61 | 1.1 | .66 | 1.4 | .70 | 1.6 | .75 | 1.8 | .79 | | | | |
| | 2 | 1.8 | 1.1 | 2.6 | 1.2 | 3.2 | 1.2 | 3.9 | 1.3 | 4.4 | 1.3 | | | | |
| 3 ×2½ | 1 | .74 | .66 | 1.0 | .71 | 1.3 | .75 | 1.5 | .79 | | | | | | |
| | 2 | 1.2 | .91 | 1.7 | .96 | 2.1 | 1.0 | 2.5 | 1.0 | | | | | | |
| 3 ×2 | 1 | .39 | .49 | .54 | .54 | .67 | .58 | | | | | | | | |
| | 2 | 1.1 | .99 | 1.5 | 1.0 | 1.9 | 1.1 | | | | | | | | |
| 2½×2 | 1 | .37 | .54 | .51 | .58 | .64 | .63 | | | | | | | | |
| | 2 | .65 | .79 | .91 | .83 | 1.1 | .88 | | | | | | | | |

* Special sections.




| | Weight per Foot | Flange b | Web t | Gauge m | Tan't T | Max. Bolt or Rivet | Moment of Inertia Axis 1-1 | Moment of Inertia Axis 2-2 | Dist'ce Base to C. of G. | Area |
|---|---|---|---|---|---|---|---|---|---|---|
| 15 | 55.00 | 3.82 | .82 | 2.50 | 12.25 | ¾ | 430 | 12.2 | 0.82 | 16.18 |
| | 50.00 | 3.72 | .72 | 2.50 | 12.25 | | 403 | 11.2 | 0.80 | 14.71 |
| | 45.00 | 3.62 | .62 | 2.00 | 12.25 | | 375 | 10.3 | 0.79 | 13.24 |
| | 40.00 | 3.52 | .52 | 2.00 | 12.25 | | 348 | 9.4 | 0.78 | 11.76 |
| | 35.00 | 3.43 | .43 | 2.00 | 12.25 | | 320 | 8.5 | 0.79 | 10.29 |
| | 33.00 | 3.40 | .40 | 2.00 | 12.25 | | 312 | 8.2 | 0.79 | 9.90 |
| 12 | 40.00 | 3.42 | .76 | 2.00 | 10.00 | ¾ | 197 | 6.6 | 0.72 | 11.76 |
| | 35.00 | 3.30 | .64 | 2.00 | 10.00 | | 179 | 5.9 | 0.69 | 10.29 |
| | 30.00 | 3.17 | .51 | 1.75 | 10.00 | | 162 | 5.2 | 0.68 | 8.82 |
| | 25.00 | 3.05 | .39 | 1.75 | 10.00 | | 144 | 4.5 | 0.68 | 7.35 |
| | 20.50 | 2.94 | .28 | 1.75 | 10.00 | | 128 | 3.9 | 0.70 | 6.03 |
| 10 | 35.00 | 3.18 | .82 | 1.75 | 8.25 | ¾ | 116 | 4.7 | 0.69 | 10.29 |
| | 30.00 | 3.04 | .68 | 1.75 | 8.25 | | 103 | 4.0 | 0.65 | 8.82 |
| | 25.00 | 2.89 | .63 | 1.75 | 8.25 | | 91 | 3.4 | 0.62 | 7.35 |
| | 20.00 | 2.74 | .38 | 1.50 | 8.25 | | 79 | 2.9 | 0.61 | 5.88 |
| | 15.00 | 2.60 | .24 | 1.50 | 8.25 | | 67 | 2.3 | 0.64 | 4.46 |
| 9 | 25.00 | 2.81 | .61 | 1.50 | 7.25 | ¾ | 71 | 3.0 | 0.62 | 7.35 |
| | 20.00 | 2.65 | .45 | 1.50 | 7.25 | | 61 | 2.5 | 0.58 | 5.88 |
| | 15.00 | 2.49 | .29 | 1.38 | 7.25 | | 51 | 2.0 | 0.59 | 4.41 |
| | 13.25 | 2.43 | .23 | 1.38 | 7.25 | | 47 | 1.8 | 0.61 | 3.89 |
| 8 | 21.25 | 2.62 | .58 | 1.50 | 6.25 | ¾ | 48 | 2.3 | 0.59 | 6.25 |
| | 18.75 | 2.53 | .49 | 1.50 | 6.25 | | 44 | 2.0 | 0.57 | 5.51 |
| | 16.25 | 2.44 | .40 | 1.50 | 6.25 | | 40 | 1.8 | 0.56 | 4.78 |
| | 13.75 | 2.35 | .31 | 1.38 | 6.25 | | 36 | 1.6 | 0.56 | 4.04 |
| | 11.25 | 2.26 | .22 | 1.38 | 6.25 | | 32 | 1.3 | 0.58 | 3.35 |
| 7 | 19.75 | 2.51 | .63 | 1.50 | 5.50 | ⅝ | 33 | 1.8 | 0.58 | 5.81 |
| | 17.25 | 2.41 | .53 | 1.50 | 5.50 | | 30 | 1.6 | 0.55 | 5.07 |
| | 14.75 | 2.30 | .42 | 1.50 | 5.50 | | 27 | 1.4 | 0.53 | 4.34 |
| | 12.25 | 2.20 | .32 | 1.25 | 5.50 | | 24 | 1.2 | 0.53 | 3.60 |
| | 9.75 | 2.09 | .21 | 1.25 | 5.50 | | 21 | .98 | 0.55 | 2.85 |
| 6 | 15.50 | 2.28 | .56 | 1.25 | 4.50 | ⅝ | 19.5 | 1.3 | 0.55 | 4.56 |
| | 13.00 | 2.16 | .44 | 1.25 | 4.50 | | 17.3 | 1.1 | 0.52 | 3.82 |
| | 10.50 | 2.04 | .32 | 1.25 | 4.50 | | 15.1 | .88 | 0.50 | 3.09 |
| | 8.00 | 1.92 | .20 | 1.13 | 4.50 | | 13.0 | .70 | 0.52 | 2.38 |
| 5 | 11.50 | 2.04 | .48 | 1.13 | 3.75 | ½ | 10.4 | .82 | 0.51 | 3.38 |
| | 9.00 | 1.89 | .33 | 1.13 | 3.75 | | 8.9 | .64 | 0.48 | 2.65 |
| | 6.50 | 1.75 | .19 | 1.13 | 3.75 | | 7.4 | .48 | 0.49 | 1.95 |
| 4 | 7.25 | 1.73 | .33 | 1.00 | 2.75 | ½ | 4.6 | .44 | 0.46 | 2.13 |
| | 6.25 | 1.65 | .25 | 1.00 | 2.75 | | 4.2 | .38 | 0.46 | 1.84 |
| | 5.25 | 1.58 | .18 | 1.00 | 2.75 | | 3.8 | .32 | 0.46 | 1.55 |
| 3 | 6.00 | 1.60 | .36 | .88 | 1.75 | ½ | 2.1 | .31 | 0.46 | 1.76 |
| | 5.00 | 1.50 | .26 | .88 | 1.75 | | 1.8 | .25 | 0.44 | 1.47 |
| | 4.00 | 1.41 | .17 | .83 | 1.75 | | 1.6 | .20 | 0.44 | 1.19 |

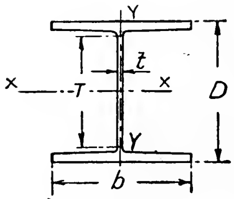
## PROPERTIES OF STANDARD I-BEAMS.



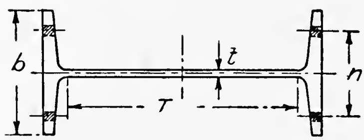

| Depth of Beam | Weight per Foot | Area of Section | Thickness of Web | Width of Flange | Moment of Inertia Axis 1–1 | Section Modulus Axis 1–1 | Radius of Gyration Axis 1–1 | Moment of Inertia Axis 2–2 | Radius of Gyration Axis 2–2 | Max. Rivet Diam. Inches | n | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| d | | A | t | b | I | S | r | I′ | r′ | | | |
| Ins. | Lbs. | Sq. Ins. | In. | Ins. | $Ins.^4$ | $Ins.^3$ | Ins. | $Ins.^4$ | In. | Ins. | Ins. | Ins. |
| 3 | 5.50 | 1.63 | .17 | 2.33 | 2.5 | 1.7 | 1.23 | .46 | .53 | 3/8 | 1 7/16 | 1 13/16 |
| 3 | 6.50 | 1.91 | .26 | 2.42 | 2.7 | 1.8 | 1.19 | .53 | .52 | | | |
| 3 | 7.50 | 2.21 | .36 | 2.52 | 2.9 | 1.9 | 1.15 | .60 | .52 | | | |
| 4 | 7.50 | 2.21 | .19 | 2.66 | 6.0 | 3.0 | 1.64 | .77 | .59 | 1/2 | 1 1/2 | 2 11/16 |
| 4 | 8.50 | 2.50 | .26 | 2.73 | 6.4 | 3.2 | 1.59 | .85 | .58 | | | |
| 4 | 9.50 | 2.79 | .34 | 2.81 | 6.7 | 3.4 | 1.54 | .93 | .58 | | | |
| 4 | 10.50 | 3.09 | .41 | 2.88 | 7.1 | 3.6 | 1.52 | 1.01 | .57 | | | |
| 5 | 9.75 | 2.87 | .21 | 3.00 | 12.1 | 4.8 | 2.05 | 1.23 | .65 | 1/2 | 1 3/4 | 3 5/8 |
| 5 | 12.25 | 3.60 | .36 | 3.15 | 13.6 | 5.4 | 1.94 | 1.45 | .63 | | | |
| 5 | 14.75 | 4.34 | .50 | 3.29 | 15.1 | 6.1 | 1.87 | 1.70 | .63 | | | |
| 6 | 12.25 | 3.61 | .23 | 3.33 | 21.8 | 7.3 | 2.46 | 1.85 | .72 | 5/8 | 2 | 4 7/16 |
| 6 | 14.75 | 4.34 | .35 | 3.45 | 24.0 | 8.0 | 2.35 | 2.09 | .69 | | | |
| 6 | 17.25 | 5.07 | .47 | 3.57 | 26.2 | 8.7 | 2.27 | 2.36 | .68 | | | |
| 7 | 15.00 | 4.42 | .25 | 3.66 | 36.2 | 10.4 | 2.86 | 2.67 | .78 | 5/8 | 2 1/4 | 5 3/8 |
| 7 | 17.50 | 5.15 | .35 | 3.76 | 39.2 | 11.2 | 2.76 | 2.94 | .76 | | | |
| 7 | 20.00 | 5.88 | .46 | 3.87 | 42.2 | 12.1 | 2.68 | 3.24 | .74 | | | |
| 8 | 18.00 | 5.33 | .27 | 4.00 | 56.9 | 14.2 | 3.27 | 3.78 | .84 | 3/4 | 2 1/4 | 6 3/16 |
| 8 | 20.25 | 5.96 | .35 | 4.08 | 60.2 | 15.0 | 3.18 | 4.04 | .82 | | | |
| 8 | 22.75 | 6.69 | .44 | 4.17 | 64.1 | 16.0 | 3.10 | 4.36 | .81 | | | |
| 8 | 25.25 | 7.43 | .53 | 4.26 | 68.0 | 17.0 | 3.03 | 4.71 | .80 | | | |
| 9 | 21.00 | 6.31 | .29 | 4.33 | 84.9 | 18.9 | 3.67 | 5.16 | .90 | 3/4 | 2 1/2 | 7 1/16 |
| 9 | 25.00 | 7.35 | .41 | 4.45 | 91.9 | 20.4 | 3.54 | 5.65 | .88 | | | |
| 9 | 30.00 | 8.82 | .57 | 4.61 | 101.9 | 22.6 | 3.40 | 6.42 | .85 | | | |
| 9 | 35.00 | 10.29 | .73 | 4.77 | 111.8 | 24.8 | 3.30 | 7.31 | .84 | | | |
| 10 | 25.00 | 7.37 | .31 | 4.66 | 122.1 | 24.4 | 4.07 | 6.89 | .97 | 3/4 | 2 5/8 | 7 15/16 |
| 10 | 30.00 | 8.82 | .45 | 4.80 | 134.2 | 26.8 | 3.90 | 7.65 | .93 | | | |
| 10 | 35.00 | 10.29 | .60 | 4.95 | 146.4 | 29.3 | 3.77 | 8.52 | .91 | | | |
| 10 | 40.00 | 11.76 | .75 | 5.10 | 158.7 | 31.7 | 3.67 | 9.50 | .90 | | | |
| 12 | 31.50 | 9.26 | .35 | 5.00 | 215.8 | 36.0 | 4.83 | 9.50 | 1.01 | 3/4 | 2 3/4 | 9 3/4 |
| 12 | 35.00 | 10.29 | .44 | 5.09 | 228.3 | 38.0 | 4.71 | 10.07 | .99 | | | |
| 12 | 40.00 | 11.76 | .56 | 5.21 | 245.9 | 41.0 | 4.57 | 10.95 | .96 | | | |
| 15 | 42.00 | 12.48 | .41 | 5.50 | 441.8 | 58.9 | 5.95 | 14.62 | 1.08 | 3/4 | 3 | 12 1/2 |
| 15 | 45.00 | 13.24 | .46 | 5.55 | 455.8 | 60.8 | 5.87 | 15.09 | 1.07 | | | |
| 15 | 50.00 | 14.71 | .56 | 5.65 | 483.4 | 64.5 | 5.73 | 16.04 | 1,04 | | | |
| 15 | 55.00 | 16.18 | .66 | 5.75 | 511.0 | 68 1 | 5.62 | 17.06 | 1,03 | | | |
| 15 | 60.00 | 17.65 | .75 | 5.84 | 538.6 | 71.8 | 5,52 | 18.17 | 1,01 | | | |
| 18 | 55.0 | 15.93 | .46 | 6.00 | 795.6 | 88.4 | 7.07 | 21.19 | 1.15 | .875 | 3 1/4 | 15 3/16 |
| 18 | 60.0 | 17.65 | .56 | 6.10 | 841.8 | 93.5 | 6.91 | 22.38 | 1.13 | 1 | | |
| 18 | 65.0 | 19.12 | .64 | 6.18 | 881.5 | 97.9 | 6.79 | 23.47 | 1.11 | 1 | | |
| 18 | 70.0 | 20.59 | .72 | 6.26 | 921.2 | 102.4 | 6.69 | 24.62 | 1.09 | 1 | | |
| 20 | 65.0 | 19.08 | .50 | 6.25 | 1169.5 | 117.0 | 7.83 | 27.86 | 1.21 | 1 | 3 1/2 | 16 15/16 |
| 20 | 70.0 | 20.59 | .58 | 6.33 | 1219.8 | 122.0 | 7.70 | 29.04 | 1.19 | | | |
| 20 | 75.0 | 22.06 | .65 | 6.40 | 1268.8 | 126.9 | 7.58 | 30.25 | 1.17 | | | |
| 24 | 80.0 | 23.32 | .50 | 7.00 | 2087.2 | 173.9 | 9.46 | 42.86 | 1.36 | 1 | 4 | 20 11/16 |
| 24 | 85.0 | 25.00 | .57 | 7.07 | 2167.8 | 180.7 | 9.31 | 44.35 | 1.33 | | | |
| 24 | 90.0 | 26.47 | .63 | 7.13 | 2238.4 | 186.5 | 9.20 | 45.70 | 1.31 | | | |
| 24 | 95.0 | 27.94 | .69 | 7.19 | 2309.0 | 192.4 | 9.09 | 47.10 | 1.30 | | | |
| 24 | 100.0 | 29.41 | .75 | 7.25 | 2379.6 | 198.3 | 8.99 | 48.55 | 1.28 | | | |

## Grey Mill Sections.

The Bethlehem Steel Company upon their Grey Mill are able to roll sections with much wider flanges than are possible with the usual methods of manufacture. These sections make better columns and can be used upon longer spans without lateral bracing. The following tables give some of these sections with their properties. In H columns under each section-number only the first 3 weights and the maximum weight are given, although a large number of intermediate weights are rolled. The beam and girder-beam sections are given almost completely.




H Columns.




Beams and Girder Beams.

### H Columns.

| Section No. | Weight per Foot | Area of Section | D | b | t | T | Axis XX: Moment of Inertia | Axis XX: Section Modulus | Axis XX: Radius of Gyration, Inches | Axis YY: Moment of Inertia | Axis YY: Section Modulus | Axis YY: Radius of Gyration, Inches |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | *Lbs.* | *Sq. Ins.* | | | | | I | S | r | I′ | S′ | r′ |
| H 14 | 83.5 | 24.5 | 13¾ | 13.9 | .43 | 11.1 | 884.9 | 128.7 | 6.01 | 294.5 | 42.3 | 3.47 |
| | 91.0 | 26.8 | 13⅞ | 14.0 | .47 | 11.1 | 976.8 | 140.8 | 6.04 | 325.4 | 46.6 | 3.49 |
| | 99.0 | 29.1 | 14 | 14.0 | .51 | 11.1 | 1070.6 | 153.0 | 6.07 | 356.9 | 51.0 | 3.50 |
| | 287.5 | 84.5 | 16⅞ | 14.9 | 1.41 | 11.1 | 3836.1 | 454.7 | 6.74 | 1226.7 | 164.7 | 3.81 |
| H 12 | 64.5 | 19.0 | 11¾ | 11.9 | .39 | 9.2 | 499.0 | 84.9 | 5.13 | 168.6 | 28.3 | 2.98 |
| | 71.5 | 21.0 | 11⅞ | 12.0 | .43 | 9.2 | 556.6 | 93.7 | 5.15 | 188.2 | 31.5 | 3.00 |
| | 78.0 | 22.9 | 12 | 12.0 | .47 | 9.2 | 615.6 | 102.6 | 5.18 | 208.1 | 34.7 | 3.01 |
| | 161.0 | 47.3 | 13½ | 12.5 | .94 | 9.2 | 1444.3 | 214.0 | 5.53 | 477.0 | 76.5 | 3.18 |
| H 10 | 49.0 | 14.4 | 9⅞ | 10.0 | .36 | 7.7 | 263.5 | 53.4 | 4.28 | 89.1 | 17.9 | 2.49 |
| | 54.0 | 15.9 | 10 | 10.0 | .39 | 7.7 | 296.8 | 59.4 | 4.32 | 100.4 | 20.1 | 2.51 |
| | 59.5 | 17.6 | 10⅛ | 10.0 | .43 | 7.7 | 331.9 | 65.6 | 4.35 | 112.2 | 22.3 | 2.53 |
| | 123.5 | 36.3 | 11½ | 10.5 | .86 | 7.7 | 790.4 | 137.5 | 4.67 | 259.3 | 49.5 | 2.67 |
| H 8 | 31.5 | 9.2 | 7⅞ | 8.0 | .31 | 6.1 | 105.7 | 26.9 | 3.40 | 35.8 | 8.9 | 1.98 |
| | 34.5 | 10.2 | 8 | 8.0 | .31 | 6.1 | 121.5 | 30.4 | 3.46 | 41.1 | 10.3 | 2.01 |
| | 39.0 | 11.5 | 8⅛ | 8.0 | .35 | 6.1 | 139.5 | 34.3 | 3.48 | 47.2 | 11.7 | 2.03 |
| | 90.5 | 26.6 | 9½ | 8.5 | .78 | 6.1 | 385.3 | 81.1 | 3.80 | 125.1 | 29.6 | 2.17 |

### Girder Beams

| Depth of Beam, Inches | Weight per Foot, Pounds | Area of Section, Square Inches | Thickness of Web, Inches | Width of Flange, Inches | Neutral Axis Perpendicular to web at center: Moment of Inertia | Radius of Gyration | Section Modulus | Neutral Axis coincident with center line of web: Moment of Inertia | Radius of Gyration | Maximum Rivet Diam. Inches | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | t | F | I | r | I/e | I′ | r′ | | n | T |
| 30 | 200.0 | 58.71 | .750 | 15.00 | 9150.6 | 12.48 | 610.0 | 630.2 | 3.28 | 1 | 11.00 | 25.2 |
| 30 | 180.0 | 53.00 | .690 | 13.00 | 8194.5 | 12.43 | 546.3 | 433.3 | 2.86 | 1 | 9.00 | 25.2 |
| 28 | 180.0 | 52.86 | .690 | 14.35 | 7264.7 | 11.72 | 518.9 | 533.3 | 3.18 | 1 | 10.25 | 23.4 |
| 28 | 165.0 | 48.47 | .660 | 12.50 | 6562.7 | 11.64 | 468.8 | 371.9 | 2.77 | 1 | 8.50 | 23.4 |
| 26 | 160.0 | 46.91 | .630 | 13.60 | 5620.8 | 10.95 | 432.4 | 435.7 | 3.05 | 1 | 9.50 | 21.6 |
| 26 | 150.0 | 43.94 | .630 | 12.00 | 5153.9 | 10.83 | 396.5 | 314.6 | 2.68 | 1 | 8.00 | 21.6 |
| 24 | 140.0 | 41.16 | .600 | 13.00 | 4201.4 | 10.10 | 350.1 | 346.9 | 2.90 | 1 | 9.00 | 20.0 |
| 24 | 120.0 | 35.38 | .530 | 12.00 | 3607.3 | 10.10 | 300.6 | 249.4 | 2.66 | 1 | 8.00 | 20.3 |
| 20 | 140.0 | 41.19 | .640 | 12.50 | 2934.7 | 8.44 | 293.5 | 348.9 | 2.91 | 1 | 8.50 | 15.7 |
| 20 | 112.0 | 32.81 | .550 | 12.00 | 2342.1 | 8.45 | 234.2 | 239.3 | 2.70 | 1 | 8.00 | 16.4 |
| 18 | 92.0 | 27.12 | .480 | 11.50 | 1591.4 | 7.66 | 176.8 | 182.6 | 2.59 | 1 | 7.50 | 14.8 |
| 15 | 140.0 | 41.27 | .800 | 11.75 | 1592.7 | 6.21 | 212.4 | 331.0 | 2.83 | 1 | 7.75 | 10.1 |
| 15 | 104.0 | 30.50 | .600 | 11.25 | 1220.1 | 6.32 | 162.7 | 213.0 | 2.64 | 1 | 7.25 | 11.1 |
| 15 | 73.0 | 21.49 | .430 | 10.50 | 883.4 | 6.41 | 117.8 | 123.2 | 2.39 | 1 | 6.50 | 12.1 |
| 12 | 70.0 | 20.58 | .460 | 10.00 | 538.8 | 5.12 | 89.8 | 114.7 | 2.36 | 1 | 6.00 | 9.0 |
| 12 | 55.0 | 16.18 | .370 | 9.75 | 432.0 | 5.17 | 72.0 | 81.1 | 2.24 | 1 | 6.00 | 9.5 |
| 10 | 44.0 | 12.95 | .310 | 9.00 | 244.2 | 4.34 | 48.8 | 57.3 | 2.10 | 7/8 | 5.50 | 7.8 |
| 9 | 38.0 | 11.22 | .300 | 8.50 | 170.9 | 3.90 | 38.0 | 44.1 | 1.98 | 7/8 | 5.25 | 6.9 |
| 8 | 32.5 | 9.54 | .290 | 8.00 | 114.4 | 3.46 | 28.6 | 32.9 | 1.86 | 7/8 | 5.00 | 6.0 |

### Beams

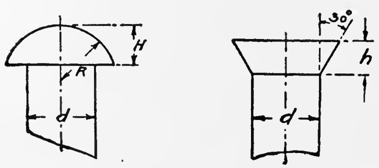
| Depth of Beam, Inches | Weight per Foot, Pounds | Area of Section, Square Inches | t | F | I | r | I/e | I′ | r′ | Maximum Rivet Diam. Inches | n | T |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| 30 | 120. | 35.3 | .54 | 10.5 | 5239. | 12.2 | 349. | 165. | 2.2 | 1 | 6.50 | 26.4 |
| 28 | 105. | 30.9 | .50 | 10.0 | 4014. | 11.4 | 287. | 131. | 2.1 | 1 | 6.00 | 24.7 |
| 26 | 90. | 26.5 | .46 | 9.5 | 2977. | 10.6 | 229. | 101. | 1.9 | 1 | 5.50 | 23.0 |
| 24 | 84. | 24.8 | .46 | 9.3 | 2382. | 9.8 | 198. | 91. | 1.9 | 7/8 | 5.25 | 21.0 |
| 24 | 73. | 21.5 | .39 | 9.0 | 2091. | 9.9 | 174. | 74. | 1.9 | 7/8 | 5.25 | 21.3 |
| 20 | 82. | 24.2 | .57 | 8.9 | 1560. | 8.0 | 156. | 80. | 1.8 | 7/8 | 5.00 | 17.1 |
| 20 | 72. | 21.4 | .43 | 8.7 | 1466. | 8.3 | 146. | 76. | 1.9 | 7/8 | 5.00 | 17.1 |
| 20 | 69. | 20.3 | .52 | 8.1 | 1269. | 7.9 | 127. | 51. | 1.6 | 7/8 | 4.50 | 17.5 |
| 20 | 64. | 18.9 | .45 | 8.1 | 1222. | 8.0 | 122. | 50. | 1.6 | 7/8 | 4.50 | 17.5 |
| 20 | 59. | 17.4 | .38 | 8.0 | 1172. | 8.2 | 117. | 48. | 1.7 | 7/8 | 4.50 | 17.5 |
| 18 | 59. | 17.4 | .50 | 7.7 | 883. | 7.1 | 98. | 39. | 1.5 | 7/8 | 4.25 | 15.7 |
| 18 | 54. | 15.9 | .41 | 7.6 | 842. | 7.3 | 94. | 38. | 1.5 | 7/8 | 4.25 | 15.7 |
| 18 | 52. | 15.2 | .38 | 7.6 | 825. | 7.4 | 92. | 37. | 1.6 | 7/8 | 4.25 | 15.7 |
| 18 | 48.5 | 14.2 | .32 | 7.5 | 798. | 7.5 | 89. | 36. | 1.6 | 7/8 | 4.25 | 15.7 |
| 15 | 71. | 20.9 | .52 | 7.5 | 796. | 6.2 | 106. | 61. | 1.7 | 7/8 | 4.25 | 11.7 |
| 15 | 64. | 18.8 | .60 | 7.2 | 665. | 6.0 | 89. | 42. | 1.5 | 7/8 | 4.00 | 12.3 |
| 15 | 54. | 15.9 | .41 | 7.0 | 610. | 6.2 | 81. | 38. | 1.6 | 7/8 | 4.00 | 12.3 |
| 15 | 46. | 13.5 | .44 | 6.8 | 485. | 6.0 | 65. | 25. | 1.4 | 7/8 | 3.75 | 12.9 |
| 15 | 41. | 12.0 | .34 | 6.7 | 457. | 6.2 | 61. | 24. | 1.4 | 7/8 | 3.75 | 12.9 |
| 15 | 38. | 11.3 | .29 | 6.7 | 443. | 6.3 | 59. | 23. | 1.4 | 7/8 | 3.75 | 12.9 |
| 12 | 36. | 10.6 | .31 | 6.3 | 269. | 5.0 | 45. | 21. | 1.4 | 3/4 | 3.50 | 9.9 |
| 12 | 32. | 9.4 | .33 | 6.2 | 228. | 4.9 | 38. | 16. | 1.3 | 3/4 | 3.50 | 10.2 |
| 12 | 28.5 | 8.4 | .25 | 6.1 | 216. | 5.1 | 36. | 15. | 1.3 | 3/4 | 3.50 | 10.2 |
| 10 | 28.5 | 8.3 | .39 | 6.0 | 134. | 4.0 | 27. | 12. | 1.2 | 3/4 | 3.25 | 8.4 |
| 10 | 23.5 | 6.9 | .25 | 5.9 | 123. | 4.2 | 25. | 11. | 1.3 | 3/4 | 3.25 | 8.4 |
| 9 | 24. | 7.0 | .36 | 5.6 | 92. | 3.6 | 20. | 9. | 1.1 | 3/4 | 3.00 | 7.5 |
| 9 | 20. | 6.0 | .25 | 5.4 | 85. | 3.8 | 19. | 8. | 1.2 | 3/4 | 3.00 | 7.5 |
| 8 | 19.5 | 5.8 | .32 | 5.3 | 61. | 3.2 | 15. | 7. | 1.1 | 3/4 | 2.75 | 6.6 |
| 8 | 17.5 | 5.2 | .25 | 5.2 | 57. | 3.3 | 14. | 6. | 1.1 | 3/4 | 2.75 | 6.6 |

**Riveting.**—The ordinary rivet sizes range from 3/8 to 1 1/8 inches inclusive, varying by eighths. 3/4 inch and 7/8 inch diameter are the most frequently used. In general practice the holes are punched 1/16 inch in diameter less than the nominal diameter of the rivet; after the assembling of the pieces the holes are then reamed to a diameter $\frac{1}{16}$th greater than the rivet diameter. This removes any material injured by punching, brings the holes in the pieces riveted together properly in line, and permits the ready insertion of the heated rivet into the hole. In estimating the strength of the riveting the calculations are based upon the nominal rivet diameter and not upon its diameter after being driven. When the following specifications are followed only the shearing and bearing value of the rivets need be considered.

## Specifications.

1. Rivets should not be spaced closer than 3 diameters of the rivet.
2. Generally the pitch should not exceed 6 inches, or 16 times the thickness of the thinnest outside plate.
3. At the ends of compression pieces the pitch should not exceed 4 diameters of the rivet for a length of at least twice the width of the pieces. (This applies to built-up compression pieces.)
4. For plates in compression the pitch in the direction of the line of stress should not exceed 16 times the thickness of the plate; the pitch at right angles to the line of stress can be from 2 to $2\frac{1}{2}$ times the above pitch.
5. The distance from the edge of a piece to the center of a rivet should not be less than $1\frac{1}{2}$ diameters of the rivets, and when possible this distance should be twice the rivet diameter, but not exceeding 8 times the thickness of the plate.
6. Where close riveting is required at least $\frac{3}{8}$ inch should be left outside the diameter of the rivet head.
7. Where single angles are used for members it is better practice to rivet both legs rather than to secure the angle by riveting to only one leg of the angle.

The dimensions of rivets will vary somewhat with the manufacturer's standards and also with the wearing of the dies. The following will give approximate dimensions:




$$H = \frac{6}{10}d.$$

$$R = \frac{3d}{4} + \frac{1}{16} \text{ inch.}$$

$$h = \frac{d}{2}.$$

The shearing strength of the rivet material is commonly taken at $\frac{3}{4}$ of its tensile strength, while the strength in bearing is generally considered as twice the shearing strength; thus material which permitted a tensile working fiber stress of 12,000 pounds per square inch would allow a shearing fiber stress of 9,000 pounds per square inch and a bearing value of 18,000 pounds per square inch.

The following table allows 1,000 pounds per square inch in shear and 2,000 pounds per square inch in bearing. To find the rivet value corresponding to any other working fiber stress it is only necessary to multiply the figures in the table by the ratio between the desired shearing fiber stress and 1,000 pounds.

SHEARING VALUE OF RIVETS AND BEARING VALUE OF RIVETED PLATES

All dimensions in inches

| | | | | | | | |
|---|---|---|---|---|---|---|---|
| Diameter of Rivet | | 3/8 | 1/2 | 5/8 | 3/4 | 7/8 | 1 |
| Area in square inches | | .1105 | .1964 | .3068 | .4418 | .6013 | .7854 |
| Single shear at 1,000 lbs. | | 110 | 196 | 307 | 442 | 601 | 785 |
| Double shear at 2,000 lbs. | | 221 | 393 | 614 | 884 | 1203 | 1571 |
| Bearing Value for different thicknesses of plate in inches at 2,000 pounds per square inch. | 1/4 | 188 | 250 | 312 | 375 | 438 | 500 |
| | 5/16 | 234 | 313 | 391 | 469 | 547 | 625 |
| | 3/8 | 281 | 375 | 469 | 562 | 656 | 750 |
| | 7/16 | ... | 438 | 547 | 656 | 766 | 875 |
| | 1/2 | ... | 500 | 625 | 750 | 875 | 1000 |
| | 9/16 | ... | ... | 703 | 844 | 984 | 1125 |
| | 5/8 | ... | ... | 781 | 938 | 1094 | 1250 |
| | 11/16 | ... | ... | ... | 1031 | 1203 | 1375 |
| | 3/4 | ... | ... | ... | 1125 | 1312 | 1500 |
| | 13/16 | ... | ... | ... | .... | 1422 | 1625 |
| | 7/8 | ... | ... | ... | .... | 1531 | 1750 |
| | 15/16 | ... | ... | ... | .... | 1641 | 1875 |
| | 1 | ... | ... | ... | .... | .... | 2000 |

The bearing values above the upper zig-zag line are less than the corresponding single shear values; those between the zig-zag lines exceed the single shear, but are less than the double shear values. The figures below the lower zig-zag line exceed the values for double shear.

The riveting should be designed with the lower of the 2 values taken from the table. An example will illustrate. 3/4 inch diameter rivets are used in 5/16 inch plates. The rivets are in double shear; the allowed shearing fiber stress is 9,000 pounds per square inch.

Shearing value $= 884 \times 9 = 7{,}956$ pounds.

Bearing value $= 469 \times 9 = 4{,}221$ pounds.

It is therefore evident that the rivet will stand 4,221 pounds pressure, this being the lower of the 2 values.

# PART II.—FRAMES AND GIRDERS.

## Columns.

The most nearly satisfactory theoretical formula for columns is Ritter's. It is:

$$p_1 = \frac{p}{1 + \frac{F}{m\pi^2 E} \times \left(\frac{l}{r}\right)^2}$$

where $p_1$ = permissible load per square inch of column section.
$p$ = maximum allowable fiber stress.
F = compressive strength of the material in pounds per square inch taken at the elastic limit.
$m$ = a constant depending upon the nature of the column ends.
$m$ = is ¼ for one end fixed and one end free.
1 for hinged ends.
4 for fixed ends.

In crane practice the columns are most frequently assumed as hinged or $m$ is 1. As the material is generally soft or very mild steel F can be taken at 30,000, and E as 30,000,000, $\pi^2 =$ 9.86, practically 10.

Making these substitutions the formula becomes

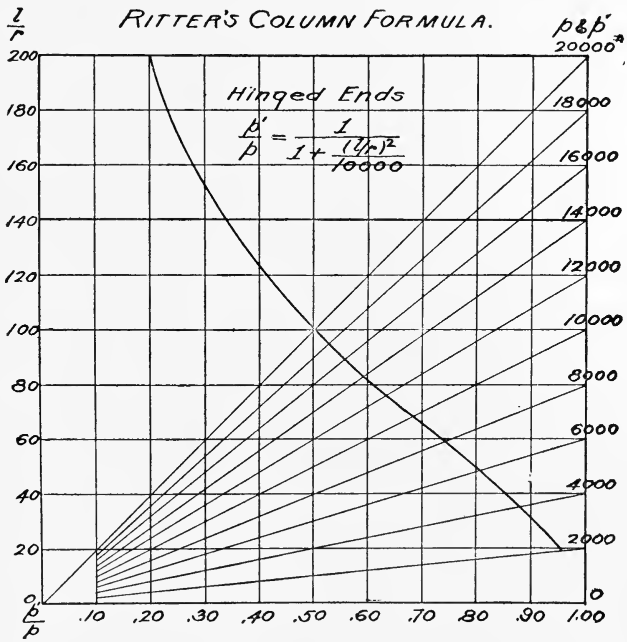
$$p_1 = \frac{p}{1 + \frac{1}{10{,}000} \times \left(\frac{l}{r}\right)^2}$$

This formula, like most other column formulæ, is cumbersome, but the use of the formula can be facilitated by the curve, Fig. 109. The curve plots the values of

$$\frac{p_1}{p} = \frac{1}{1 + \frac{1}{10{,}000} \times \left(\frac{l}{r}\right)^2}.$$

These are expressed as percentages and are plotted as abscissas, the values of $l/r$ being laid off as ordinates. The diagonals represent maximum allowable fiber stress; making use of similar triangles, these furnish a means of reading off the permissible load per square inch at once. An example will illustrate. A soft steel column whose $l/r$ is 120 is to have a maximum fiber stress of 16,000 pounds per square inch, what load per square inch will it carry?

FIG. 109.




$$\text{Ritter's Formula } p^1 = \frac{p}{1 + \frac{F}{m\pi^2 E} \times \left(\frac{l}{r}\right)^2}$$

The line at $l/r = 120$ crosses the curve at 41 per cent.; following down the 41 per cent. ordinate it crosses the diagonal marked 16,000 at a point 6500 above the horizontal axis, hence $p_1 = 6500$.

In the diagram a heavy line is intended to draw attention to the fact that general practice limits $l/r$ not to exceed 140.

Higher values should only be used with care and discretion, *i.e.*, when perhaps the actual load is but a small fraction of that allowed by the curve and the piece infrequently loaded and then not subjected to shock.

## Beams.

The general formula applicable to the theory of beams is, $M=\frac{pI}{e}$

where M = bending moment in inch-pounds.

$p$ = maximum working fiber stress in pounds per square inch.

I = moment of inertia of section about which bending is taken.

$e$ = distance from neutral axis to extreme fibers in inches.

$I/e$ is commonly called the section modulus, and given in manufacturers' hand-books for the usual rolled sections.

For cast, special or built-up sections it must be calculated. The table p. 11 gives data concerning bending moments, deflections, etc., for several classes of beams and different loadings, also the properties of the usual geometrical sections.

In a beam of common type either uniformly loaded or loaded with concentrated loads, one flange will be in compression.

The flange in compression ordinarily being long compared to its width, acts as a column, hence unless it is supported laterally at intervals the fiber stress will have to be reduced.

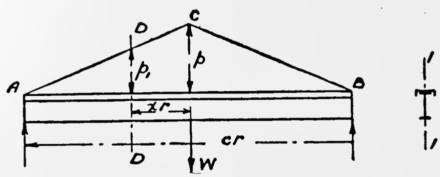



FIG. 110.

In ordinary structural specifications it is customary to limit the laterally unsupported width of flanges to 1/20 the span and up to this point to permit the same flexural stress in tension and compression. Beyond this limit the fiber stress is reduced in accordance with some column formula.

So far as the writer is aware no very satisfactory discussion of the lateral strength of beams exists. The following is offered as in a measure filling the deficiency:

Let AB be a beam supported at the ends and carrying a central load W. This will produce an extreme flexural fiber stress at the middle of $p$ and the extreme fiber stress will vary across the beam as shown by the triangle ABC so that the extreme fiber stress at any section DD will be $p_1$. Now if the beam is of considerable depth and the flange not of unusual thickness the horizontal force acting on the flange area may be assumed at

$$\text{Flange area} \times p_1 = F.$$

The compression flange will tend to buckle, due to the action of this force F, and the increase in the stress at the middle of the beam can be estimated by a proper column formula.

If $r$ = radius of gyration of the flange about an axis at right angles to the flange width. In the section drawn $r$ is the radius of gyration of the channel about 1-1.

$c = l/r$, where $l$ is the length of the beam.

$xr$ = distance of the section from the middle of the beam.

$p_m$ = maximum fiber stress at the center of the beam due to combined beam and column action.

$p_c$ = fiber stress in the flange at the middle of the beam due to column action.

$$p_1 = \frac{p_c}{1 + k\left(\frac{l}{r}\right)^2} \quad \text{General form of column formula.}$$

$k$ = constant depending upon character of ends, coefficient of elasticity, and strength of material at the elastic limit.

From the figure $$p_1 = p\left(1 - \frac{2x}{c}\right)$$

Since $$p_1 = \frac{p_c}{1 + k\left(\frac{l}{r}\right)^2} = \frac{p_c}{1 + 4kx^2}$$

It follows that the increase in stress due to this column action at the center of the beam span is

$$p_c - p_1 = p_1(1 + 4kx^2) - p^1 = 4kx^2 p_1$$

and $$p_c - p_1 = p\left(4kx^2 - \frac{8k}{c}x^3\right)$$

Determining the distance $x.r$ that will make this a maximum by putting $\frac{d(p_c - p_1)}{dx} = o$ we find $x = \frac{1}{3}c$.

The maximum fiber stress at the middle of the beam then is

$$p_m = p_c - p_1 + p = \left(1 + 4kx^2 - \frac{8k}{c}x^3\right)p$$

Since for the maximum value $x = \frac{1}{3}c$

$$\frac{p_m}{p} = 1 + \frac{4}{27} kc^2 \text{ or } p = \frac{p_m}{1 + \frac{4}{27} kc^2}$$

Here $p$ is the maximum flexural fiber stress for which the beam should be designed, the expected total maximum being $p_m$. In a similar manner the following formulæ can be derived.

Beam centrally loaded when no stiffening beam or channel is used and flange has a width $b$.

The maximum column effect will be produced in the flange when the column is assumed as ⅔ the length of the beam.

If $l = c_1 b$

$$p = \frac{p_m}{1 + \frac{48}{27} kc_1^2}.$$

Cantilever beam, load at end.

First, with stiffening beam. The maximum column effect is produced by a column ⅔ the length of the beam.

$$p = \frac{p_m}{1 + \frac{4}{27} kc^2}$$

Where no stiffening beam is used and only the width $b$ of the flange is considered, as before $l = c_1 b$

$$p = \frac{p_m}{1 + \frac{48}{27} kc_1^2}$$

Beam with uniform load, stiffening beam used, $r$ being the radius of gyration of this stiffening beam about its axis 1-1 as before. The column producing maximum effect will be $\frac{7}{10}$ the length of the beam.

$$p = \frac{p_m}{1 + \frac{1}{4} kc^2}$$

When no stiffening beam is used and $l = c_1 b$, $b$ as before being the flange width,

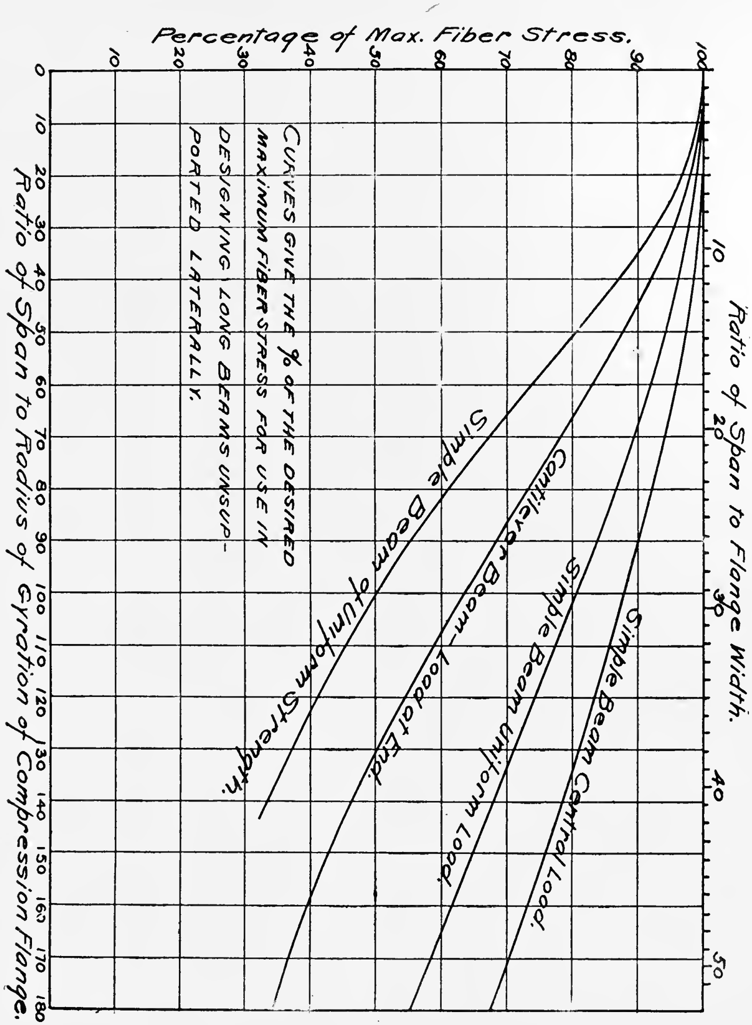
$$p = \frac{p_m}{1 + 3kc_1^2}$$

The following curves illustrate the reduction in fiber stress that are made with the following assumptions.

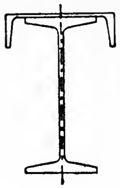
Ritter's formula $\quad p_1 = \dfrac{p_c}{1 + \dfrac{F}{m\pi^2 E} \times \left(\dfrac{l}{r}\right)^2}$

For a beam supported at ends, $k=\frac{F}{m\pi^2E}\backsim\frac{1}{10{,}000}$

For a cantilever beam, $m=\frac{1}{4}$ and $k\backsim\frac{1}{2{,}500}$




**Problem.**—The following built-up section is used to carry a central load on a span of 24 feet; what should the maximum fiber stress in flexure be so that the combined maximum fiber stress shall not exceed 12,000 pounds per square inch?



←7″ channel 14.75 pounds.

←10″ I beam 25 pounds.

$r$ of channel axis $1-1=2.50$.

$$c=\frac{l}{r}=\frac{24\times 12}{2.50}=115.$$

$$p=\frac{p_m}{1+\frac{4}{27}kc^2}=\frac{12000}{1+\frac{4\times 115^2}{27\times 10{,}000}}=10{,}150 \text{ pounds.}$$

If the 10-inch I beam had been used upon this span without the stiffening beam, what fiber stress could have been carried? The flange width is 4.66 inches.

$$p=\frac{12000}{1+\frac{48}{27}\times\frac{62^2}{10000}}=\frac{12{,}000}{1.683}=7150 \text{ pounds.}$$

## Frames for Hoists.

The simplest frames are those attached to walls or posts and used to carry hoists.

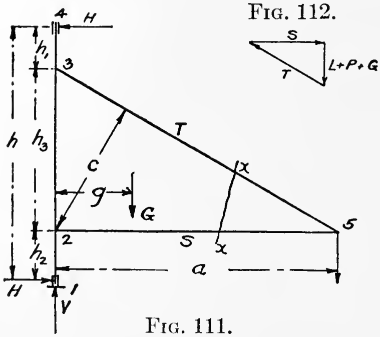



Fig. 111.

Fig. 112.

L = load being raised in pounds.
P = pull in pounds to raise load.
G = weight of frame in pounds.
$G_1$ = load in pounds acting at 5 and producing the same moment on part P as the frame.
H = horizontal reaction in pounds against bearings.
V = vertical reaction in pounds on step bearing.

$g=$ approximately $\frac{a}{4}$ to $\frac{a}{5}$.

$$G_1=\frac{Gg}{a}.$$

Determining the stresses algebraically, we have by taking moments around 1

$$H=\frac{(L+P)\,a+Gg}{h}$$

$$V=L+P+G.$$

Equating internal and external moments to the right of the section *xx*, by taking moments about 2

$$T=\frac{(L+P+G_1)a}{c} \quad \text{(tension)}$$

Similarly taking moments about 3

$$S=\frac{(L+P+G_1)a}{h_3} \quad \text{(compression)}$$

The stresses S and T are also readily found by Fig. 112. P is the vertical loads L, P and G; T and S are drawn respectively parallel to T and S in Fig. 111. The magnitude of the stresses is determined by measuring S and T by the same scale to which P is laid-off.

The post is subjected to the direct compression $V = L + P + G$ and also to bending due to the horizontal forces H. Analyzing the stresses in the post, the bending at 2 is

$$M_2=H\times h_2$$

and at 3 $\quad M_2=H\times h_1$

If $\frac{I}{e}$ =section modulus of the post section ıeferred to an axis through its center of gravity and at right angles to the force $H_1$.

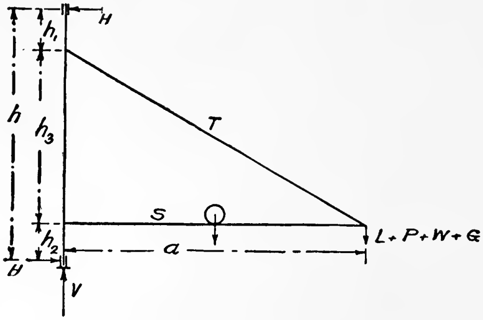
$A$ = the area of the section in square inches.

$p_3$ and $p_2$ = the fiber stresses at 3 and 2 respectively.

Then
$$p_3=\frac{H\times h_1\times e}{I}+\frac{V}{A}$$

and
$$p_2=\frac{H\times h_2\times e}{I}+\frac{V}{A}.$$

In other cases the hoist is suspended from a trolley which allows the load to be moved along the strut. Here the maximum stresses in the tie and the post will be found similarly to the frame just described, while the stress in the strut S can be obtained as follows:




Let W = weight of the trolley in pounds.

$$S=\frac{(L+P+W+G)a}{h_3}$$

The bending moment under the load at the middle of $a$ is

$$M=\frac{(L+P+W)a}{4}$$

The maximum compression fiber stress can then be assumed as approximately

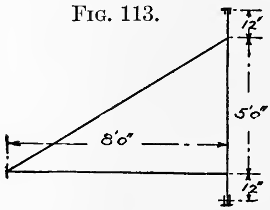
$$p_1=\frac{S}{2A}+\frac{Me}{I}$$

Here A = area of strut in square inches.

$\frac{I}{e}$ = section modulus of section referred to axis through center of gravity and at right angles to load.

**Problem.** — Design a wall bracket to carry a load of 4300 pounds at any point along the strut. This load includes the useful load, the trolley and hoist weight and chain pull to raise load. The weight of the frame will be assumed at 600 pounds.

FIG. 113.




Trying a 7-inch I beam 15 pounds per foot, $\frac{I}{e}=10.4$; area = 4.42 square inches; flange width = 3.66 inches;

$$\frac{\text{Span}}{\text{Flange width}}=\frac{96}{3.66}\backsim 26$$

and according to curves p. 111, the allowable working fiber stress should not exceed 90 per cent. of the maximum stress desired. If the maximum fiber stress desired is 12,000 pounds the allowable working fiber stress is $12000\times0.90=10{,}800$ pounds per square inch.

The direct stress in the strut S is $S=\frac{7840}{4.42}=1770$ pounds.

The combined direct compression and bending stress is

$$p_1 = \frac{S}{2A} + \frac{M_e}{I}$$

$$p_1 = \frac{7840}{2 \times 4.42} + \frac{4300 \times 96}{4 \times 10.4} = 10{,}805 \text{ pounds.}$$

Stress in tie T—. The equivalent weight of the frame at 5 is

$$G. = \frac{600}{4} = 150$$

$$T = \frac{(4300 + 150) \times 8}{4.25} = 8380 \text{ pounds.}$$

Making this of two round rods and allowing a fiber stress of 10,000 pounds per square inch at the root of the threads will require an area here of 0.84 square inch or will take 2—¾ inch diameter rods.

Post $H = \frac{(L + P + W + G.) \times a}{h_1 + h_3 + h_2} = \frac{4450 \times 8}{7} \backsim -5100$ pounds.

$$M_2 = M_3 = 5100 \times 12 = 61{,}200 \text{ inch-pounds.}$$

Direct load $= V = L + P + W + G = 4300 + 600 = 4900$ pounds.

If the fiber stress is not to exceed 12,000 pounds when properly reduced by Ritter's formula, the maximum allowable fiber stress is:

$$p_1 = \frac{12000}{1 + \frac{1}{10000} \times \left(\frac{l}{p}\right)^2} = \frac{12000}{1 + \frac{30^2}{10000}} = 11{,}000 \text{ pounds.}$$

The channels will be latticed. Trying 6-inch channels 8 pounds per foot, area $(2 \times 2.38) = 4.76$ square inches. Section modulus $= \frac{I}{e} = 8.6$.

Radius of gyration 2.34.

Fiber stress due to bending $\frac{61{,}200}{8.6} \backsim 7120$ pounds.

Direct fiber stress $\frac{4900}{4.76} = 1030$.

Combined fiber stress $= 7120 + 1030 = 8150$ pounds.

The 5-inch channels at 6½ pounds give a combined fiber stress of about 11,500 pounds, or slightly in excess of that to be allowed, so that 6-inch channels at 8 pounds have been chosen.

## Frames.

In fully considering the stresses produced in a frame we must not only consider the external forces acting upon the frame but also the forces acting on its individual members due to the method of applying the external forces to the frame and of the forces required by the several crane motions. The following instances will illustrate these statements:

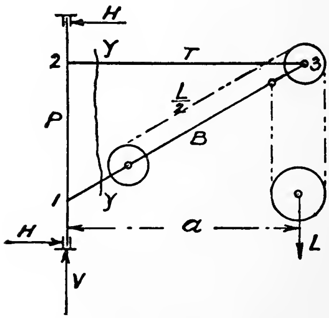



Fig. 114.

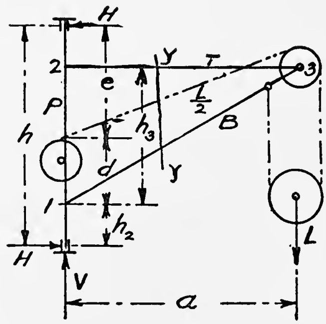



Fig. 115.

In Fig. 114 the load L produces the horizontal reaction H and the vertical reaction V. These external forces create the direct stresses in T, B and the bending and direct stresses in P. Neglecting friction the pull $\frac{L}{2}$ required to hold the load L modifies the direct stress in B and produces bending in it also. The stress in T, however, is unaffected by $\frac{L}{2}$. This can be seen from the fact that the section *yy* can be drawn cutting T and B and not cutting $\frac{L}{2}$, hence the internal moment due to the forces in T and B about any point must equal the external moment due to L about the same point (equating moments to the right of the section). It is seen that the forces T, B and P are inter-dependent but are independent of the pull $\frac{L}{2}$.

Taking moments about 1 we have

$$(B \times 0) - (T \times \overline{12}) + L \times a = 0 \therefore T = \frac{L \times a}{\overline{12}}.$$

In the second case, Fig. 115, the pull $\frac{L}{2}$ is held by the drum attached to P; in this case any section *yy* cuts T, B and $\frac{L}{2}$, showing that they are inter-dependent. The stresses in T and B can be found algebraically as above or graphically as follows. Since the sum of the external moments must be zero

$$H \times h = L \times a \qquad \therefore H = \frac{L \times a}{h}$$

Bending moment at 2 is $-M_2 = -H \times h_1$

Bending moment at 1 is $+M_1 = +H \times h_2$

The bending moment due to the horizontal component of $\frac{L}{2}$ on the post P is $-M = -\frac{F \times d \times e}{h_3}$.

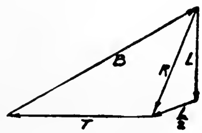
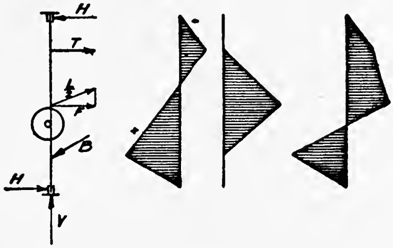



Fig. 116.

Fig. 116 shows the determination of the forces acting at the apex point 3.

Fig. a shows the mast with the external forces acting on it.

Fig. b is a diagram representing the bending moment due to the forces H.




a b c d

Fig. c shows the bending moment due to the horizontal component of $\frac{L}{2}$.

Fig. d is the sum of diagrams b and c. From diagram d the extreme fiber stress of any trial section can be found, since $p = \frac{Me}{I}$, and to this the direct stress at the corresponding point of P can be added, thus giving the total fiber stress.

## Jib Cranes.

Three types of jib cranes are illustrated.

Fig. 117.—This is the commonest and simplest type and can generally be constructed at lower cost than the other two; the objections to it are its limited reach and insufficient clearance under the jib.

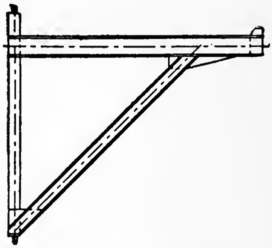


Fig. 117.—Single Under-Braced Jib Cranes.

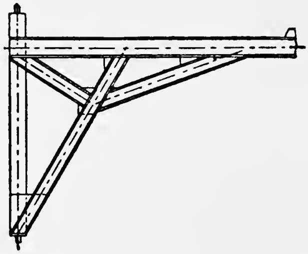


Fig. 118.—Triple Under-Braced Jib Cranes.

Fig. 118.—In this type of crane the objections raised to No. 117 are reduced, as it is suited to longer radii and affords increased clearance under the jib.

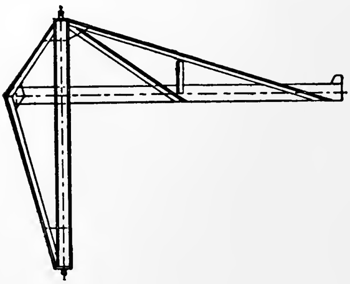


Fig. 119.—Top-Braced Jib Cranes.

Fig. 119.—This type is applicable where considerable head room is to be had. It practically makes available the entire floor space under the crane.

The determination of the frame stress will now be considered.

## Crane Stresses Determined Algebraically.

Nomenclature:

$L$ = live load, block, trolley and tackle, in pounds.

$W$ = total weight of crane frame, pounds.

$W_s$ = total weight of strut, pounds.

$W_2$ = vertical reaction of struts on jib J.

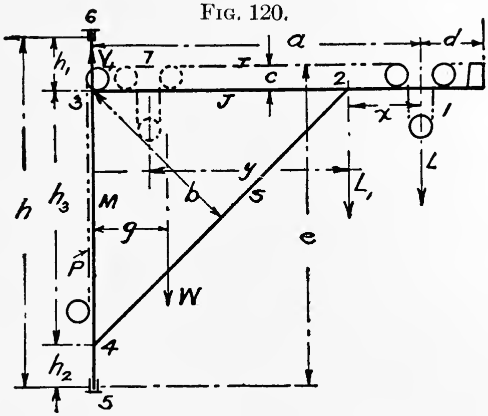
$g$ = distance from center line of post to center of gravity of crane frame.

$T$ = pull in pounds on rope.

$H$ = horizontal forces acting at top and bottom of post.

M = bending moment in inch-pounds, on sections as specified by subscripts.
$p$ = fiber stress, pounds per square inch.
$p_t$ = fiber stress, pounds per square inch, in tension.
$p_b$ = fiber stress, pounds per square inch, in flexure.
$p_c$ = fiber stress, pounds per square inch, in compression.
I = moment of inertia.
$e$ = distance from center of gravity to extreme fibers in inches.
A = area of section in square inches.

FIG. 120.




First find the center of gravity of the crane frame. To determine the distance $g$ take moment about the point 4.

$$g=\frac{1}{2\,\mathrm{W}}\left[w(a+d)^2+\mathrm{W}_s\left(\frac{a-x}{2}\right)\right]$$

Here $w$ is the weight per lineal foot of the jib. The reaction at 2 due to the crane weight equals

$$\mathrm{W}_2=\frac{\mathrm{W}g}{a-x}$$

The stress S in the strut is found by cutting members J and S and taking moments about point 3, then

$$\mathrm{S}=\frac{(\mathrm{L}\times a)+\mathrm{W}_2(a-x)-\mathrm{T}\times c}{b}$$

If the pull in the rope or chain is not considered, then T.$c$ drops out.

Stress in the Jib.—To determine the stress in the jib cut members J and S and take moments about 4, then

$$J = \frac{(L \times a) + W_2(a - x) - (T \times e_1)}{h_3}$$

Here again if the pull T is not considered $T \times e_1$ drops out.

The jib in this case is also subjected to bending both in the position of the load shown and as the trolley carrying the load is moved toward the mast. The maximum bending moment when the load is fully out is

$$M_1 = L \,.\, x - T \,.\, c.$$

The maximum bending when the trolley is located centrally on 23 is

$$M_8 = \frac{L(a - x)}{4} + T \,.\, c.$$

The maximum fiber stress then will be for the load at 1

$$p = p_b + p_t = \frac{M_1 e}{I} + \frac{J}{A}$$

The maximum fiber stress when the load is centrally on 23 will be

$$p = p_b + p_t = \frac{M_8 e}{I} + \frac{J_8}{A}$$

The stress $J_8$ here is that corresponding to the load at the middle and is given by

$$J_8 = \left[L\left(\frac{a - x}{2}\right) + W_2(a - x) - T \,.\, e_1\right] \div h_3$$

In designing this member the combined fiber stress in compression $p_b - p_t$ must be reduced to provide for the proper lateral strength (see p. 111).

Stress on Mast.—The stress upon the mast is first taken for the load fully out, and then as close to the mast as it travels. To find H take moments about point 6

$$H = \frac{(L \times a) + (W \times b)}{h}$$

To find $V_1$ take moments about 2, members M and S being cut, by which

$$V_1 = \frac{L\,x + W\,g - H\,h_1}{a - x} - P - W.$$

When the trolley and the load are as close to the mast as they can be pulled, then

$$H_{11} = \frac{L(a - x - y) + W\,g}{h}$$

and $$V_1 = \frac{(H_{11} \times h_1) - Ly + Wg}{a - x} + P + W.$$

The maximum stress at the foot of the crane will be

$$p = p_b + p_c = \frac{H \times h_2 \times e}{I} + \frac{L + W}{A}$$

Here I/e = section modulus of the mast.
A = area of mast section.

If $h_1$ is greater than $h_2$ it may happen that the bending at this point 3 may be greater than that at 4 and hence this may determine the mast section.

## Graphical Method of Determining Crane Stresses.

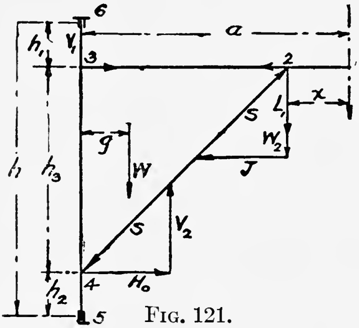
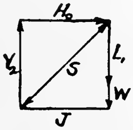



FIG. 121.

FIG. 122.




The graphical method of determining crane stresses possesses the advantages of being both simpler and making a better record of the problem.

First find the equivalent load at 2

$$L_1 = \frac{L \times a}{a - x}$$

To this must be added the dead load at 2, equal to

$$W_2 = \frac{W \times g}{a - x}$$

Now the sum of the two forces $L_1 + W_2$ acting at 2 is held in equilibrium by the forces in the jib and strut; completing the force triangle by drawing a line from the extremity of $L_1 + W_2$ parallel to the jib until it intersects the brace gives these stresses J and S. If the 3 forces are in equilibrium they must act around the triangle in the same sense, *i.e.*, here clock-wise. These arrows placed properly at the point 2 show the direction of the forces and the consequent character of stresses at this point. S pushes against 2 and must be in compression. J pulls on 2 and must be in tension. The diagram is completed as shown in Fig. 122.

To find the bending moment on the mast the forces H acting at the bearings must be first obtained.

$$H = \frac{H_o \times h_3}{h}$$

The stresses on the several parts can now be represented.

Fig. 123 represents the fiber stress due to bending and direct stress upon the mast, at the top

$$p_b = \frac{H_o \times h_1 \times e}{I}$$

and similarly at the bottom

$$p_b = \frac{H_o \times h_2 \times e}{I}$$

I/e being the section modulus of the mast section, whose area is = A

$$V_1 = \frac{L\,x}{a - x} - \frac{W\,(a - g - x)}{a - x}$$

$$p_t = \frac{V_1}{A}$$

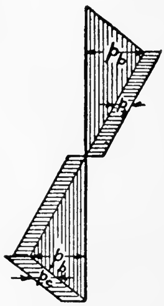


Fig. 123.

Fig. 124.

Fig. 124 represents the mast with the load moved as close to it as possible, assumed as directly over it, and the bending due to the frame weight neglected.

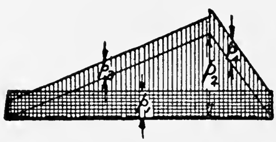


Fig. 125.

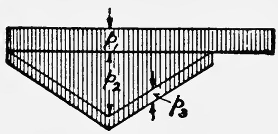


Fig. 126.

Fig. 125 shows the fiber stress on the jib. $p_1$ is the fiber stress due to the bending moment resulting from the rope or chain pull.

$p_2$ is the fiber stress due to bending resulting from the full load $a$ feet from the mast.

$p_3$ the algebraic sum of tension in jib due to full load at the outer position and the chain or rope pull, this result being divided by the area of the jib section.

$p_4$ pull in chain or rope divided by the area of the jib section. Here the bending due to the chain or rope pull being opposite to that due to the load, the fiber stresses $p_2$ and $p_1$ are subtracted.

Fig. 126 is a diagram similar to Fig. 125, drawn for the load in the middle of the distance $(a-x)$. Here the bending moments due to the chain or rope pull and the load L are in the same direction and the resulting fiber stresses are added.

A few examples will illustrate the general methods suggested.

**Crane Frames.**

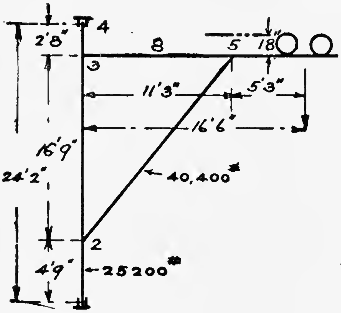



Fig. 127.

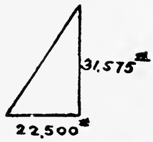



Fig. 128.

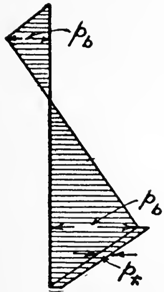



Fig. 129.

**Example.**—First make a rough estimate of the weight of the frame based upon the following assumptions:

Capacity 10 tons. Jib weighs 80 pounds per foot.

Post and strut weigh 50 pounds per foot.

The vertical reaction at foot of mast equals the frame weight.

| | | |
|---|---|---|
| Jib | $18.5 \times 80 =$ | 1480 pounds. |
| Strut | $20 \times 50 =$ | 1000 pounds. |
| Post | $24 \times 50 =$ | 1200 pounds. |
| | | 3680 |

The vertical dead load reaction at 5 is due to the jib and ½ the weight of the strut.

$$r_5 = \frac{1480 \times 9.25}{11.25} + \frac{1000}{2} = 1720 \text{ pounds.}$$

The vertical live load reaction at 5 is

$$R_5 = \frac{(20000 + 1520) \times 16.5}{11.25} = 31{,}575 \text{ pounds.}$$

The 1520 pounds is the assumed weight of the trolley, the upper and lower blocks and the tackle. The horizontal reactions are (see p. 120),

$$H = \frac{[W_J \times \frac{1}{2}(a + d)] + [W_s \times \frac{1}{2}(a - x)] + (L \times a)}{h}$$

$$H = \frac{(1480 \times \frac{1}{2} \times 16.5) + (1000 \times \frac{1}{2} \times 5.62) + (21520 \times 16.5)}{24.16}$$

$$H = \frac{12{,}210 + 2810 + 355{,}080}{24.16} = 15{,}300 \text{ pounds.}$$

The direct stresses in the post below the strut will be the sum of the dead and live loads. Direct stress, post, $=3680+1520+20000=25{,}200$ pounds.

Fig. 128 gives the direct stress in the frame due to combined dead and live load. The bending stresses in the several frame members will be found algebraically. Assuming the load carried by 4 chains, and that 2 of these chains run to the hoisting drum, neglecting friction and adding 1520 pounds for the weight of the block, trolley and tackle, the pull T in the 2 chains parallel to the jib is $\frac{2 \times 21520}{4} = 10{,}760$ pounds

The bending moment at 2 due to the cantilevered portion of the jib is (see p. 120),
$M_2 = Lx - T_1c = (21520 \times 63) - (10760 \times 18) = 1{,}162{,}080$ inch-pounds.

The bending due to hoisting-chain pull and that due to the chain pull to move the trolley will both increase the bending at 8 when both hoisting chain and trolley wheels are above the jib center. At 8 the bending moment at the middle due to hoisting-chain pull is

$$M = \frac{10760 \times 18}{2} = 96{,}840 \text{ inch-pounds.}$$

To find the bending due to the other chain pull, the amount of this pull must be determined. It is equal to the force required to overcome rolling and journal friction of the trolley plus the force required to move the trolley and block sheaves through the loaded chain, as the trolley is moved upon the jib.

Force for trolley friction, wheels being taken 6 inches in diameter and the axle journals 2¼ inches in diameter.

$$F = \frac{(f + \mu r)\,W}{R} = \frac{[.003 + (0.08 \times 1.125)]\,20{,}000}{3} = 620 \text{ pounds.}$$

Force to move sheave wheels through tackle, assume efficiency of 2 fixed blocks at 96 per cent. each, and the efficiency of the floating block at 98 per cent.

Combined efficiency $= .96^2 \times .98 = 90$ per cent.

$$\text{Pull} = \frac{20000}{2 \times .90} = 11{,}100 \text{ pounds.}$$

Total force to move trolley $620 + 1100 = 1720$ pounds.

The bending due to this force is $M = 1720 \times 7.5 = 12{,}900$ inch-pounds.

The total bending on the jib at 8 is

$$M_8 = \frac{21520 \times 135}{4} + \frac{10760 \times 18}{2} + \frac{1720 \times 7.5}{2} = 836{,}040 \text{ inch-pounds.}$$

The section for the jib can now be selected. It will be braced laterally at 5. This divides it into 2 parts, making one a cantilevered beam and the other part 53 a simple beam supported at its ends. Try 2—15 inch channels 45 pounds per foot, having a flange width of 3.62 inches, which makes $\frac{\text{length of span}}{\text{flange width}} = \frac{l}{b}$.

Cantilevered portion $\frac{l}{b} = \frac{63}{3.62} = 17.4.$

Simple beam $\frac{l}{b} = \frac{135}{3.62} = 37.3.$

If the maximum allowable fiber stress is 14,000 pounds per square inch, then, from the curves p. 111, the reduced working fiber stresses become:

Cantilevered beam $\frac{l}{b} = 17.4.$ $p^1 = 14000 \times .82 = 11{,}500$ pounds.

Supported beam $\frac{l}{b} = 37.3.$ $p^1 = 14000 \times .80 = 11{,}200$ pounds.

The bending moment being much greater on the cantilevered beam it will determine the section.

2 – 15 inch channels 45 pounds – $2 \times 13.24$ square inches = 26.48 square inches.

Direct unit stress $\frac{22500 - 10760}{26.48} = 445$ pounds.

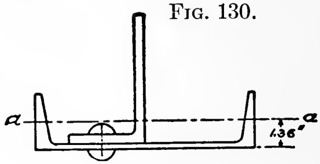
Allowable flexural fiber stress $11500 - 445 \sim 11{,}000$ pounds.

The resistance required for bending is $\frac{I}{e} = \frac{M}{p} = \frac{1{,}162{,}080}{11000} = 105.6.$

This being for 2 channels, the resistance for 1 channel is $\frac{105.6}{2} = 52.8$; this demands 2—15 inch channels 50 pounds per foot.

**Strut.**—The strut is subjected to 40,400 pounds stress, and since 2 channels are to be used the load is 20,200 pounds on each channel. The strut is 20 feet long. Assuming $\frac{l}{r} \leqq 140$, it will be necessary to use channels reinforced laterally. Assuming that the channels can be tied across for a distance of 2 feet from the post, this makes the column length 18 feet = 216 inches. The best section will have to be found by trial. We will try 1—10 inch channel 15 pounds per foot, 4.46 square inches, web thickness 0.24 inches. I = 2.30, $x$ = 0.64 = back to center of gravity, and 1—6×3½×⅜ inches angle at 11.7 pounds—area 3.43 square inches, I = 3.34, $x$ = 2.04 (back of short leg to center of gravity).

Fig. 130.




The center of gravity, moment of inertia and radius of gyration of the composite section must now be found.

*Center of Gravity:*

| | Area. | | Moments. |
|---|---|---|---|
| 1—10 inch channel ...... | 4.46×0.64 | = | 2.86 |
| 1—6×3½×⅜ angle ....... | 3.43×2.28 | = | 7.83 |
| | 7.89 | | 10.69 |

$$x = \frac{10.69}{7.89} = 1.36 \text{ inches.}$$

*Moment of Inertia:*

| | |
|---|---|
| Inertia of 10 inch channel .................. | 2.30 |
| Area×(distance between axes)² = 4.46×0.72².. | 2.32 |
| Inertia of 6×3½×⅜ angle .................. | 12.86 |
| Area×(distance between axes)² = 3.43×0.92.. | 2.92 |
| Inertia ........................ | 20.40 |

Radius of gyration axis $a-a$.

$$r = \sqrt{\frac{\text{I}}{\text{A}}} = \sqrt{\frac{20.4}{7.89}} = 1.61.$$

$$\frac{l}{r} = \frac{216}{1.61} = 134.$$

Allowable compressive fiber stress (see p. 103),

$$p = \frac{14000}{1 + \frac{1}{10000} \times \left(\frac{l}{r}\right)^2} = \frac{14000}{1 + \frac{134^2}{10000}} = \frac{14000}{2.8} = 5000 \text{ pounds.}$$

The total load this strut will carry is

$$P = 7.89 \times 5000 = 39{,}450 \text{ pounds.}$$

This is about twice the amount that will come upon it, but although it is a trifle heavy it will be used. Any single section would be much heavier.

*Post:* $M_2 = 15300 \times 57 = 872{,}100$ inch-pounds.
$M_3 = 15300 \times 32 = 489{,}600$ inch-pounds.

At 2 the section must resist a direct stress of 25,200 pounds and a bending moment of 872,100 inch-pounds. This assumes the hoisting and traversing drums attached to the post so as not to introduce bending of any consequence in it.

Trying 2–15 inch channels 33 pounds per foot.

$$\frac{I}{e} = 2 \times 41.7 = 83.4.$$

Area $= 2 \times 9.90 = 19.8$ square inches.

$$\text{Direct stress} = \frac{25200}{19.8} = 1275 \text{ pounds.}$$

$$\text{Flexural stress} = p = \frac{Me}{I} = \frac{872000}{83.4} = 10{,}450 \text{ pounds.}$$

Combined fiber stress $= 1275 + 10450 = 11{,}725$ pounds.

The fiber stress in the post when the crane carries its full load at the maximum radius is shown in Fig. 129.

When the trolley is back all the way the post will be subjected to very little bending but more direct stress. As it can be latticed, or the 2 channels properly tied together, the radius of gyration of the 2 channels will be the radius gyration of a single channel referred to its principal axis, *i.e.*, 5.62.

$$\frac{l}{r} = \frac{258}{5.62} \backsim 46.$$

According to Ritter's formula for a soft steel column the reduced fiber stress is:

$$p^1 = \frac{14000}{1 + \frac{1}{10000} \times \left(\frac{l}{p}\right)^2} = \frac{14000}{1 + \frac{46^2}{10000}} = \frac{14000}{1.21} = 11{,}600 \text{ pounds.}$$

As the direct stress is only 1275 pounds the post should prove amply strong.

## Under-Braced Jib Frame.

Capacity of crane 5 tons (10,000 pounds) at a radius of 20 feet. Assume the weight of the trolley, blocks, tackle and hook at 500 pounds. The equivalent load at $g$ is

$$L_1=\frac{10500\times ch}{cg}=\frac{10500\times 20}{16}=13{,}125 \text{ pounds.}$$

Fig. 132 gives the direct stresses in the frame when the load is at its maximum radius. The stress in CD according to this diagram is zero. The maximum stress will occur in this member

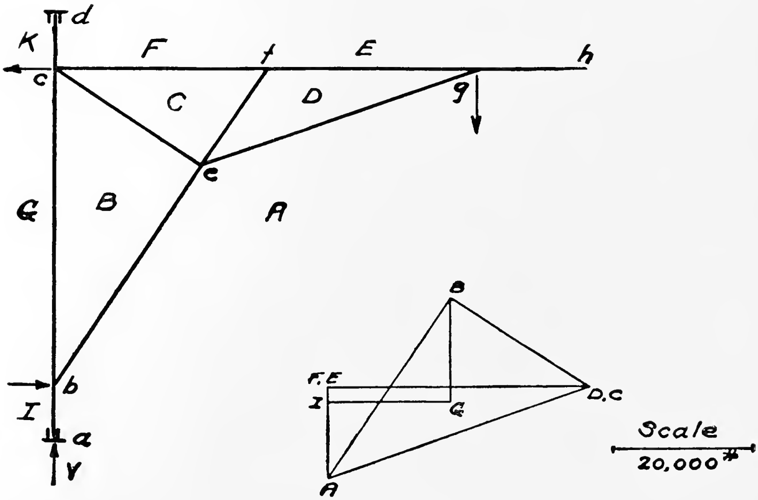



Fig. 131. Fig. 132.

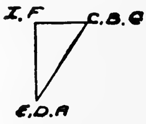



Fig. 133.

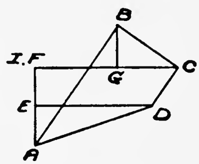



Fig. 134.

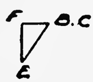



Fig. 135.

when the load is at $f$, and the magnitude of this stress is given by Fig. 133. The maximum stress on the jib will be that due to combined bending and tension. Fig. 134 gives the direct stresses when the load is midway between $f$ and $g$. Fig. 135 gives the stresses when the load is midway between $c$ and $f$.

Fig. 136 gives the stresses due to the weight of the frame members. Although the several frame members vary considerably in weight they have been uniformly assumed at 60 pounds per lineal foot. The apex loads have been taken at one-half the weight of the several members meeting at that point, thus the apex load at $e$ is ½ the weight of AB, BC, CD, and DA. This will be slightly in error for apex points $c$, $f$ and $g$ on the jib, but will be sufficiently accurate.

The estimated apex loads are:

| Apex | Load in pounds |
|---|---|
| $g$ | 810 |
| $f$ | 615 |
| $c$ | 1050 |
| $e$ | 975 |
| $b$ | 1000 |
| Total | 4450 |

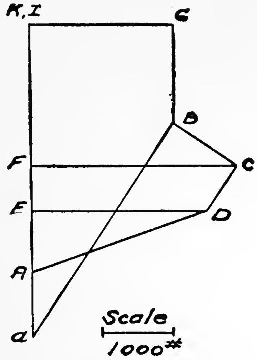



FIG. 136.

| Member | Dead Load Stress | Live Load Stress | Total Stress | Character of Stress |
|---|---|---|---|---|
| AB | 3600 | 31,200 | 34,800 | Compression |
| AD | 2600 | 39,000 | 41,600 | Compression |
| BG | 1400 | 15,000 | 16,400 | Tension |
| BC | 1050 | 23,800 | 24,850 | Compression |
| CF | 2950 | 37,000 | 39,950 | Tension |
| CD | 750 | 13,000 | 13,750 | Compression |
| DE | 2500 | 37,000 | 39,950 | Tension |
| *CF | 2950 | 20,600 | 23,550 | Tension |
| *DE | 2500 | 2,000 | 4,500 | Tension |
| *GK and GI | 2000 | 17,100 | 19,100 | |

*Direct forces in CF and DE when the load produces the maximum bending in these pieces.

$$V = 10000 + 500 + 4450 = 14,950.$$

$$H = \frac{19,100 \times 12}{16} = 14,325 \text{ pounds.}$$

Bending moment at apex points $b$ and $c$

$$M = 14325 \times 24 = 343,800 \text{ inch-pounds.}$$

Use a working fiber stress not exceeding 14,000 pounds per square inch.

Member AB, length 10 feet. Assuming that 8 feet of this cannot be braced laterally and that $\frac{l}{r} \leqq 160$, $r \geqq \frac{96}{160} = 0.60$.

Trying 2—10 inch channels at 4.46 square inches.

$$\frac{l}{r} = \frac{96}{0.72} = 133.5$$

According to Ritter's formula the working fiber stress will be:

$$p_1=\frac{14000}{1+\frac{133.5^2}{10000}}=5040 \text{ pounds.}$$

The 2−10 inch channels will then carry 2×4.46×5000=44,600 pounds. This is well above the load on the member of 34,800 pounds.

Member $\overline{AD}$−Length 11 feet 0 inch. Try 2−12 inch channels, $r=0.81\ \frac{l}{r}=\frac{120}{0.81}=148.$

Allowable working fiber stress according to Ritter's formula

$$p_1=\frac{14000}{1+\frac{148^2}{10000}}=4350 \text{ pounds.}$$

The load carried by 2−12 inch channels will be 2×6.03×4350=52,500 pounds, which is well above the 41,600 pounds coming on it.

Member $\overline{BC}$, length 6 feet 8 inches. Try 2−7 inch channels.

$$r=0.59, \frac{l}{r}=\frac{80}{0.59}=136.$$

Allowable working fiber stress according to Ritter's formula

$$p_1=\frac{14000}{1+\frac{136^2}{10000}}=4900 \text{ pounds.}$$

Load carried by 2 channels, 2×2.85×4900=27,900 pounds; the load actually coming on the channels is 24,850 pounds, so that these channels will be amply strong. The stresses and length of the member $\overline{CD}$ being less than $\overline{BA}$ the 10 inch channels used for $\overline{BA}$ will be continued for $\overline{CD}$. The jib $\overline{CF, DE}$ and the cantilever extension will be a continuous section, so that it will have to be designed to meet the worst conditions in the 3 spans, *cf*, *fg* and *gh*. Cantilever span 4 feet, the bending moment at *g* is

$$M=(10000+500)\times 4\times 12=504{,}000 \text{ inch-pounds.}$$

Direct stress 39,950 pounds.

Supported span *fg*−8 feet.

Bending moment at the center of the span

$$M=\frac{WL}{4}=\frac{10500\times 8\times 12}{4}=252{,}000 \text{ inch-pounds.}$$

Direct stress $\overline{CF}$ – 23,550 pounds.
Direct stress $\overline{DE}$ – 4500 pounds.

Evidently the cantilevered portion will determine the section, trying two 12 inch channels 20½ pounds per foot. The allowable compressive fiber stress corresponding to a ratio of

$$\frac{\text{Span}}{\text{Flange width}}=\frac{4\times 12}{2.94}=16.3.$$

From the curves p. 111, the working stress should not exceed 84 per cent. of the maximum desired, or $14000\times 0.84=11{,}750$ pounds.

$$\text{Section modulus }=\frac{504000}{2\times 11750}=21.5,$$

so that two 12 inch channels 20½ pounds per foot will be ample.

Post – $\overline{BG}$. The bending moment at b is 343,800 inch-pounds, the direct stress is 14,500 pounds.

Trying two 10 inch channels 15 pounds per foot, area of each being 4.46 square inches; section modulus 13.4.

$$\text{Direct stress }=\frac{14500}{2\times 4.46}=1620\text{ pounds.}$$

$$\text{Flexural fiber stress }=\frac{343{,}800}{2\times 13.4}=12{,}800\text{ pounds.}$$

$$\text{Combined stress }=1620+12800=14{,}420\text{ pounds.}$$

This is sufficiently close to that desired.

### Top-Braced Jib Crane Frame.

Design a top-braced frame for a jib crane to carry 3 tons (6000 pounds) at a radius of 25 feet, the jib to have a clearance above floor of 17 feet 6 inches.

The force polygon Fig. 138 gives the maximum stresses in all members but FG and the jib. The latter is subjected to combined bending and compression. The equivalent load that must be suspended at 2 to produce the same stresses upon the crane members as the load L (6000 pounds) at 1 is,

$$\frac{L\times \overline{14}}{\overline{24}}=\frac{6000\times 25}{23}=6500\text{ pounds.}$$

$$\text{The horizontal forces are, } H=\frac{6000\times 25}{25}=6000\text{ pounds.}$$

The maximum stress will occur in $\overline{FG}$ when the load is at point 3 and is given by Fig. 139.

Fig. 140 is the stress diagram corresponding to the load held at point 5.

The point 5 is 17 feet 3 inches from the mast and the horizontal forces $\overline{BC}$ and $\overline{CH}$ equal to

$$\frac{6000\times 17.25}{25}=4140 \text{ pounds.}$$

The reactions at 2 and 3 are each 3000 pounds.

**Direct Stresses in Pounds.**

| Member | Live Load | Dead Load | Combined | Character |
|---|---|---|---|---|
| *a*G | 20,000<br>* 3,000 | 1800 | 24,800 | Compression |
| AF | 20,000<br>* 3,000 | 3500 | 26,500 | Compression |
| DE | 20,000 | 3500 | 23,500 | Compression |
| CE | 23,000 | 4000 | 27,000 | Tension |
| EF | 24,600 | 7000 | 31,600 | Compression |
| FG | 11,000 | 1900 | 12,900 | Tension |
| GJ | 21,000 | 2000 | 23,000 | Tension |
| AD | 24,600<br>* 3,000 | 7000 | 34,600 | Compression |
| DC | 20,000 | 3500 | 23,500 | Tension |

* The chains or ropes are assumed as acting along the axis of the members and the load is carried by a floating block; neglecting friction the pull in the members AD, *a*G and AF due to the pull from the chain or rope is

$$\frac{6000}{2}=3000 \text{ pounds.}$$

Design of the Jib.—The worst condition will occur on *a*G when the load is at 5. The direct stress is given by Figs. 140 and 141.

| | |
|---|---|
| Live load stress in *a*G | 9200 pounds |
| Live load chain pull | 3000 pounds. |
| Dead load stress in *a*G | 1800 pounds. |
| | 14,000 pounds. |

The bending moment will be

$$M=\frac{WL}{4}=\frac{6000\times \overline{34}}{4}=\frac{6000\times 11.5}{4}=17{,}250 \text{ foot-pounds.}$$

Trying two 12 inch channels 20½ pounds, area 6.03 square inches each.

$$\text{Direct stress }=\frac{14000}{2\times 6.03}=1160 \text{ pounds.}$$

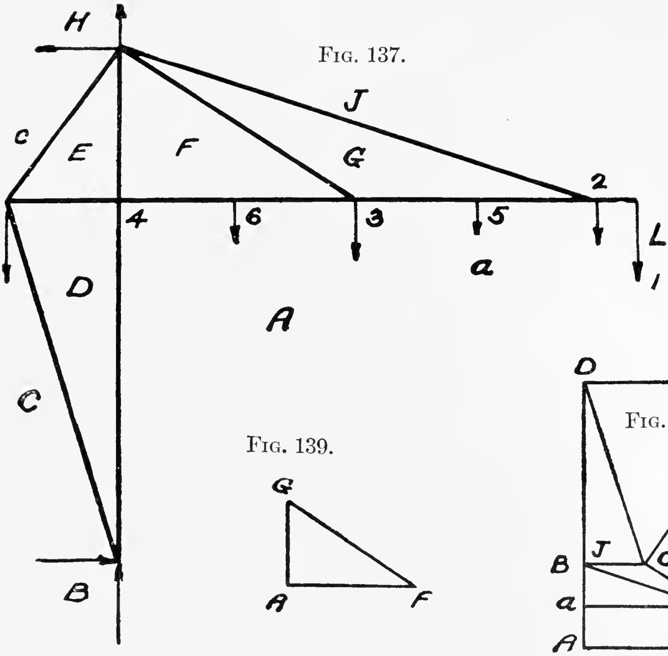



Fig. 137.

Fig. 139.

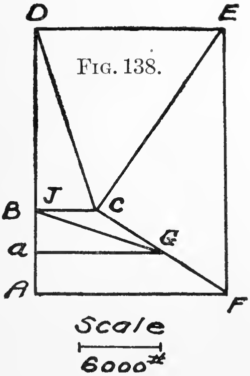



Fig. 138.

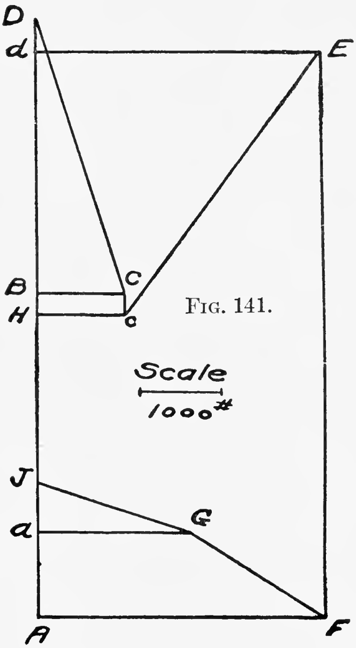



Fig. 141.

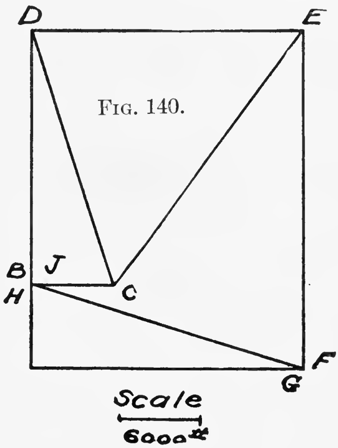



Fig. 140.

Considering the beam as affected by the relation between its span and flange width, we have

$$\frac{\text{Span}}{\text{Flange width}}=\frac{138}{2.6}=53.$$

The allowable fiber stress is given for a simple beam centrally loaded at

$$p=\frac{11000}{1+\frac{48}{27}kC_1^2}=\frac{11000}{1+\frac{48}{27}\times\frac{1}{10000}\times 53^2}=7350 \text{ pounds.}$$

Net allowable fiber stress 7350−1160=6190 pounds.

Since $$\frac{I}{e}=\frac{M}{p}=\frac{17250\times 12}{2\times 6190}=16.7.$$

The resistance of a 12 inch channel 20½ pounds is $\frac{I}{e}=21.4$ and is ample. The 10 inch channels 20 pounds per foot having an $\frac{I}{e}$ value of 15.7 are too light to fill the specified requirements. The cantilevered portion of the jib should be checked to see that the section is amply strong for it.

The bending moment is M=6000×24=144,000 inch-pounds. Direct stress 24,800 pounds.

$$\frac{\text{Span}}{\text{Flange width}}=\frac{24}{2.6}=9.25.$$

The allowable working fiber stress (see p. 111) is 0.90 of the maximum desired. 11,000×0.90=9900 pounds.

Direct unit stress $=\frac{24,800}{2\times 6.03}=2030$ pounds.

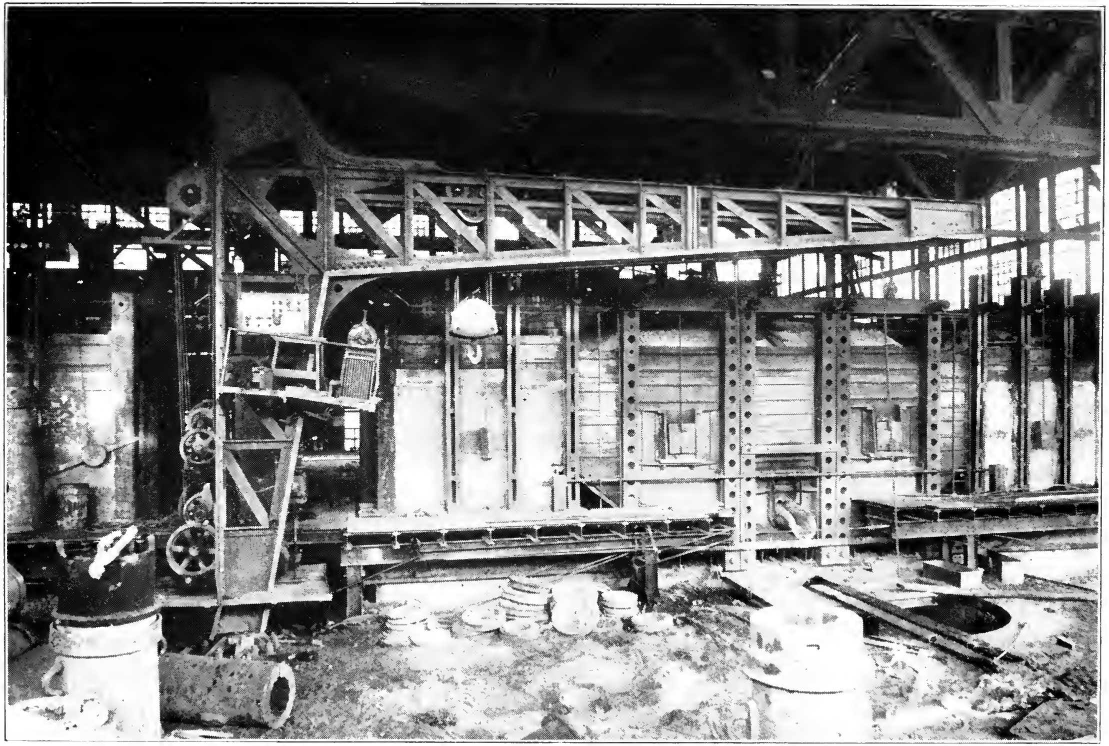
Allowable flexural fiber stress 9900−2030=7870 pounds.

Resistance required $$\frac{I}{e}=\frac{M}{p}=\frac{144,000}{2\times 7870}=9.15.$$

This being less than the resistance of the 12 inch channels used, the section is satisfactory.

Tension Pieces.—According to the diagram the maximum stress in a tension piece is 27,000 pounds and this requires a net section of $\frac{27000}{11000}=2.45$ square inches. Assuming material ¼ inch

Fig. 142.



Ten-ton Electric Jib Crane.

thick and ¾ inch diameter rivets with 2 rivets in line, the gross section would have to be 2.45+0.40=2.85 square inches, or 2 angles 1.43 square inches each. Hence 3½×2½×¼ inch angles will do for all tension pieces excepting FG, which can be 2½×2½×¼ inch angles, these being used because 2½ inches is about the smallest leg that should have ¾ inch diameter rivets driven in it.

Mast.—Assuming that the length 17 feet 6 inches is at least braced at 2 points, making the unsupported length 5 feet 10 inches =70 inches.

$$\text{If } \frac{l}{r} \text{ is limited to 120 then } r \geqq \frac{70}{120} = 0.59.$$

Trying two—9 inch channels at 13¼ pounds per foot.

According to Ritter's formula the allowable unit stress upon this column is

$$p_1 = \frac{11000}{1+\frac{1}{10000}\times\left(\frac{l}{r}\right)^2} = \frac{11000}{1+\frac{1}{10000}\left(\frac{70}{.67}\right)^2} = \frac{11000}{2.09} = 5250 \text{ pounds.}$$

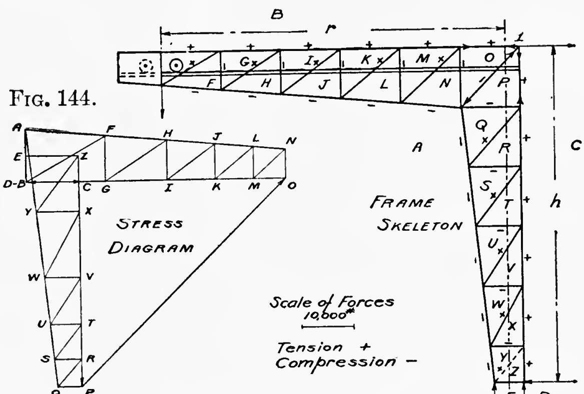
Total allowable load on two—9 inch channels 2×3.89×5250 =40,900 pounds.

The maximum stress on $\overline{AD}$ is 34,600 pounds and providing there is no bending introduced in the mast by the attachment to top or step bearing this should prove ample.

**Latticed Frame Jib Crane.**

Fig. 143.

Fig. 144.




Latticed Frame Jib Crane.

Where considerable clearance is required under a jib crane and the head room is limited a latticed frame crane can be used.

Fig. 142 illustrates a 10-ton electric jib crane, hook radius 30 feet, lift 18 feet, made by Pawling & Harnischfeger.

The skeleton of such a crane is shown in Fig. 143 and the stress diagram due to the live load is given in Fig. 144.

The principles used are those previously given under Graphics, page 76.

## Crane Girders.

### *Overhead Electric Travelling Cranes.*

There is considerable variation in the spans of cranes, depending upon the situation in which they are placed, inside cranes being limited by the building clearances, while outside cranes will vary somewhat with the materials they are to handle and the ground they are to cover. In general the spaces will vary from 20 to 60 feet and in some cases reaching 80 to 100 feet. Light cranes handle loads up to 10 tons, while heavier cranes are capable of raising loads of 50, 75 or even 150 tons (2000 pounds).

The wheel base of the bridge will average from $\frac{1}{6}$ to $\frac{1}{7}$ the span.

As I beams are rolled in depths up to 24 inches, and Grey Mill sections to 30 inches, rolled sections can be used for bridge girders on spans up to 20 or 30 feet.

To secure proper lateral stiffness the flange width of the section chosen should be at least equal to one-fortieth of the span. In addition to this the fiber stress allowed should be reduced to provide for the column action in the compression flange, for the limiting value of flange width given above the beam should be selected so that the flexural fiber stress due to the load acting vertically is only 75 per cent. of the maximum desired flexural fiber stress. (See p. 111.)

Girder, Rolled Section.—When a rolled section is used for a girder it is only necessary to determine the maximum bending on the girder. This is the sum of the bending moments due to the uniform dead load and the travelling or live load. In the case of the two-wheeled trolley the bending will be a maximum when the load is in the middle of the girder. The total bending moment is $M=\frac{Wl}{4}+\frac{Gl}{8}$.

M = total bending moment in inch-pounds.

$l$ = length of the girder span in inches.

G = weight of girder in pounds.

W = weight of live load in pounds. (Useful load, trolley, etc.)

If the bridge consists of two girders, then W will be ½ the total live load.

Four-Wheeled Trolley.—If the two wheel loads are equal and the wheels are $a$ inches apart the maximum bending due to live load will occur under a wheel when it is a distance $\frac{a}{4}$ from the center of the girder. The maximum bending due to the dead load occurs at the center of the girder. Usually no error of consequence results from assuming that the maximum bending on the girder is the sum of the two bending moments just given.

The resulting maximum bending moment then is,

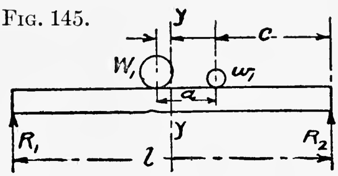
$$M=\frac{W}{8l}\left(l-\frac{a}{2}\right)^2+\frac{Gl}{8}$$

W = weight in pounds of the live load, including trolley, chains and block. Each wheel load then is $\frac{W}{4}$.

If the two wheel loads on the girder are not equal the maximum bending will occur when the center of the girder span bisects the distance between the center of gravity of the two wheel loads and the nearest wheel, the maximum bending occurring under this wheel.

Four-wheeled trolley, wheel loads unequal.

Fig. 145.




$$b=\frac{a}{2}\left(\frac{w_1}{W_1+w_1}\right)$$

$$c=\frac{l}{2}-a+b$$

$$M=\left[\left(\frac{w_1c+W_1(c+a)}{l}\right)\times(l-a-c)\right]+\frac{Gl}{8}$$

$W_1$ = the maximum wheel load on one girder, pounds.

$w_1$ = the minimum wheel load on one girder, pounds.

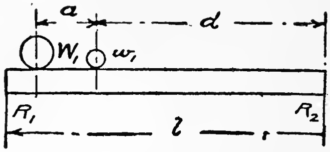
G = weight of one girder in pounds.

$b$ = distance of $W_1$ from center of girder.

All dimensions in inches, and representing the distances indicated on the drawing.

In girders of this type the reactions will usually be required only to determine the maximum wheel load of the bridge on the runway girder and can be determined by placing the loads to the right or left of the girder and then taking moments about the other support.

FIG. 146.




$$R_1=\frac{(w_1d)+W_1(d+a)}{l}+\frac{G}{2}$$

If the loads are unequal or do not approach the girder ends equally $R_2$ should be determined when the loads are close to that end, and the larger of the two reactions used. Inspection will usually suffice to determine which reaction will be required.

The depth of a beam girder will generally be $d \geqq \frac{l}{20}$. The fiber stress is commonly figured at 12,000 pounds per square inch. Having found the bending moment it is only necessary to equate the external bending moment with the resisting moment of the beam, *i.e.*,

$$M=\frac{pI}{e}$$

where M = external bending moment in inch-pounds.

$p$ = allowable fiber stress in pounds per square inch.

I = moment of inertia of the section (inches$^4$).

$e$ = distance in inches from the neutral axis to the extreme fibers.

**Problem.**—Select a beam suitable for a light crane girder whose span is 26 feet. The trolley wheel base is 4 feet and the trolley with live load weighs 14,000 pounds.

$$M=\frac{W}{8l}\left(l-\frac{a}{2}\right)^2+\frac{Gl}{8}$$

$$M=\frac{14000}{8\times 312}(312-24)^2+\frac{55\times 26\times 312}{8}$$

In assuming the weight of the girders the beam depth was taken as approximately $\frac{l}{20}$ or $\frac{312}{20}=15.6$ inches.

This suggested trying an 18 inch beam.

$$M=465{,}000+55{,}770=520{,}770 \text{ inch-pounds.}$$

The flange width of an 18 inch I beam is 6 inches, so that

$$\frac{\text{Span}}{\text{Flange width}}=\frac{312}{6}=52.$$

To allow for the compression in the upper flange (see p.111) the working fiber stress should be $12{,}000 \times .70 = 8400$ pounds.

Trying these values in the general equation for a beam,

$$M = \frac{pI}{e} \therefore \frac{I}{e} = \frac{M}{p} = \frac{520{,}770}{8400} = 62.0$$

According to manufacturers' hand-books, the value of $\frac{I}{e}$ for an 18 inch I beam weighing 55 pounds per foot is 88.4, and this beam should prove ample.

Built-up Girders.—Where girder depths greater than 30 inches are required or where the rolled girders have not sufficient flange width it becomes necessary to use sections made by building them up from the usual rolled sections.

Where the beam simply lacks flange width this can be cared for by riveting a beam or channel to the compression flange of the vertical beam. Thus suppose the span was 40 feet, the allowable fiber stress in tension 12,500 pounds. The central load is 18,000 pounds. If the girder depth is one-twentieth of the span the girder must be 24 inches deep. If the flange width is limited to 1/40 the span, its width must be 12 inches. The trial section then will be one 24 inch I beam 80 pounds, and one 12 inch channel 20½ pounds.

Fig. 147.

The center of gravity of this section must be found. Taking moments about the back of the channel.

| | Area. | | Lever Arm. | | Moment. |
|---|---|---|---|---|---|
| Channel | 6.03 | × | 0.70 | = | 4 22 |
| I beam | 23.32 | × | 12.28 | = | 286.37 |
| | 29.35 | | | | 290.59 |

$$a = \frac{290.59}{29.35} = 9.9$$

The moment of inertia of the section is then found as follows:

| | |
|---|---|
| Inertia of channel, axis parallel to web ..... | 3.91 |
| Area × square of distance from axis ........ $6.03\,(9.9-0.70)^2$ | 510.38 |
| Inertia of I beam, axis parallel to flange .... | 2087.20 |
| Area × square of distance from axis ........ $23.32\,(12.28-9.9)^2$ | 132.09 |
| Inertia = | 2733.58 |

$\frac{I}{e}$ for compression $\frac{2733.6}{9.9} = 276$ $\frac{I}{e}$ for tension $\frac{2733.6}{14.38} = 190.$

According to article p. 107 the allowable fiber stress in the compression flange is $12{,}000 \times 0.78 = 9360$ pounds. The bending moment due to the central load of 18,000 pounds plus that due to the uniform load of the girder weight taken at 100 pounds per foot of span is

$$M = \frac{18{,}000 \times 40 \times 12}{4} + \frac{4000 \times 40 \times 12}{8} = 2{,}400{,}000 \text{ inch-pounds.}$$

The maximum tensile stress then is

$$\frac{2{,}400{,}000}{190} = 12{,}600 \text{ pounds.}$$

This fiber stress is satisfactory.

The maximum compressive fiber stress is

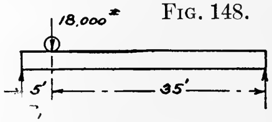
$$\frac{2{,}400{,}000}{276} = 8700 \text{ pounds,}$$

and this is satisfactory.

The rivet spacing for the rivets securing the channel to the I beam could be determined as shown hereafter (see p. 150), or can be done as follows:

The theoretical rivet spacing would be closest near the girder ends. As the function of the rivets is to transmit the change in flange force to the web, we can determine the change in flange force by finding the fiber stress a short distance, say 5 feet, from the end of the girder; and since the flange force is zero at the end of the girder, it is evident that the approximate flange force found 5 feet from the end of the girder represents the change in flange force.

FIG. 148.




$$R_1 = \frac{18{,}000 \times 35}{40} + \frac{40 \times 100}{2}.$$

$$R_1 = 17{,}750 \text{ pounds,}$$

The bending moment at this section

$$M = (R_1 \times 5 \times 12) - (5 \times 100 \times 2.5 \times 12)$$

$$M = (17.750 \times 60) - (500 \times 30) = 1{,}050{,}000 \text{ inch-pounds.}$$

The fiber stress in compression, since $p = \frac{Mc}{I}$, is

$$p = \frac{1{,}050{,}000}{276} = 3950 \text{ pounds.}$$

The approximate total flange force in the channel is

Area of the channel $\times 3950 = 6.03 \times 3950 = 23{,}820.$

The average change of flange force per inch of span in the first 5 feet is $\frac{23.820}{60}=397$ pounds. Allowing 9000 pounds per square inch in shear on the rivets we have: Area of ¾ inch rivets $\times 9000 = 0.44 \times 9000 = 3960$ pounds.

Rivet spacing for shear $= \frac{3960.}{397} \frown 9.97.$

The bearing strength of the ¾ inch rivets should also be considered. Allowing twice the bearing fiber stress that we allowed in shear would give an allowable bearing stress of 18,000 pounds per square inch. The web of the channel being thinner than the flange of the I beam, the bearing strength of the joint will depend upon the rivet bearing in the channel web.

Web thickness 0.28 inch, the projected rivet area is

$$0.75 \times 0.28 = 0.21 \text{ square inch.}$$

The bearing value of the rivet in the channel web then is

$$18{,}000 \times 0.21 = 3780 \text{ pounds.}$$

The rivet spacing based upon the bearing value is

$$\frac{3780}{397} = 9.52 \text{ inches.}$$

Usual practice would limit this rivet spacing to 6 inches, and for 12 inches to 18 inches from the ends of the beam only ½ of this spacing, or 3 inches, should be used.

## Crane Girders.

When in the case of built-up girders it is necessary to know the bending moments for its entire length, they are most readily obtained by the graphical methods previously explained. (See p. 78.)

An example here will illustrate. The trolley wheels are 5 feet apart. Each trolley wheel load is 6500 pounds, the bridge (2 girders) weighs 30,000 pounds, *i.e.*, each girder weighs 250 pounds per foot of span. Span 60 feet.

The girder with the trolley upon it is shown in Fig. 149. Beginning at the left support, 5 foot intervals have been laid-off on the girder. These have been numbered from 1 to 7, this latter being the middle of the span.

Fig. 150 is the force diagram, and $hg$ and $gi$ each represents a wheel load of 6500 pounds and has been drawn to a scale of 1 inch equals 10,000 pounds. The pole O has been taken so that O$g$ is at right angles to $hgi$ and the distance O$g$=8000 pounds taken to the same scale as the forces, *i.e.*, 1 inch=10,000 pounds.

In Fig. 151 $8x$ is drawn parallel to O$g$ in Fig. 150, intersecting the lines $\overline{KK}$ and $\overline{LL}$, which are the direction of the wheel load forces acting upon the girder. 1–8 is parallel to O$h$ and $x1^1$ is parallel to O$i$, each being drawn through 8 and $x$ respectively.

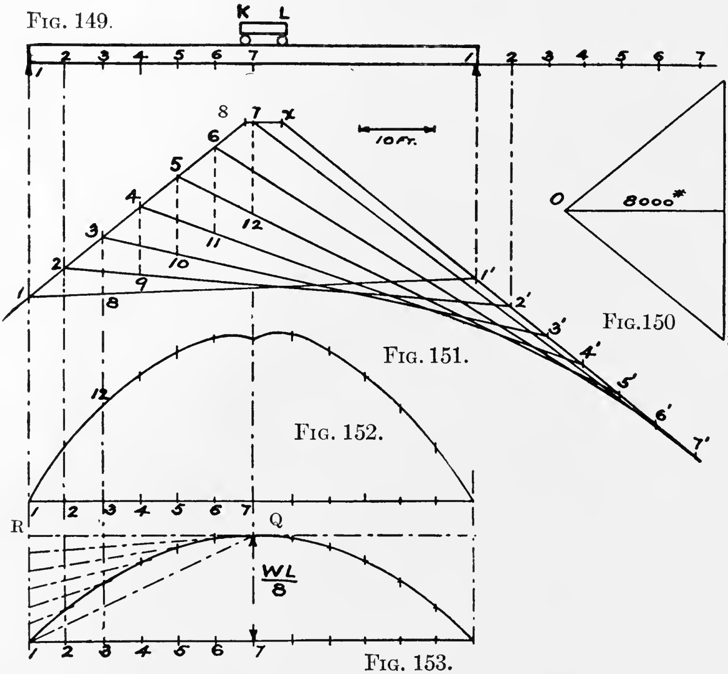



FIG. 149.

FIG. 150

FIG. 151.

FIG. 152.

FIG. 153.

Fig. 151 completed shows 7 equilibrium polygons, each one of which is a bending-moment diagram for one position of the trolley. Thus equilibrium polygon $1\text{–}8\,x\,1^1$ is a bending-moment diagram with the trolley as shown upon the girder. $2\text{–}8\,x\,2^1$ gives the bending-moment diagram with the trolley moved 5 feet toward the left support, etc. If these several bending-moment diagrams are laid-off upon a common base line and a curve drawn through the outside points Fig. 152 results. Instead of drawing in all these equilibrium polygons, it is more commonly done by means of ordinates in the several equilibrium polygons for different distances from a support. Thus suppose we want to know the maximum bending moment 10 feet from the left support produced by the above moving load. If the left support is at 1 the equilibrium polygon is 1–8 $x$ $1^1$. The vertical intercept in this polygon 10 feet from 1 is the broken line 3–8. When the left support is at 2 or the trolley 5 feet nearer the left support than it was in the first case, the bending will be proportional to the ordinate 4–9, as the equilibrium polygon in this case is 2–8 $x$ $2^1$. After trial it is found that 7 − 12 is the maximum ordinate and the line 3 − 12 is laid-off in Fig. 152 equal to 7 − 12 in Fig. 151 and similarly with the other ordinates.

In this diagram Fig. 152, since in Fig. 150 the pole distance O$g$ is 8000 pounds and since the scale of lineal dimensions in Fig. 149 is 1 inch equals 16 feet, the scale of Fig. 152 is 1 inch = 16 × 8000 = 128,000 foot-pounds.

Fig. 153 is a parabola representing the bending moment due to the uniform load of the girder, *i.e.*, 15,000 pounds.

The middle ordinate Q7 is $\frac{M}{8000}$.

$$M = \frac{WL}{8} = \frac{15000 \times 60}{8} = 112{,}500 \text{ foot-pounds.}$$

$$\overline{Q7} = \frac{M}{8000} = \frac{112{,}500}{8000} = 14.06 \text{ feet.}$$

Q7 is laid-off to the scale 1 inch = 16 feet.

QR being taken equal to ½ the span.

The parabola is drawn by dividing QR and R1 into the same number of equal parts, in this case 6, and then determining the points in the parabola by drawing the slanting and vertical lines as shown.

The total bending moment on any section of the girder will be the sum of the moments found in Figs. 152 and 153. If desired the two diagrams can be combined by drawing Fig. 153 inverted on the same base line as is used for Fig. 152. The bending moment at any section will then be the vertical intercept between the curves.

### Diagram of Maximum Shears.

To determine the web area and the rivet spacing of the rivets connecting the web and flange, a diagram of maximum shears is useful. Taking the data just used for the diagrams of maximum bending, we proceed as follows:

First lay off the bridge girder having the span KM drawn to scale and the wheel loads K and L at the extreme left of the bridge. Then draw the force polygon by laying off *hg* and *gi* each equal to a wheel load of 6500 pounds, draw O*h* parallel and equal to the span KM, connect O with *g* and *i*. Now under the bridge take

FIG. 154.

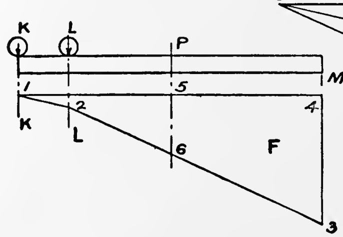
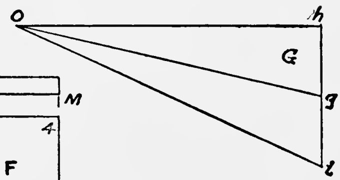
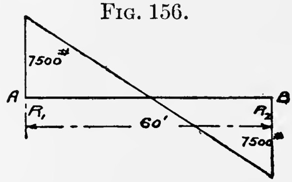



FIG. 155.




any point 1 in K–K the line representing the wheel load, draw 1–4 parallel to O*h* and 1–2 parallel to O*g*. 1–2 intersects the force of the wheel load L–L in 2 and through this point draw 2–3 parallel to O–*i*. 4–3 is the maximum shear corresponding to the left end reaction, when K and L are as shown. To find the maximum shear at any intermediate point between M and P, the middle of the girder, draw a line at that section parallel to 4–3, the intercept between the sides of the equilibrium polygon will be the required maximum shear. 5–6 is the maximum shear at the middle of the girder.

FIG. 156.




To this maximum live load shear must be added the dead load shear. This is shown in Fig. 156. The dead load per girder being 15,000 pounds, each reaction is 7500 pounds; these reactions being laid-off as shown, and their extremities connected by the diagonal line, the vertical shear at any section is the vertical intercept between the horizontal line A–B and the diagonal line.

### Built-Up Bridge Girders.

The following figures illustrate types of bridge girder sections.

Fig. 157.—This is probably the commonest type, being the box girder. The trolley runs on the upper flanges of the girder. This girder is generally designed for uniform strength. The

Fig. 157.

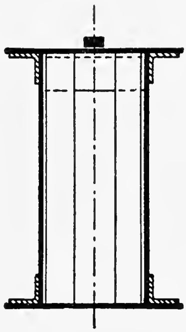
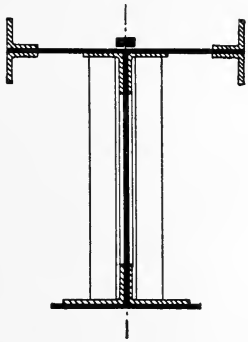


Fig. 158.



width of the flange is made $\frac{1}{30}$ to $\frac{1}{40}$ of the span of the girders. The unit stress is reduced in the compression flange in accordance with the theory given on p. 107.

Fig. 159.

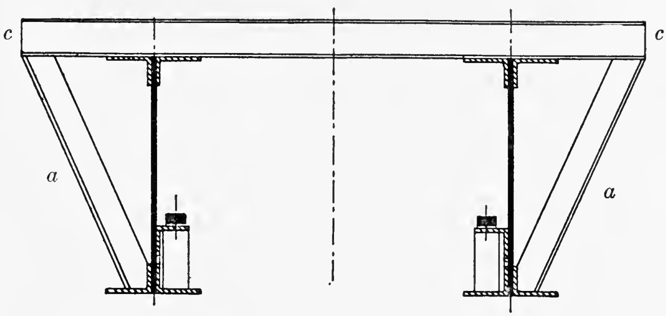


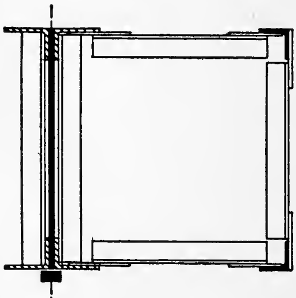

Fig. 158.—The general design of this girder is similar to that of Fig. 157. The lateral strength is furnished by the horizontal stiffening girder.

Fig. 159.—This type shows the girders of uniform depth and the trolley located in the space between the girders. The

load coming upon the girders through the trolley wheels is eccentric to the vertical axis of the girder and it consequently puts a twisting moment on the girder; this is counteracted by the channels *cc* and the struts *aa*. The bridge is stiffened laterally by the channels *cc* and diagonals which brace them in plan.

Fig. 161.—This is also a common type. Here the main girder is stiffened by a horizontal stiffening girder. See drawing p. 152.

Fig. 161.



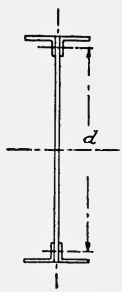
It will be noted that these several types of crane girders are a result of attempts to reduce the height occupied by the crane, to keep down the height of a building, or to give as much lift as possible in an existing building. In addition to this the girders are designed for lateral stiffness.

The general method of girder design will now be illustrated, taking first one of the main girders of type Fig. 161.

**Problem.**—Design a girder for a crane bridge, the capacity of the crane being 40,000 pounds, and span 60 feet. Assume the trolley wheel base as 6 feet, and the live load and the weight of the trolley as carried equally by the 4 trolley wheels. Take the bridge weight as 27,000 pounds (13,500 pounds per girder) and the trolley weight as 12,500 pounds.

As the section and depth will be kept the same the entire length of the girder, no bending-moment diagram will be required. The maximum bending moment can be calculated for the girder and a diagram of maximum shears drawn to determine the riveting and web section.

Fig. 162.




The maximum bending is given by the formula (see p. 138).

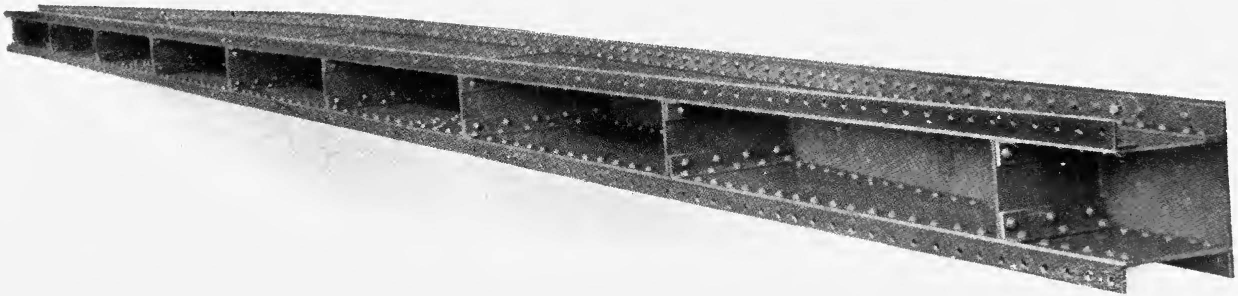
$$M=\frac{W}{8l}\left(l-\frac{a}{2}\right)^2+\frac{Gl}{8}$$

$$M=\left[\frac{40{,}000+12{,}500}{8\times60\times12}\left(720-\frac{72}{2}\right)^2\right]+\left(\frac{13{,}500\times60\times12}{8}\right)$$

$$M=4{,}264{,}400+1{,}215{,}000=5{,}479{,}400 \text{ inch-pounds.}$$

If the girder is taken $\frac{1}{12}$ of the span its depth is $\frac{60}{12}=5$ feet.

Fig. 160.



**Crane Girders.**—The accompanying cut illustrates a bridge girder of the box type. The plate diaphragms show the method of stiffening up the girder.

In a girder of considerable depth relative to its flange depth it will generally be sufficiently accurate to assume the distance between the centers of gravity of the two flanges about 2 inches less than the girder depth. If $d$ is this distance and

M = external bending moment in inch-pounds,

$p$ = mean fiber stress in flange.

A = area of 1 flange in square inches. The flanges being alike in this case,

$$M = A \times p \times d, \text{ from which } A = \frac{M}{p \times d}$$

$$d = 60 - 2 = 58 \text{ inches.}$$

Since the extreme fiber stress is specified and the center of gravity is assumed as 29 inches from the axis of the girder, then the fiber stress $p$ is

$$p \backsim \frac{11{,}000 \times 29}{30} = 10{,}600 \backsim 10{,}500.$$

$$A = \frac{5{,}479{,}400}{58 \times 10{,}500} = 9\ 00 \text{ square inches.}$$

This is the net section and should be increased to allow for the rivet holes. If we assume angles $\frac{5}{8}$ inch thick with $\frac{3}{4}$ inch rivets, the holes for these rivets will be reamed $\frac{1}{16}$ inch full, making the diameter of the hole $\frac{13}{16}$ inch. The area removed by the rivet holes then is $0.81 \times 1.25 = 1.01$ square inches, and the gross area must be $9.00 + 1.00 = 10.00$ square inches.

The flange being made of two angles, each angle will be 5.00 square inches in section; this will take $2 - 6 \times 4 \times \frac{9}{16}$ inches, each of which has an area of 5.31 square inches.

If it is now desired to check this by means of the moment of inertia of the section, it can be done as follows:

Assuming as before that the web is for the shearing strength of the girder, the inertia will be due to the flange angles only. The inertia of a built-up section is the sum of the inertias of the parts of which the complete section is made, each being referred to an axis through its own center of gravity, and parallel to the axis of the completed section about which the inertia is being determined, and to these must be added the area of each section times the square of the distance between the axes just referred to.

**Moment of Inertia of Section.**

4 angles referred to their own axes, $6 \times 4 \times \frac{9}{16}$ inches $4 \times 6.91$ .......................... = 28

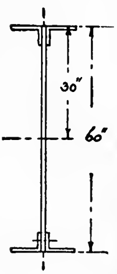
Area of angles times the distance between axes squared, $4 \times 5.31 \times 29.00^2$ ........ = 17,860

17,888

Deducting the value of the $Ah^2$ of rivet holes $(0.81 \times 0.56) \times 4 \times 29.69^2$ .............. = 1600

Inertia = 16,288

FIG. 163.




$$\frac{I}{e} = \frac{16,288}{30} = 542.9.$$

$$p = \frac{Me}{I} = \frac{5,479,000}{542.9} = 10,100.$$

Before determining the riveting, the stiffeners and the web section, it will be necessary to draw diagram of maximum shears. This diagram is drawn in Fig. 164 and its construction is in accordance with the explanation given on p. 145.

The pole distance $Oa$ in the force polygon is taken equal to the span. 1–2 is drawn parallel to the line $Ob$, $ab$ representing

FIG. 164.

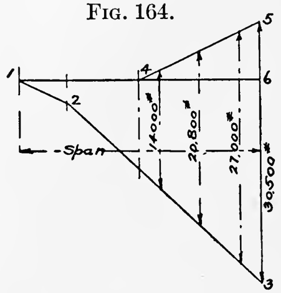
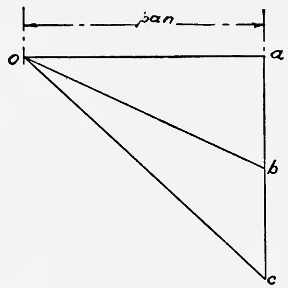



FIG. 165.




one trolley wheel load. In the same way 2–3 is drawn parallel to $Oc$. The reaction 5–6 at the right of the span is that due to the weight of the girder and is equal to ½ its weight. 4 is the middle of the girder span. The maximum shear occurring along the girder between the points 4 and 6 is the vertical intercept between the lines 4–5 and 2–3 measured by the same scale to which the forces $ab$ and $bc$ are laid-off.

Web Plate.—At the ends of the girders the depth of the web may be cut down to ½ the web depth at the middle of the span; this is done to make room for the bridge truck. The depth at the ends in this case then would be 60/2=30 inches. The shearing stress in the web plate is usually figured low, say 5000 pounds per square inch.

Web area required=30,000/5000=6 square inches. The specifications would generally limit this web to from 3/16 inch to ¼ inch plate; either of these should prove ample in this problem.

Stiffening Angles.—Girder design requires stiffening angles where the web thickness is less than 1/60 of the girder depth. These stiffeners are spaced at intervals equal to the girder depth but not exceeding 5 feet along the girder.

Although not usually calculated, their size may either be estimated by a column formula, or since their length divided their radius of gyration referred to an axis mid-way between their parallel flanges will be small, their area can be found by dividing the maximum end shear by the allowable fiber stress in tension. The maximum end shear in this girder being 30,500, and the allowable tensile fiber stress being 11,000 pounds per square inch, the area of the end stiffeners will be 30,500/11,000 =2.78 square inches.

Allowing 0.4 square inch for rivet holes makes the gross section 3.18 square inches. Referring to any structural manufacturer's hand-book, it will be found that if 2 angles are used each must have an area of 1.59 square inches, the material must be at least ¼ inch thick and that legs of angles to have ¾ inch rivets driven in them should not be less than 2½ inches long. It will therefore be found that 2—3×2½×5/16 inch angles will be needed.

In general girder design it is not uncommon to specify the intermediate stiffeners about as follows:

| Depth of Web Feet. | Stiffening Angles |
|---|---|
| 3 | 2—3 ×2½×5/16 inch |
| 4 | 2—3 ×3 ×5/16 inch |
| 5 | 2—3½×3½×5/16 inch |
| 6 | 2—4 ×3½×3/8 inch |
| 7 | 2—5 ×3½×3/8 inch |

The short legs are placed back to back and riveted to the web of the girder.

Rivets in End Stiffeners.—The value of ¾ inch rivets double shear, allowing 8000 pounds per square inch in shear is

$$2\times0.44\times8000=7500 \text{ pounds.}$$

The value of ¾ inch rivets bearing on ¼ inch web plate, using a bearing value of 16,000 pounds per square inch, is

$$0.25\times0.75\times16{,}000=3{,}000 \text{ pounds.}$$

It is therefore seen that the bearing value will determine the number of the rivets.

$$\text{Number of rivets}=30{,}500/3000=10 \text{ rivets.}$$

Should it be found necessary the number of rivets should be increased, so that the rivet spacing shall not exceed 6 inches.

Rivet Spacing in the Flange Angles.—It is generally only necessary to consider the rivet spacing in the upper flange, the spacing in the lower flange being made the same. The rivets must provide sufficient strength to transmit the concentrated wheel load to the web plate through the angles, and also to care for the change in the flange force due to the differing bending moments upon adjacent sections of the girder.

If the rail is of usual depth the wheel load can be assumed as uniformly distributed over the flange for a distance of about 36 inches. Let this value for 1 inch be $S_1$. The change of flange force per inch of span is given by $S_2=\frac{V}{H}$, where V = vertical shear in pounds. The mean value of the section for which the riveting is being determined should be used.

H = vertical distance in inches between rivets in the upper and lower flanges.

If S is the shearing stress upon rivets per 1 inch of span then

$$S=\sqrt{S_1^2+S_2^2}$$

and the rivet spacing is

$$p=\frac{\text{Rivet value}}{S}$$

where p is the pitch of the rivets measured horizontally. The rivet value will be the lower of the values for shearing and bearing. Working out the rivet spacing for the first 10 feet of the girder under consideration, we have

$$\text{The trolley wheel concentration} = \frac{40{,}000+12{,}500}{4} = 13{,}125 \text{ pounds.}$$

$$S_1 = \frac{13{,}125}{36} = 365 \text{ pounds.}$$

$$S_2 = \frac{V}{H} = \frac{27{,}000}{60-(2\times 2.5)} = \frac{27{,}000}{55} = 490$$

$$S = \sqrt{365^2+490^2} = 612 \text{ pounds.}$$

Using the same values for ¾ inch rivets previously found would make the pitch of the rivets $p = \frac{3000}{612} = 4.90$ inches.

From this it is seen that the rivet spacing in the first 10 feet should not exceed 4⅞ inches; similar calculations to the above will show how the rivet spacing can be increased toward the middle of the girder, but in no case should it exceed 6 inches.

The calculated pitch for the distance from 10 to 20 feet from the girder end is $S_2 = \frac{20{,}800}{55} = 380.$

$$S = \sqrt{365^2+380^2} = 527. \qquad p = \frac{3000}{527} = 5.7 \text{ inches.}$$

The pitch between 20 and 30 feet from the ends will be 6.75 inches. In this latter case the pitch should be limited to 6 inches.

Lateral Stiffening Girders.—It is necessary that girders of this type should be strengthened laterally. We will use a horizontal stiffening girder in the plane of the compression flange. It will be necessary to support the outer flange of this truss, and the two girders are then braced diagonally across the lower or tension flanges. The designing of the horizontal stiffening girder is largely empirical. The lateral strength is necessitated by the following causes:

1. The upper flange of the main girder being in compression is liable to buckle and increase the fiber stress as in a column. In the case of a crane this is increased by the crane motions.

2. Starting and stopping the bridge travel introduces strain on the girders.

3. Although a crane is built only for vertical hoisting, still it is not infrequently used to pull loads along in the direction of its bridge travel.

Some designers assume that the girders must withstand a horizontal loading of 1/10 of its vertical loading. We will design the girder upon this assumption and adding the following conditions:

1. The compression flange of the main girder will be stiffened laterally at intervals not exceeding 12 times the flange width.

2. The width of the horizontal girder shall equal approximately the depth of the main girder.

3. The length divided by the least radius of gyration of any compression member shall not exceed 120.

4. Allowable fiber stress the same as in the main girder.

The maximum vertical load upon one girder is,

| | |
|---|---|
| Assumed girder weight ............ | 13,500 pounds. |
| One-half trolley weight ............ | 6,750 pounds. |
| One-half maximum load ........... | 20,000 pounds. |
| Total ...................... | 40,250 pounds. |

One-tenth of this acting horizontally would be 4000 pounds and would be assumed as acting at the middle of the girder. Fig. 166 represents a diagram of the horizontal truss. Fig. 167 is a stress diagram with the load of 4000 pounds acting in the middle of the span. This will give the maximum stress in the outer flange IC. The maximum stress in the diagonals will occur when the trolley and load are at one end of the bridge girder, making the assumed horizontal reaction 3350 pounds and the diagonal force 4750 lbs. These are quite readily found algebraically.

Fig. 166.

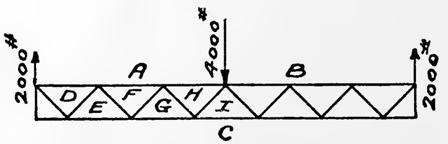



Fig. 167.

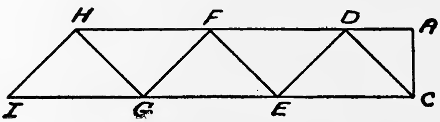
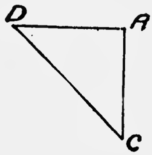



Fig. 168.




$$\text{Force in } \overline{CI} = \frac{2000 \times 30}{5} = 12{,}000 \text{ pounds.}$$

$$\text{Maximum force in } \overline{DC} = \frac{3350}{\text{Cos. } 45^\circ} = 4725 \text{ lbs.}$$

The length of CI is 10 feet = 120 inches.

The length of DC is 7 feet = 84 inches.

Since the force of 4000 pounds may act in either direction upon the horizontal girder these pieces may be in either tension or compression, and must therefore be designed as compression pieces.

Design of ci.—Since l/r must not exceed 120, r must be greater than 1. This will require a 6×6×⅜ inch angle. This angle is much too heavy, but the next smaller standard section does not fill the specification requirements, *i.e.*, the least radius of gyration is under 1, being 0.79.

How much in excess the 6 inch×6 inch angle is can be shown by the following calculation:

$$\frac{l}{r}=\frac{120}{1.19}\backsim 100.$$

By Ritter's column formula

$$p^1=\frac{p}{1+\frac{F}{m\pi^2E}\times\left(\frac{l}{r}\right)^2}=\frac{12,000}{1+\frac{30,000}{10\times 30,000,000\times 1}\times(100)^2}=6000 \text{ lbs.}$$

A 6 inch×6 inch ⅜ inch angle is therefore good for

Load = 4.36×6000 = 26,160 pounds.

For the diagonal, r must be greater than $^{96}/_{120}$ = 0.80. This angle calculated as the preceding will require a 4×4×$^5/_{16}$ inch having an area of 2.41 square inches and will carry a load per square inch of

$$p^1=\frac{12000}{1+\frac{F}{m\pi^2E}\times\left(\frac{l}{r}\right)^2}=\frac{12000}{1+\frac{120^2}{10000}}=\frac{12000}{2.44}=4900 \text{ pounds.}$$

The total load will be 2.41×4900 = 11,850 pounds and is greatly above the load of 4750 pounds estimated as coming upon it.

For the design of the usual box girder see the design of the girder of the 20 ton O.E T. crane p. 320.

### Latticed Crane Girders.

Latticed girders are used upon loads ranging from 5 to 100 tons and upon spans exceeding 60 feet. The determination of the stresses in such a girder will be illustrated by an example.

**Problem.**—Span 72 feet; capacity 15 tons; trolley weight 10,000 pounds; bridge weight $W_b$ = 0.55 L+400 S−9000.

$$W_b=(0.55\times 30,000)+(400\times 72)-9000=36,000.$$

$$\text{Weight of 1 girder } \frac{36,000}{2}=18,000.$$

The girder depth will be assumed at $^1/_{12}$ the span, $^{72}/_{12}$ = 6.

$$\text{Apex loads }=\frac{18,000}{6}=3000 \text{ pounds.}$$

The bridge girder will be divided into 6 panels and perpendicular struts shown dotted in Fig. 169 will be used to reduce the bending upon the members of the upper chord, CF, DH, EJ, etc.

Dead Load Diagram.—Fig. 170. The apex loads CD, DE, $EE^1$, etc., have been found to be 3000 pounds, while the end ones, BC and $B^1.C^1.$, are 1500 pounds each. The reaction AB is ½ the dead load on one girder, or $\frac{18,000}{2} = 9000$ pounds. The diagram follows directly as drawn in Fig. 170, using the methods explained on p. 79. These stresses have been measured and placed in table p. 156.

Live Load Stresses.—The live load stresses are due to the trolley load travelling across the girder. The trolley wheel loads, assuming the load carried centrally with the 4 wheels, will be,

| | |
|---|---|
| Weight of trolley................ | 10,000 pounds. |
| Live load...................... | 30,000 pounds. |
| Total ...................... | 40,000 pounds. |

$$\text{Wheel load } \frac{40,000}{4} = 10,000 \text{ pounds.}$$

It is first necessary to draw the diagram of maximum shear as shown in Fig. 171. A force diagram is drawn as indicated in the dotted lines. Its pole distance must be taken equal to the span $BB^1$. Lay-off $B^1L$ and LM each equal to a trolley wheel load (10,000 pounds) and draw the dotted lines BL and BM. Draw a perp ndicular line a distance $a$ from the pole B. $a$ is the trolley wheel base. Through the intersection of this perpendicular line with the broken line BL draw a full line parallel to the broken line BM. The intercepts $R_1$, $R_2$, $R_3$, and $R_4$ give the maximum shears at the apex points produced by the moving loads passing from the ends to the middle of the girder.

It will be noted that $R_2$ is the reaction AB when the left hand wheel is 12 feet to the right of support B or at CD as shown.

The apex load at CD is produced by the left hand wheel carrying 10,000 pounds which is over CD and ½ of the other wheel load of 10,000 pounds which is midway between CD and DE. This makes CD = 15,000 pounds and DE = 5,000 pounds. With these data diagram No. 1 of live load stresses follows as shown. In the same way when the loads are moved 12 feet farther to the right the reaction AB = $R_3$, DE = 15,000 pounds and $EE^1$ = 5,000 pounds. Diagram No. 2 follows similarly to No. 1. For diagram

## BRIDGE GIRDER STRESSES.

Fig. 169.

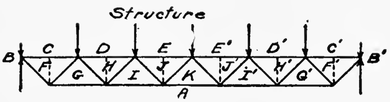



Fig. 170.

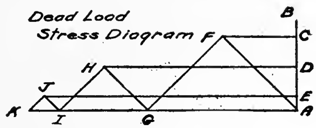



Fig. 171.

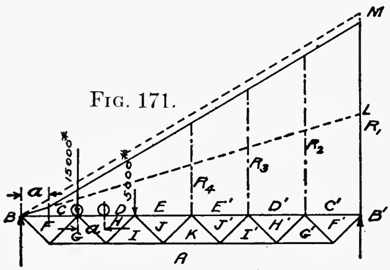



Fig. 172.

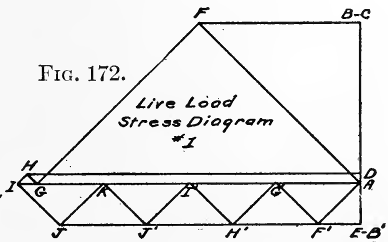



Fig. 173.

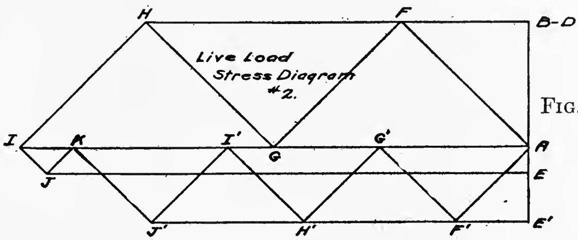
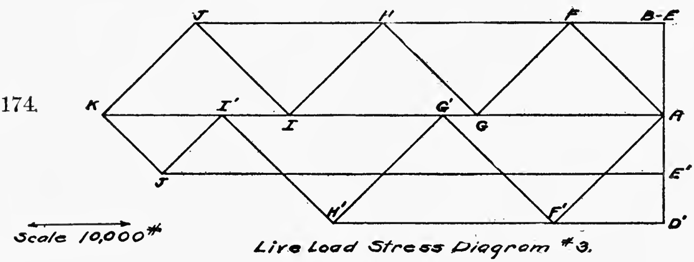



Fig. 174.



No. 3 the wheel loads are moved 12 feet still farther to the right. This makes apex load $EE^1 = 15{,}000$ pounds and $ED = 5{,}000$ pounds. These 3 diagrams being drawn to the same scale the greatest stress acting in any member will be given by the longest line representing the stress in that member. Thus the stress in AF is found in diagram No. 1, AF in this diagram being longer than in either of the other two diagrams. The character of the stress, whether tension or compression, can be determined as shown on p. 79. In this case it will be sufficient to note that all members of the upper chord are in compression, while all members of the lower chord, together with AF and $AF^1$, are in tension. The diagonals in the center of the truss are liable to a reversal of stress. In this case diagonals IJ, JK, $KJ^1$, and $J^1I^1$ are subject to such reversal as is shown in the table of stresses.

| Name | Dead Load Stress | Live Load Stress | Total Stress, pounds |
|---|---|---|---|
| CF | + 7,500 | +16,000 | +23,500 |
| DH | +19,500 | +37,500 | +57,000 |
| EJ | +25,500 | +47,500 | +73,000 |
| AF | −10,600 | −22,600 | −33,200 |
| FG | +10,600 | +22,600 | +33,200 |
| GH | − 6,400 | −17,800 | −24,200 |
| HI | + 6,400 | +17,800 | +24,200 |
| IJ | − 2,100 | { + 5,900<br>−13,200 | { + 3,800<br>−15,300 |
| JK | + 2,100 | { − 5,900<br>+13,200 | { − 3,800<br>+15,300 |
| GA | −15,000 | −32,000 | −37,000 |
| IA | −24,000 | −50,000 | −74,000 |
| KA | −27,000 | −55,500 | −82,500 |

When this type of truss is used for a crane bridge the track is usually placed upon the top chord. This introduces bending in addition to the direct compression just found. Assuming the vertical members shown dotted in Fig. 169 to be in place, the span from an apex point to the vertical member in this case will be 6 feet.

According to Johnson the total combined stress in such a member is given by

$$p = p_1 + p_2 = \frac{Me}{I - \frac{Pl^2}{\alpha E}} + \frac{L}{A}$$

For a simple beam uniformly loaded $\alpha = 10$.
For a simple beam centrally loaded $\alpha = 12$.

Here M = bending moment due to central load (inch-pounds).
$e$ = distance extreme fibers in compression to neutral axis.
I = moment of inertia, inches.
P = load in pounds.
$l$ = span in inches.
E = modulus of elasticity, pounds.
L = direct compressive force in member, pounds.
A = area of section in square inches.

Having found this combined stress it should be under the stress permitted upon the upper chord as a strut, considering its $\frac{l}{r}$ value for the bridge span. The girder may be braced laterally by a horizontal stiffening girder, in which case the horizontal girder should be designed to resist the lateral forces upon the girders (see p. 152).

Pratt Truss.—The stresses in a Pratt truss can be determined similarly to those in the Warren truss just completed. Fig. 175, p. 158, gives the outline of a Pratt truss. Fig. 176 is the dead load diagram. Fig. 177 is the diagram of maximum shears. Fig. 178 is a live load stress diagram giving the stresses when the left wheel is 6 feet to the right of the left support, as shown in the figure. Figs. 176 and 178 are drawn to the same scale. All diagonals are in tension according to Fig. 176. The nature of the stresses in the diagonals due to the live loads must be ascertained from the live load diagrams. It can be noted that all members of the upper chord are in compression, while all lower chord members are in tension. An examination of Fig. 178 will show that LM, NO, PQ, and RS will be in compression, due to the live load in this position. In LM and NO the dead load stresses exceed the live load stresses, so that the resultant leaves these members in tension. In the case of the other two, PQ and RS, the live load or compression stresses are the greater, so that these members will have to be designed for both compression and tension or counter diagonals inserted as shown in the dotted lines. After drawing the remaining live load stress diagrams the maximum stresses can be tabulated as was previously done for the Warren truss.

The more usual sections for these girders are shown in Fig. 179. Section A requires a horizontal stiffening girder, similar to that shown for an ordinary plate girder (see p. 152). The other sections are usually designed so as not to require this stiffening girder.

Fig. 175.

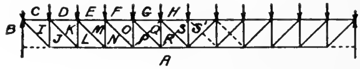


Fig. 176.

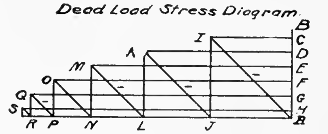



Fig. 178.

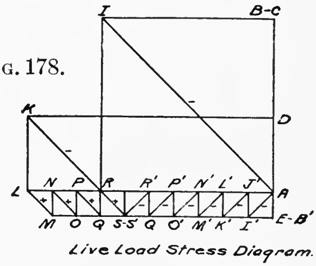



Fig. 177.

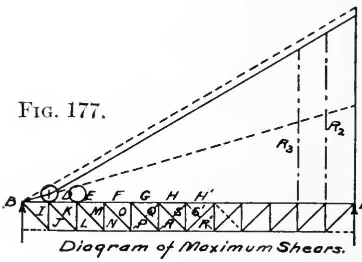



Fig. 179.

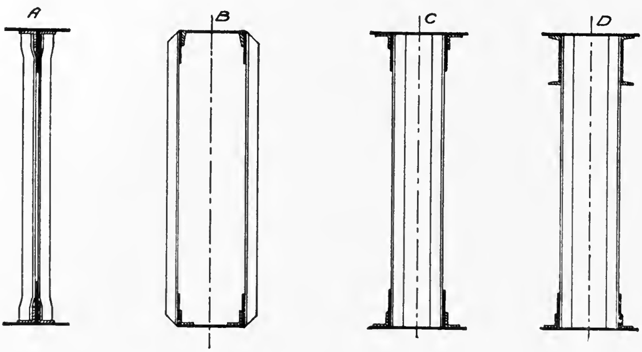



Fig. 182.

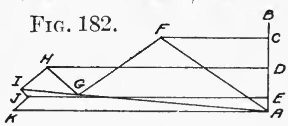


Fig. 180.

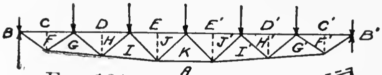


Fig. 181.

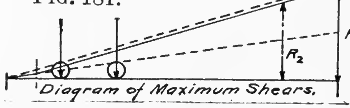
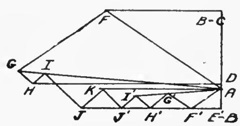
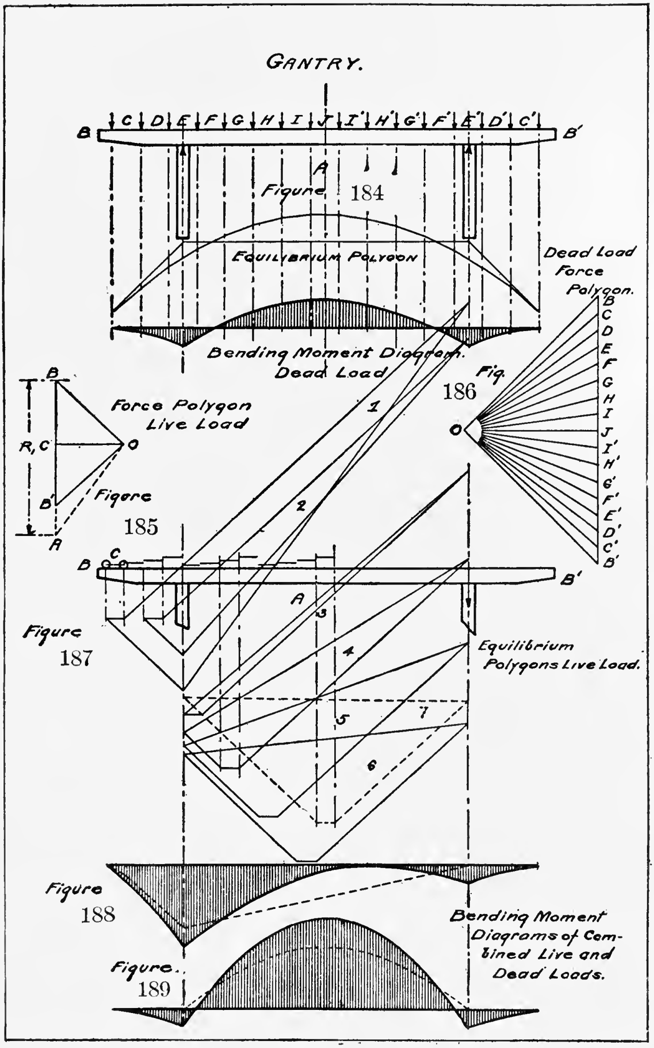



Fig. 183.




GANTRY.
B C D E F G H I J I' H' G' F' E' D' C' B'
A
Figure 184
Equilibrium Polygon
Dead Load Force Polygon.
Bending Moment Diagram. Dead Load
Fig. 186
Force Polygon Live Load
R, C
O
Figure 185
Figure 187
Equilibrium Polygons Live Load.
1
2
3
4
5
6
7
Figure 188
Figure 189
Bending Moment Diagrams of Combined Live and Dead Loads.

The case of the Warren truss of varying depth, with its diagram of maximum shears, is shown in Fig. 180. The analysis of the stresses is quite similar to the Warren truss previously discussed. The dead load stress diagram and one live load stress diagram are shown respectively in Figs. 182 and 183.

Gantry.—The determination of the stresses in a gantry when an I or box girder section is used is given by the diagram on p. 159.

Fig. 184 is the bending-moment diagram due to dead load. This diagram is made by the aid of the force polygon Fig. 186 from which the equilibrium polygon in Fig. 184 is drawn. This equilibrium polygon is now plotted upon a horizontal base line and the bending moment at any point is the vertical intercept in the shaded portion of this diagram, measured in the same scale to which the span of the girder is drawn and multiplied by the pole distance OJ (in pounds) in Fig. 186.

Live Load Bending.—This is found by the method shown in Fig. 187. Fig. 185 is the live load force polygon. The bending over the supports will be a maximum when the trolley is farthest out on the cantilever portion of the girder. This is shown by Figure 1 (equilibrium polygon) of Fig. 187. The shaded portion of Fig. 188 represents the algebraic sum of live load bending (trolley in extreme position), indicated by the dotted lines, and the dead load bending Fig. 184.

Fig. 189.—Here the several equilibrium polygons 3, 4, 5, 6 and 7 in Fig. 187, representing the bending due to different positions of the trolley in girder span between the supports, have been plotted upon a horizontal line, and a curve, shown dotted, drawn through the outside points. The dead load bending is then added algebraically to these ordinates, resulting in the curve bounding the shaded portion. At any section the vertical intercept in the shaded portion measured to the scale to which the span is drawn, and multiplied by the pole distance OC in Fig. 185 measured in pounds, gives the bending moment at the section.

The principles of design illustrated in the common types of crane girders just shown are applicable to other types of girders.

# PART III.—BRAKES AND CLUTCHES

## Friction Brakes.

Friction brakes are used to control the action of machines by absorbing their kinetic energy, changing it through frictional work into heat. When the machines are at rest brakes may be applied to prevent motion.

In the design of a brake the following points must be considered.

1. Coefficient of friction between material of brake-wheel and brake shoes, due consideration being given to the existing state of lubrication, or lack of it, at which the brake operates.

2. The heat generated by the working of the brake must be dissipated so as not to injure the brake.

3. The parts of the brake must be designed of sufficient strength to withstand the stresses that will come upon them during its operation.

The following are a few of the many types of brakes:

1. Block brakes.
2. Band brakes.
3. Friction cone brakes.
4. Multiple disc brakes.
5. Coil brakes.

The "safety mechanical brakes" or "lowering brakes" used on cranes are generally disc or coil brakes, the former being the more common.

Coefficients of Friction.—The coefficient of friction is highest for static conditions and decreases with motion and increase of velocity.

The following table gives coefficients of friction, according to Morin, for pressures of 15 to 20 pounds per square inch and low velocities, of the materials stated.

| | Dry | Slightly greased |
|---|---|---|
| Bronze on bronze | 0.20 | .... |
| Bronze on cast-iron | 0.21 | .... |
| Bronze on wrought-iron | .... | 0.16 |
| Cast-iron on cast-iron or bronze | .... | 0.15 |
| Wrought-iron on cast-iron or bronze | 0.18 | .... |
| Wrought-iron on wrought-iron | 0.44 | .... |
| Cast-iron on oak | 0.49 | 0.19 |
| Leather on oak | 0.27 | .... |
| Leather on cast-iron | 0.56 | .... |

According to Galton, the coefficient of friction of cast-iron brake shoes on steel tires varies with the velocity as follows:

| Velocity, feet per minute | Coefficient of friction |
|---|---|
| Beginning | 0.330 |
| 440 | 0.273 |
| 880 | 0.242 |
| 2200 | 0.166 |
| 4000 | 0.127 |
| 5250 | 0.074 |

According to Wichert, this effect of velocity upon the coefficient of friction for cast-iron brake shoes upon steel-tired car wheels is approximated by the formula

$$\mu = B\frac{1+0.002\,V}{1+0.0011\,V}$$

where B is the coefficient of friction at rest under the given condition of the surfaces, and V is the velocity of the surfaces relative to each other, in feet per minute.

The coefficients of friction for materials used for brakes, for velocities ranging from 180 to 3900 feet per minute, and pressures from 7 to 140 pounds per square inch, according to L. Klein, are as follows:

| | Cast-iron | Wrought-iron |
|---|---|---|
| Beech | 0.29 to 0.37 | 0.54 |
| Oak | 0.30 to 0.34 | 0.51 to 0.40 |
| Poplar | 0.35 to 0.40 | 0.65 to 0.49 |
| Elm | 0.36 to 0.37 | 0.60 to 0.49 |
| Willow | 0.46 to 0.47 | 0.60 to 0.63 |

Heating of Brakes.—Since 1 British thermal unit corresponds to 778 foot-pounds of energy, the heat produced by a weight falling any distance can readily be calculated. Unfortunately, so many assumptions have to be made regarding conduction, convection and radiation constants, areas of radiating surfaces and weights of parts heated that a continuation of the calculations along these lines is of little use. The general method employed is to design the brake by proportions that have been found satisfactory in brakes already in operation.

According to C. F. Blake (*Machinery*, August, 1906), brakes with wooden shoes on iron drums give satisfaction if one square inch of friction surface is allowed for each 200–250 foot-pounds of energy absorbed. In the case of car brakes, cast-iron on cast-iron, the brake must often absorb 10,000 to 15,000 foot-pounds of energy per minute.

When wooden shoes are used, if the radiating surface is insufficient the destruction of the shoe may result. In the case of metal on metal too great a rise in temperature in some cases may result in greatly increased pressure and cause seizure and cutting of friction faces, or the temperature rise not being uniform throughout the piece may produce unequal expansion and failure of some portion of the brake mechanism.

The design of the brakes for strength will be considered under the several discussions of the individual brakes.

## Brake Design.

M = twisting moment on brake shaft (inch-pounds).

$r$ = radius of brake wheel in inches.

$F=\frac{M}{r}$ = force at the circumference of brake wheel in pounds.

P = pull on the brake lever in pounds.

$\mu$ = coefficient of friction between brake wheel and shoe.

B = pressure between the brake wheel and shoe, normal to the brake wheel. Measured in pounds.

## Block Brakes—Figs. 190 to 194.

With a pressure B between the brake wheel and shoe, the tangential resistance at the brake wheel due to the brake will be $\mu$B; if this is to overcome the turning moment on the brake shaft, then

$$F \overline{\overline{<}} \mu B$$

$$\frac{M}{r} \overline{\overline{<}} \mu B$$

Inspection of the formula would indicate that P becomes zero when $\frac{a}{b}$ is $\mu$. This would make the brake automatic, however; the brake would not be under as satisfactory control in lowering the load as when the value $\frac{a}{b}$ was taken higher.

The brake shown in Fig. 194 works the same whichever direction the brake wheel rotates.

In block brakes, the block may be made wedge shape, and run in a corresponding groove in the brake wheel. In this case the normal pressure between the friction faces is increased by the wedge

Fig. 190.

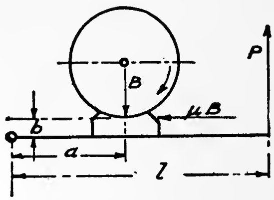
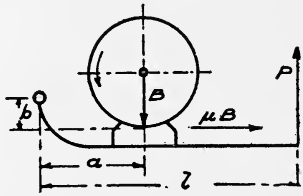



Fig. 191.




$$Pl = Ba + \mu Bb \qquad B \geqq \frac{F}{\mu} \qquad P = F\left(\frac{1}{\mu} - \frac{b}{a}\right)\cdot\frac{a}{l}$$

Fig. 192.

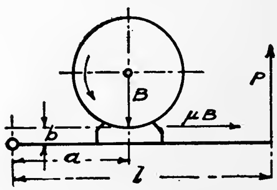
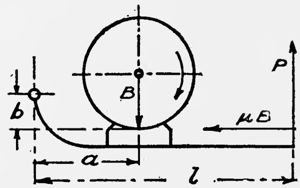
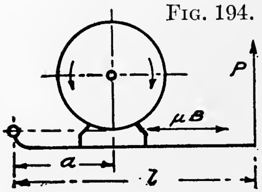



Fig. 193.




$$Pl = Ba + \mu Bb \qquad B \geqq \frac{F}{\mu} \qquad P = F\left(\frac{1}{\mu} + \frac{b}{a}\right) \times \frac{a}{l}$$

Fig. 194.

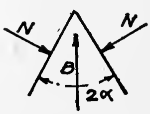



$$Pl = Ba$$

$$B \geqq \frac{F}{\mu}$$

$$P = \frac{F}{\mu} \times \frac{a}{l}.$$

action so that if N = normal pressure in pounds on each due to the pressure B produced by the pull P on the brake lever, then




$$2N = \frac{B}{\sin \alpha + \mu \cos \alpha}$$

$$F = 2\mu N \quad \text{and} \quad F = \frac{\mu B}{\sin \alpha + \mu \cos \alpha}$$

This value of F is then to be substituted in the formula for P that corresponds to the design of brake proposed. The angle of the wedge is $2\alpha = 20° — 30°$.

Double block brakes are shown in Figs. 195 and 197. The formulæ for these are given below.

Blocks without wedges

$$B = \frac{P \times h}{e} \times \frac{l}{a} \qquad B = \frac{F}{\mu}$$

$$P = \frac{F}{\mu} \times \frac{e}{h} \times \frac{a}{l} \qquad F = \frac{T}{r}$$

Blocks with wedges

$$F = \frac{M}{r} \qquad B = \frac{F(\sin\alpha + \mu\cos\alpha)}{\mu}$$

$$P = \frac{F.e}{h} \times \frac{a}{l}\left(\frac{(\sin\alpha + \mu\cos\alpha)}{\mu}\right)$$

$$2\alpha = 20° - 30°$$

FIG. 195.

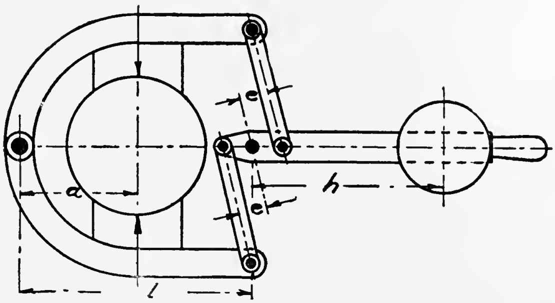



FIG. 196.

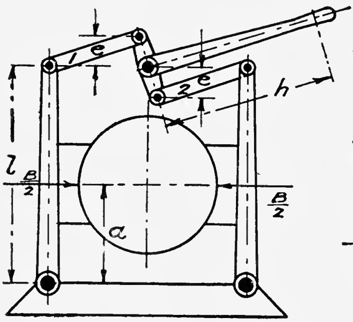
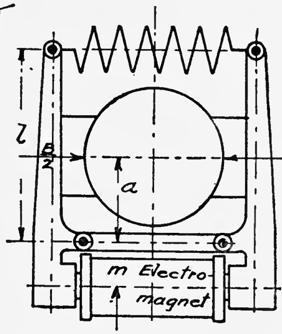



FIG. 197.



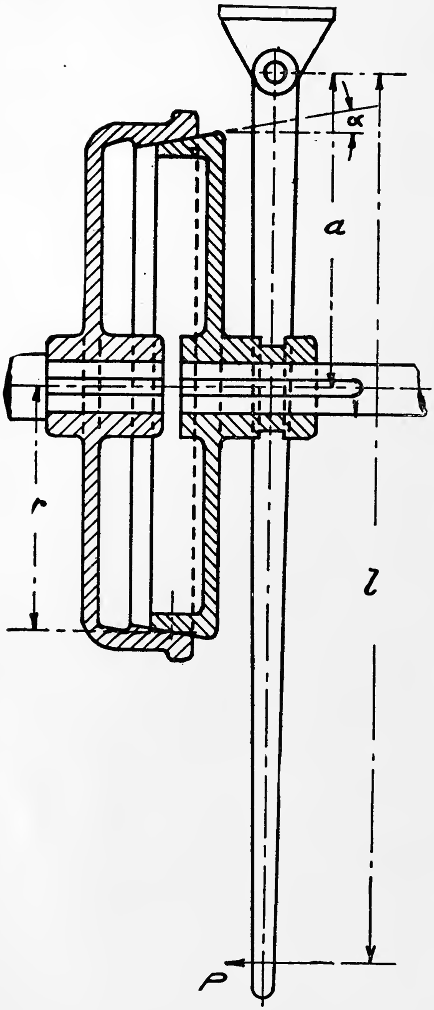

Where double brakes are used, the mechanism should be designed so that the pressure can be readily equalized on the two blocks, and at the same time adjusted for wear. In Fig. 196 sleeve-nuts can be put on the short parallel links 1 and 2.

## Cone Brakes and Clutches.

The nomenclature being as before $F=\frac{M}{r}$.

If N is the normal pressure on the cone, in order that the braking action shall overcome the turning moment

Fig. 198.




$$F \overline{\gtrless} 2\mu N$$

If A is the axial pressure along the cone shaft forcing the cones together,

$$A=2N\ (\sin\alpha+\mu\cos\alpha)$$

from which

$$N=\frac{A}{2\,(\sin\alpha+\mu\cos\alpha)}$$

and

$$F \overline{\gtrless} \frac{\mu A}{\sin\alpha+\mu\cos\alpha}$$

hence

$$A \overline{\gtrless} \frac{F\ (\sin\alpha+\mu\cos\alpha)}{\mu}$$

From this the force P at the end of the operating lever is

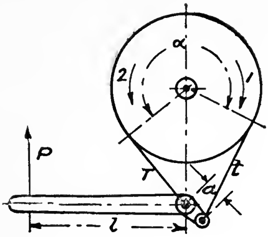
$$P \geqq F \times \frac{a}{l} \times \frac{\sin\alpha+\mu\cos\alpha}{\mu}$$

In brakes or clutches of this type the angle $\alpha$ ranges from 10 to 15 degrees.

The coefficient of friction for cast-iron on cast-iron can be taken at $\mu=0.18$.

## FRICTION BRAKES.

FIG. 199.




The law of belt friction gives $T = t\,e^{\mu\alpha}$

$T$ = tension on the side of the band that begins contact with the brake wheel.

$t$ = tension on the band where it leaves the wheel.

$e$ = 2.718, the base of the Naperian system of logarithms.

$\mu$ = coefficient of friction between band and wheel.

$\alpha$ = arc of contact in radians.

Equation 1 is generally written

$$\frac{T}{t} = e^{\mu\alpha} \text{ or } \mu.\alpha = \text{Nap. log.} \frac{T}{t} \qquad (1)$$

As it is more convenient to use this in the common system of logarithms, we have

$$\frac{T}{t} = 0.434 \times \mu \times \alpha \qquad (2)$$

As the arc $\alpha$ is usually some fraction of the circumference it is an added convenience to have it so expressed instead of in radians. Formula No. 2 becomes

$$\text{Common log. } \frac{T}{t} = 2.727 \times \mu \times \alpha \qquad (3)$$

The following table gives the values of $e^{\mu\alpha}$ found by formula No. 3.

| | Arc of Contact—Fraction of Circumference (360°). | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|
| $\mu$ | .10 | .20 | .30 | .40 | .50 | .60 | .70 | .80 | .90 | 1.00 |
| .10 | 1.06 | 1.13 | 1.21 | 1.29 | 1.37 | 1.46 | 1.55 | 1.65 | 1.76 | 1.87 |
| .18 | 1.12 | 1.25 | 1.40 | 1.57 | 1.76 | 1.97 | 2.21 | 2.48 | 2.77 | 3.10 |
| .20 | 1.13 | 1.29 | 1.46 | 1.65 | 1.88 | 2.13 | 2.41 | 2.74 | 3.11 | 3.52 |
| .30 | 1.21 | 1.46 | 1.76 | 2.13 | 2.52 | 3.10 | 3.74 | 4.53 | 5.45 | 6.61 |
| .40 | 1.29 | 1.65 | 2.13 | 2.74 | 3.14 | 4.52 | 5.77 | 7.51 | 9.65 | 10.20 |
| .50 | 1.37 | 1.88 | 2.57 | 3.14 | 4.82 | 6.59 | 9.04 | 10.20 | 16.90 | 22.95 |

The symbols being the same as those used before,

$$F = M/r \text{ and } F = T - t.$$

For rotation in the direction No. 1, we have $P \times l = t \times a$.
Let $e^{\mu \propto} = k$, then $F = T - t = t(k-1)$.

$$\text{and} \quad F = \frac{Pl}{a}(k-1), \text{ and } P = F \times \frac{a}{l} \times \frac{1}{k-1}.$$

Rotation in direction 2 gives,

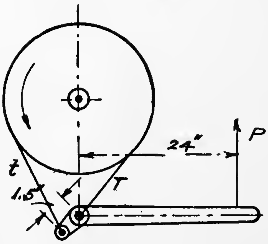
$$F = t - T \text{ and } t = Te^{\mu \propto}.$$

Calling $e^{\mu \propto} = k$, we have $T = \frac{t}{k}$ and $F = t - \frac{t}{k} = t\left(\frac{k-1}{k}\right)$,

and $F = \frac{Pl}{a}\left(\frac{k-1}{k}\right)$, since $P \times l = t \times a$ we have $P = F\frac{a}{l}\left(\frac{k}{k-1}\right)$.

**Example.**—A brake wheel shaft is subjected to a twisting moment of 2000 inch-pounds, it is 18 inches in diameter. Assume the arc of contact, $\propto = \frac{6}{10}$, and $\mu = 0.18$.

Fig. 200.




$$F = \frac{M}{r} = \frac{2000}{9} = 222 \text{ pounds.}$$

From the tables $k = 1.97$,

$$\text{hence } P = 222 \times \frac{1.5}{24}\left(\frac{1}{1.97-1}\right) = 14.3 \text{ lbs.}$$

$$t = \frac{F}{k-1} = \frac{222}{1.97-1} = 229 \text{ pounds.}$$

$$T = F + t = 222 + 229 = 451 \text{ pounds.}$$

Allowing a working fiber stress of 8000 pounds per square inch the band would be

$$\text{Area of band section} = \frac{451}{8000} = 0.0565 \text{ square inch.}$$

If the band is 1.5 inches wide, the thickness will be

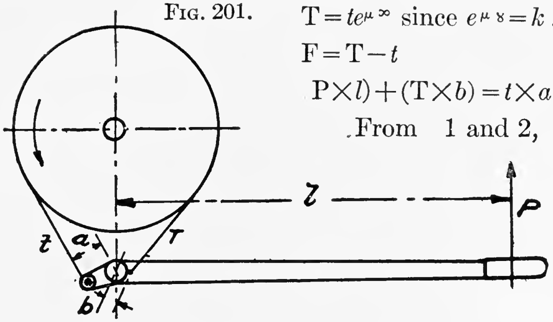
$$\frac{0.0565}{1.5} = 0.0376 \text{ inch.}$$

On account of the wear it will be better to use a greater thickness, say 1/16 inch to 3/32 inch. The band will generally not exceed 1/8 inch in thickness. A greater thickness than this stiffens it and prevents the belt action upon which the brake depends for its efficiency. When the band exceeds 3 inches in width two should be used.

The band-brake is much more efficient than the previously mentioned brakes. This will be evident from an inspection of the values of $k$ given in the table.

## Differential Band Brake.

Fig. 201.




$$T = te^{\mu\infty} \text{ since } e^{\mu\theta} = k \therefore T = tk \quad (1)$$

$$F = T - t \quad (2)$$

$$P \times l) + (T \times b) = t \times a \quad (3)$$

From 1 and 2, $F = t(k-1)$

Eliminating T and $t$ from equation 3

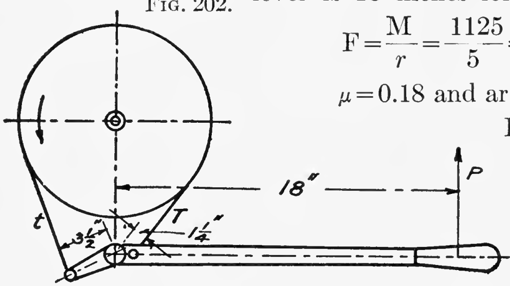
$$(P \times l) + \frac{F.k.b}{k-1} = \frac{F \times a}{k-1} \qquad P \times l = F\left(\frac{a-bk}{k-1}\right) \qquad P = \frac{F}{l} \times \frac{a-bk}{k-1} \quad (4)$$

When $a \overline{<} kb$, $P = O$ or is negative, *i.e.*, the brake works automatically. If the arc of contact of the band with the brake wheel is $\frac{7}{10}$, then $a/b$ equals from 2.5 to 3, *i.e.*, in equation 4, $a$ approaches $b.k$ but should always exceed it. If $a$ equals $b.k$ the brake action will be equal in both directions, and $P = \frac{F\,a}{l} \times \left(\frac{k+1}{k-1}\right)$

In this type of brake P is very large compared with F, so that it is not much used.

**Example.**—The twisting moment on a brake wheel is 1125 inch-pounds, the brake wheel diameter is 10 inches, the hand-lever is 18 inches long, design the brake.

Fig. 202.




$$F = \frac{M}{r} = \frac{1125}{5} = 225 \text{ pounds.}$$

$\mu = 0.18$ and arc of contact $= \frac{6}{10}$.

From table $k = 1.97$.

$$P = \frac{F}{l} \times \left(\frac{a-bk}{k-1}\right) = \frac{225}{18}\left[\frac{3.50 - (1.25 \times 1.97)}{1.97 - 1.00}\right] = 13.4 \text{ pounds.}$$

$$t = \frac{F}{k-1} = \frac{225}{.97} = 232 \text{ pounds.}$$

$$T = F + t = 225 + 232 = 457 \text{ pounds.}$$

The area of the band cross section if the tensile stress is 8000 pounds per square inch would be,

$$A = \frac{457}{8000} = 0.057 \text{ square inch.}$$

Making the band 1.5 inches wide the thickness would be

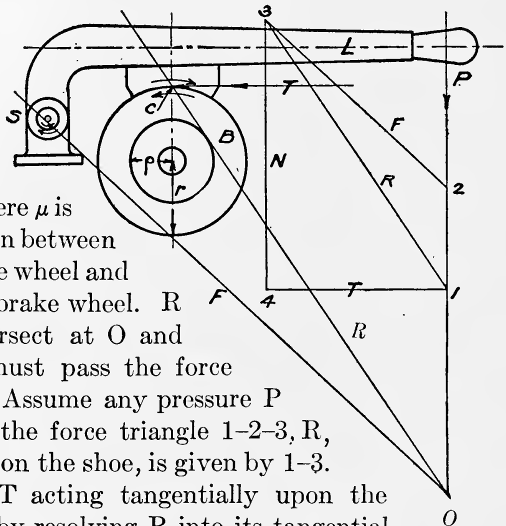
$$\frac{0.057}{1.5} = 0.039 \text{ inch.}$$

Allowing for wear would require a band about 1/16 inch thick.

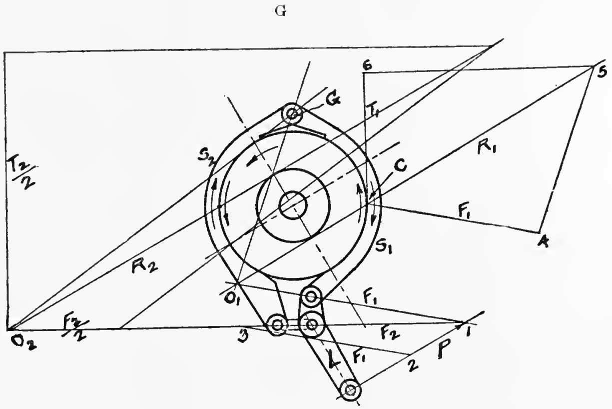
## Graphics Applied to Brakes.

Graphics may be occasionally used in the analysis of brakes; this will be illustrated in the following cases. In Fig. 203 the lever turns on the pin S, the eye being in the lever. The brake wheel B turns as shown by the arrow. The resultant pressure between shoe and brake wheel passes through C and tangent to the friction circle. The radius of the friction circle is $\rho = \mu r$, where $\mu$ is the coefficient of friction between the shoe and the brake wheel and $r$ is the radius of the brake wheel. R and P produced intersect at O and through this point must pass the force acting in the pin S. Assume any pressure P as 1–2 and complete the force triangle 1–2–3, R, the resultant pressure on the shoe, is given by 1–3. The retarding force T acting tangentially upon the brake wheel is found by resolving R into its tangential and radial components at C. The ratio of T to P is found by dividing 1–4 by 1–2.

Fig. 203.




A somewhat more complicated although common form of brake is shown in Fig. 204. The lever L is acted upon by the 3 forces $F_1$, $F_2$, and P; neglecting pin friction, the direction of P is known, and $F_2$ must act in the center-line of the bolt B. P and $F_2$ intersect in 1 and $F_1$ must also pass through the same point 1. Assume a value 1–2 for P and draw the force triangle 1–2–3; this gives the relation between P, $F_1$, and $F_2$; the strap $S_1$ is acted upon by 3 forces—$F_1$; $R_1$, acting through the assumed center of pressure C and tangent to the friction circle, and a force acting through the pin G. $F_1$ and $R_1$ produced intersect at $O_1$ and through this point the force acting through G must also pass. $F_1$ being known in 2–3, draw the force triangle C–4–5 and as in the previous case resolve $R_1$ into its tangential C–6 and normal 5–6 components.



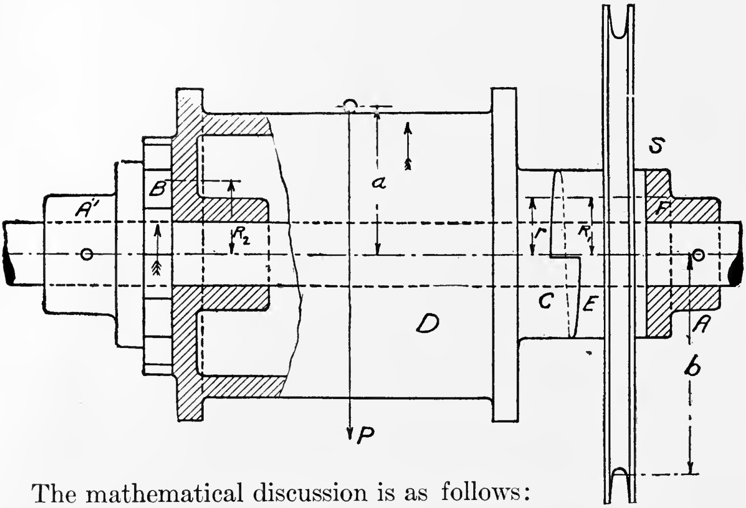

The strap $S_2$ must be treated in a similar way; the forces $F_2$, $R_2$ and a force through G acting upon the brake wheel are found for this side. The braking effect then is $\frac{T_1+T_2}{P}$.

The action must now be analyzed for rotation in the opposite direction, if the shaft runs in either direction, and the lower of the 2 ratios used.

## Brakes.

The action of a Weston or plate brake will be best understood by first explaining the brake in Fig. 205. The two collars A are rigidly attached to the shaft. The ratchet-wheel B is fastened to the collar $A^1$. At the right end of the drum is an extension whose face is a helix which mates with a corresponding helix on a collar attached to the sheave wheel S. The ratchet-wheel B turns freely in the direction shown by the arrow, but is held by the pawl from rotating in the opposite direction. The drum D turns on journals on the shaft. The load P tends to turn the drum in the opposite direction to that shown on the top of the drum by the arrow. Suppose the load acting to run the drum backwards the forces acting on the helix produce an axial pressure along the shaft which makes the right hand face of the ratchet wheel bear against the adjacent side of the drum. The ratchet wheel is kept from rotating by the pawl and the axial pressure also makes the right hand face of the sheave wheel hub bear against the adjacent collar. If the frictional resistance between B and D plus that between F and E is sufficient the load will be sustained.

Fig. 205.




The mathematical discussion is as follows:

The symbols are—

$a$ = pitch radius of drum.

P = load in pounds on the drum.

$R_2$ = mean radius of the friction faces at B.

$R_1$ = mean radius of the friction faces at F.

$\mu_2$ = coefficient of friction between faces at B. $\mu_2 = \tan \Theta_2$.

$\mu_1$ = coefficient of friction between faces at F. $\mu_1 = \tan \Theta_1$.

$\alpha$ = angle of helix. Measured at the pitch line.

$\rho$ = angle of friction between helix faces.

K = axial pressure due to helix faces.

$r$ = pitch radius of helix or screw.

If we now consider that while the shaft is held by the pawl and ratchet wheel that the load tries to turn the drum back over the friction face at B, and the helix face, we have

$$Pa = Kr \tan(\alpha + \rho) + \mu_2 KR_2 \qquad (1)$$

This will clamp the drum C and the helix face E and the load will be held providing the frictional moment at B plus that at F is equal to or greater than the moment due to the load on the drum.

$$Pa \overline{\gtrless} (\mu_2 K R_2) + (\mu_1 K R_1) \qquad (2)$$

If the angle of the helix was too steep the helix attached to the sheave wheel S would back off, requiring a pull on the sheave wheel to retain the load, hence we have the turning moment produced by the axial pressure K on the helix must be less than zero.

Turning moment on helix collar $\left.\right\} Kr \tan(\alpha - \rho) - (\mu_1 K R_1) \overline{\lessgtr} 0$ (3)

from which $r \tan(\alpha - \rho) \overline{\lessgtr} \mu_1 R_1$ (4)

In the case where $r = R_1$

$$\tan(\alpha - \rho) \overline{\lessgtr} \mu_1$$

Since the angles are very small and $\mu_1 = \tan \Theta_1$

$$\alpha \overline{\lessgtr} \rho + \Theta_1 \qquad (5)$$

In designing such a brake it will be necessary to take the angle of the helix less than the sum of the angles of friction between the helix faces and the collar faces at F, thus fulfilling condition 5.

The radii of the friction faces must be assumed and then K can be found by equation 1. Having determined K, the values must be substituted in equation No. 2 to see if the sum of the frictional moments exceeds $Pa$. If necessary, fiber can be used at the friction faces at F to increase the frictional moment here and decrease the tendency for the helix attached to the sheave wheel to back off. The unit pressure on the E friction faces must be determined to see that this pressure does not exceed that suited to the materials, so that the heat will be properly radiated. In an article in *Machinery* for July, 1906, the author states that, to allow for proper radiation in a plate or Weston brake, the friction surface should be one square inch for each 2000 to 3300 foot-pounds of energy absorbed by the brake per minute. Böttcher allows 3750 foot-pounds per minute per square inch of total effective brake surface in plate brakes.

Figs. 206 and 207 illustrate a lowering brake which can be used with a worm drive. If the worm has so small a pitch that the efficiency of the drive is under 50 per cent. no brake will be needed. Practically this efficiency should be well below 50 per cent. if a brake is not used, as vibration may cause the load to drop when there is not sufficient margin of security..

Fig. 206.

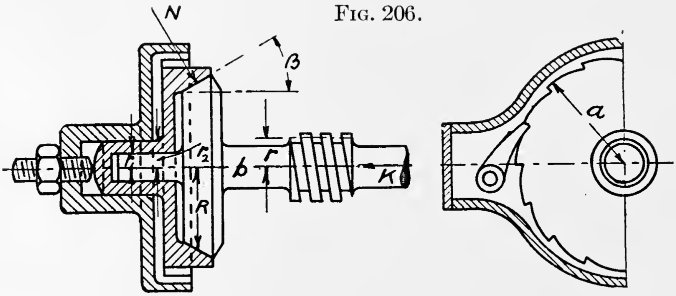
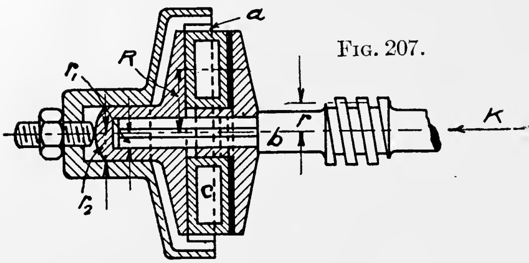



Fig. 207.




The calculation of the efficiency is more or less in doubt owing to the difficulty in determining the proper coefficients of friction. It is better practice to use a rapid worm and add the proper lowering brake.

In raising the load the ratchet wheel *a* passes freely under the pawl. When the driving forces stop, the load will start in the opposite direction; this motion will be immediately stopped by the pawl engaging with the ratchet wheel, and further lowering of the load must overcome the frictional twisting moment due to the friction cone faces *c* in Fig. 206 or the 2 flat faces in Fig. 207. As the axial pressure K will be proportional to the load, the friction faces can be designed to hold the load so that an additional turning moment will be needed to lower it.

The theory of Fig. 206 is as follows:

K = axial pressure due to worm.
$r$ = pitch radius of worm.
$\alpha$ = angle of worm helix at pitch circle.
$\rho$ = angle of friction between worm and wheel. $\tan \rho = \mu$.
$Fa$ = turning moment on worm due to K.
$r_1$ = radius of bearing.
$r_2$ = radius of step bearing.
R = mean radius of friction faces.
$\beta$ = angle of cone.
$\rho_1$ = angle of friction between cone faces. $\tan \rho_1 = \mu_1$.
N = normal pressure on cone surfaces.
$W = \mu_1 N$.
$a$ = radius of ratchet wheel.

$$F.a = Kr \tan (\alpha - \rho) - \mu F r_1 - \mu K r_2.$$

To prevent the lowering of the load

$$Fa \leqq W R \therefore F \leqq \frac{W R}{a}.$$

$$\frac{W R}{a} \geqq \frac{Kr \tan (\alpha - \rho) - \mu K r_2}{a + \mu r_1}.$$

$$W \geqq \frac{K[r \tan (\alpha - \rho) - \mu r_2]}{a + \mu r_1} \times \frac{a}{R}.$$

From the properties of the friction cone

$$W = \mu N = \frac{K \tan \rho_1}{\sin (\beta + \rho_1)}.$$

Equating the 2 values of W,

$$\operatorname{Sin} (\beta + \rho_1) \leqq \frac{(a + \mu r_1) \tan \rho_1 R}{[r \tan (\alpha - \rho) - \mu r_2] a}.$$

According to Bethman, $\beta$ is generally taken 21 degrees with 28 degrees a maximum, and the solution made for R.

$$R \geqq \frac{\sin (\beta + \rho_1) [r \tan (\alpha - \rho) - \mu r_2] a}{(a + \mu r_1) \tan \rho_1}.$$

Bethman gives also the following values:

$$\alpha = 22^\circ \quad \rho = 7^\circ \quad \mu = 0.08 \quad \mu_1 = \tan \rho_1 = 0.08.$$

The width of the cone surfaces should be determined in the same way as the faces of cone clutches.

In the straight faced friction surfaces Fig. 207 there will be 2 surfaces preventing lowering, so that

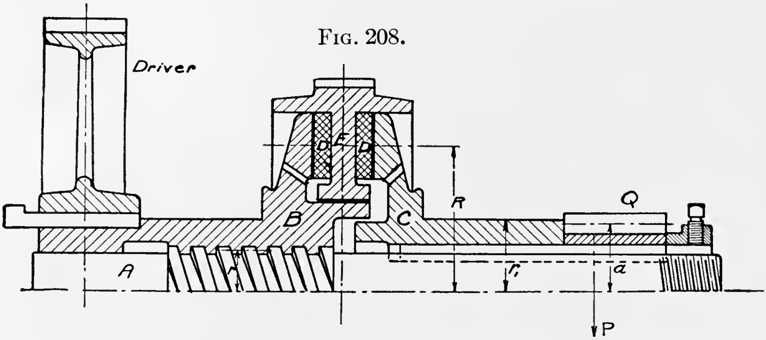
$$Fa \leqq 2\mu\ KR \text{ and } F \leqq \frac{2\mu\ KR}{a}.$$

From which it follows that $R \geqq \dfrac{[r \tan(\alpha - \rho) - \mu r_2]a}{2\mu(a + \mu r_1)}$.

## Mechanical Brakes.

Weston brake. One type of this brake is shown in Fig. 208. The driver meshes with the motor pinion. The load is raised by gearing meshing with the pinion Q. In raising the load, the

Fig. 208.




driver through the nut B, to which it is keyed, advances on the screw A, clamping the friction faces D–D to the ratchet wheel E, which during the raising of the load rotates freely through its pawls, driving the sleeve C to which the pinion Q is keyed. In lowering the load the driver must be made to rotate in the opposite direction, thus moving the nut B back upon the screw A, reducing the pressure between the faces D–D and E and permitting the load to back down the pinion Q.

The theoretical analysis is as follows:

$P$ = force on pinion Q due to load in lowering.
$a$ = pitch radius of pinion Q.
$r$ = pitch radius of screw.
$\alpha$ = angle of screw thread. Measured at pitch circumference.
$\mu$ = coefficient of friction between friction faces.
$\rho$ = angle of friction at thread.
$K$ = axial force along screw.
$R$ = mean radius of friction plates.
$r_1$ = radius of bearing at pinion Q.

Considering the moment produced by the load in trying to run down, we have

$$Pa - \mu r_1 P = Kr \tan(\alpha + \rho) + \mu_1 KR \tag{1}$$

In order that the load shall not drop,

$$Pa - \mu r_1 P \leqq 2\mu_1 KR \tag{2}$$

Equating these two values,

$$Kr \tan(\alpha + \rho) \leqq 2\mu_1 KR - \mu_1 KR.$$

$$r \tan(\alpha + \rho) \leqq \mu_1 R.$$

In designing a brake it is necessary to assume a number of the several variables, and the following values may be taken:

$\alpha = 8°$ to $15°$.

$\rho = 4°$

$R = 6$ inches to 9 inches.

$\mu_1 = 0.06$ to 0.10, depending upon the lubrication. This should be taken low to insure safety.

K can now be determined for the brake, using the formulæ just given; if this pressure reaches an excessive amount, allowing for sufficient radiation, either larger friction plates can be tried or a brake designed with a greater number of plates tried.

**Problem.**—In a crane brake $P = 2400$ pounds, $a = 3.35$ inches, $r = 2$ inches, $R = 7$ inches, $\alpha = 11$ degrees, $r_1 = 1\frac{1}{2}$ inches. Take $\mu_1$ at 0.05 and $\mu$ 0.10. Can a brake similar to the one just described be designed to do the work so that if it has to absorb work at the rate of 800,000 foot-pounds per minute it shall have a friction face surface of at least 1 square inch per each 3000 foot-pounds of work absorbed per minute?

From $Pa - \mu r_1 P \leqq 2\mu KR$

$$(2400 \times 3.35) - (0.05 \times 1.5 \times 2400) \leqq 2 \times 0.10 \times K \times 7.$$

$$K \geqq 5600$$

and $\quad r \tan(\alpha + \rho) \leqq \mu_1 R \therefore 2 \tan(11 + 3) \leqq 0.10 \times 7$

or $0.50 \leqq 0.70$, which shows brake will hold the load.

Assuming outer radius of friction plate 8½ inches, inner 5½ inches, then D=17 inches and $d$=11 inches. Area of plate 226−95= 131 square inches.

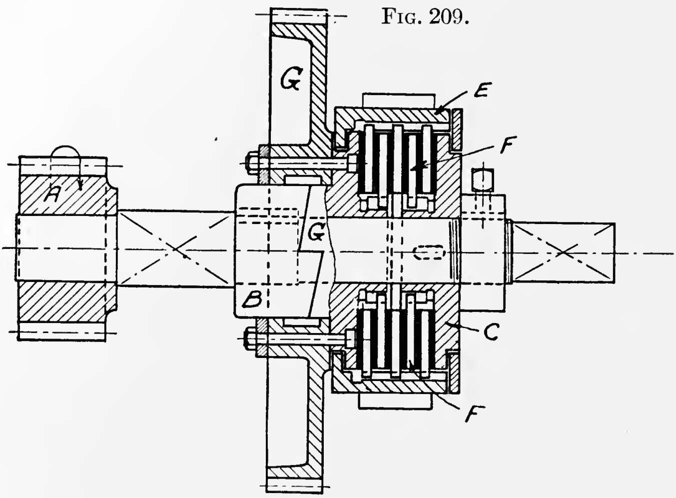
Surface required $\frac{800,000}{2\times 3,000}=135$ square inches. The above is near enough.

The direct pressure K per square inch is $\frac{5600}{131}=42.8$ pounds, which is sufficiently low.

When a greater number of friction plates than 2 is required the accompanying sketch shows a modification of the brake just described with additional plates.

In this brake the load in lowering tends to turn the pinion A, as shown by the arrow, carrying clutch B and disc C with it. The

FIG. 209.




drum is prevented lowering by a ratchet wheel and pawl or some other locking device. The disc C having the friction face F keyed to it, in running down brings 3 disc faces in sliding contact, from which we have, if

$c$=faces in sliding contact when disc C is moved relative to the drum E,

$c_1$=faces in sliding contact when discs attached to both G and C move relative to the drum E.

$$Pa=Kr\tan(\alpha+\rho)+c\mu KR.$$

Now if the load falls it must carry the gear G and the attached discs with it, from which

$$Pa \leqq c_1 \mu K R.$$

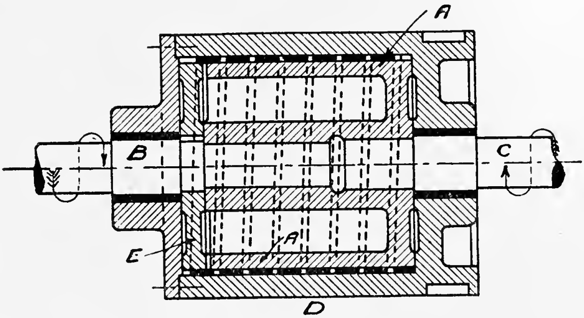
Hence to prevent the load from dropping, we have,

$$r \tan (\alpha + \rho) \leqq (c_1 - c)\mu R.$$

The other symbols are the same as used in the brake previously described.

Among the numerous types of safety lowering brakes the coil brake is considerably used. This is simply an internal form of band brake, and the ordinary band brake theory applies to its design. It makes use of several coils instead of a single coil as in the band brake.

FIG. 210.




The drum or collar A is placed over the driving shaft B and the driven shaft C being keyed to A. The coil is a flat helix. To insure contact with the cylinder D the external diameter of the coil is ground slightly larger than the internal diameter of the cylinder. The flange E which is keyed to the shaft B has one end of the coil fastened to it, while the other end of the coil is secured to the drum A.

The coil being the vital part of the brake is usually made either of special bronze bars coiled and then ground to proper diameters, or from a forged steel cylinder, turned to the proper dimensions and then cut into a helix.

The objection to this type of brake is that due to the pressure between the coil and the strap the wear is greater at the strap ends and that the action of the brake is materially affected by

Fig. 211.

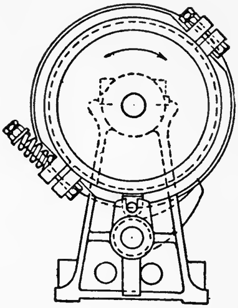
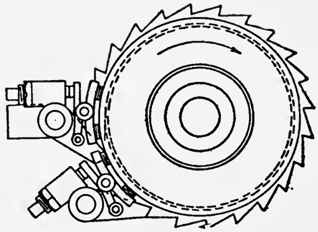
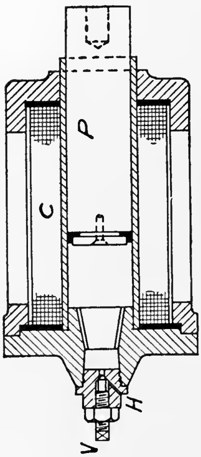


Fig. 212.



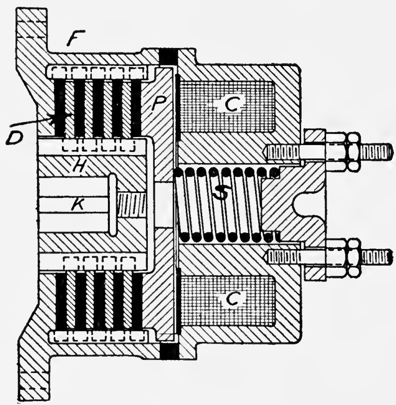
Friction devices for operating Pawls.

Fig. 213.




Solenoid.

Fig. 214.



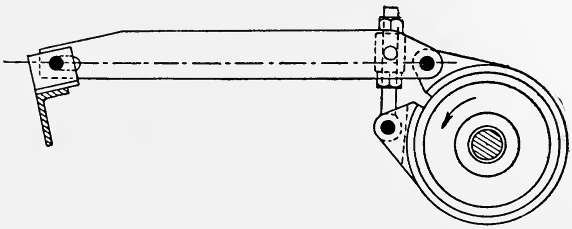

Magnetic Plate Brake.

this wear. A positive drive clutch should be placed between the driver and the driven parts so that before the wear becomes excessive this clutch will come into action.

In brakes requiring ratchet wheels and pawls the riding of the pawls over the ratchet-wheel teeth creates an objectionable noise, which is avoided by having the pawls removed from the ratchet wheel and when required brought into action automatically, as shown in Figs. 211 and 212. In Fig. 215 a design is shown in which the ratchet wheel and pawls are replaced by a brake which clamps in brake wheel when it attempts to rotate, as shown by the arrow.

It is frequently desirable to apply brakes by springs and release them with solenoids. The brake is generally a block brake, as illustrated in Figs. 195 and 197; the solenoid is shown in Fig. 213.

Fig. 215.



The solenoid consists of a plunger P sliding in a brass tube about which is rolled a coil of insulated copper wire C, the whole covered with the metal frame. The plunger is arranged with a packing, so that when it is drawn rapidly into the coil it forces the air behind it through the valve V and out of the hole H. This permits the valve to be so adjusted that the plunger is not drawn back too rapidly.

The application of an electrical release to a plate brake is shown in Fig. 214. In this case the spring S pushes the pole-piece P against the adjoining disc; this creates the necessary pressure between the several brass friction discs, a number of which are secured to the hub H, which rotates with the motor shaft being keyed at K. The other discs are held stationary by the housing F. When the motor is started the coils C being in series with it pull back the pole-piece P, thus relieving the pressure between the discs and releasing the brake from the motor.

### Electro-Magnets.

If a U-shaped steel or iron bar has insulated wire wound upon the limbs, passing current through the wire makes a magnet of the U. The holding or lifting power in pounds is

$$P = \frac{B^2 a}{72{,}134{,}000}$$

B = number of lines of force per 1 square inch of iron section, 110,000 for wrought-iron or soft steel.

$a$ = area of 1 pole face in square inches.

The ampere turns to produce the pull P is

$$nI = 2661 \frac{l}{\mu} \sqrt{\frac{P}{a}}$$

$l$ = length of magnetic circuit in inches.

$\mu$ = permeability.

Where the pull is exerted across an air space let $z$ be the width of the air gap in inches, then

$$nI = 2zB \times 0.3133$$

Generally the ampere turns required to force the flux through the metal can be neglected when an air-gap is considered.

### Solenoid and Plunger.

According to C. M. Sames, a solenoid and plunger can be calculated with sufficient accuracy by the following formula:

$$nI = 96\,P\,(L+1) \text{ and } A = 0.01\sqrt{In}$$

$nI$ = ampere turns.

P = 1.1 × pull in pounds desired.

L = length of solenoid in inches.

The pull can be assumed practically uniform through a distance 0.5L.

The current density over the gross section for momentary work, the magnet having ample surface and being well ventilated, should not exceed 2000 to 3000 amperes per square inch. For continuous work the current density on the gross section should be limited to from 300 to 400 amperes. The gross section equals the length of the solenoid L multiplied by the depth of the winding T.

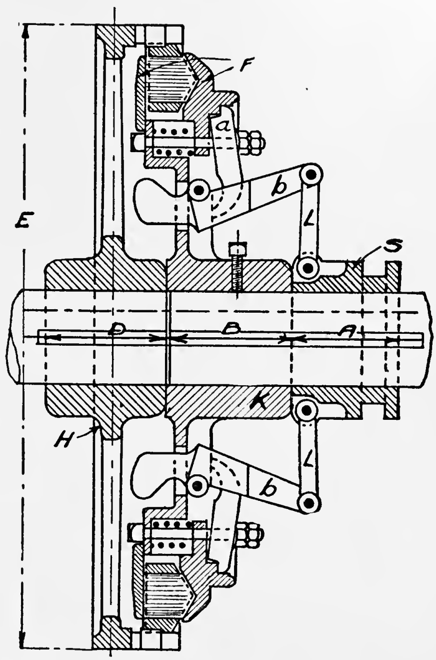
If D is the gross density in amperes $D = \frac{LT}{nI}$.

The coil should be designed so that the watts it consumes should not exceed from 1¼ to 3 watts (accordingly as its service is more or less intermittent) per square inch of its lateral surface.
See page 301 for properties of copper wire.

## Friction Clutches.

The design of friction clutches is quite similar to that of brakes. A frictional moment is produced in both cases; in a brake it opposes an equal turning moment, thus sustaining the load, while in the case of the clutch this frictional moment overcomes a turning moment, thus driving the machine. There is this important difference, however: the clutch is merely called upon to transmit motion, there preferably being little or no slipping between the clutch parts, whereas the mechanical brake, in permitting the load to be lowered, must absorb all this energy, transforming it into heat. Hence the clutch need not be designed as carefully against heating as a brake.

Fig. 216.




Two illustrations of regularly manufactured friction clutches are shown, the first of the Moore & White Company, Philadelphia.

In this clutch the wooden friction blocks are inserted in the disc D, which travels with the hub H. The follower K carrying the friction faces F is keyed to the other shaft and the cut-off sleeve S also slides on a feather on this shaft. Throwing the cut-off sleeve into the position shown raises the levers *a* and *b* by the links L, thus clamping the friction faces.

In the Cresson clutch the friction faces are circumferential instead of disc surfaces, otherwise understanding the Moore & White clutch will make the action of the Cresson clutch evident.

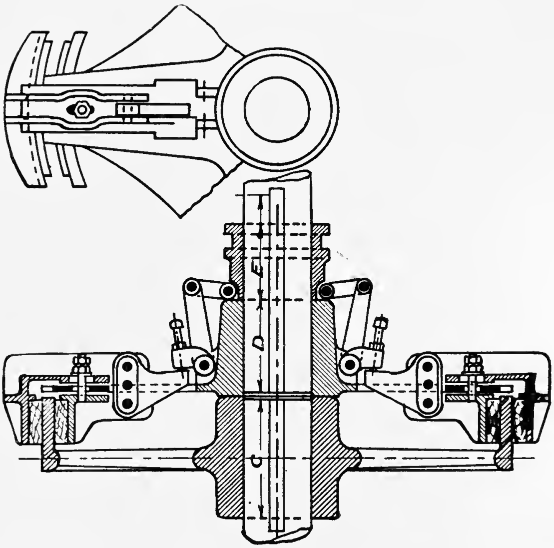
CLUTCH PROPORTIONS. (Fig. 216.)

| Size | Outside Diameter | Space required including movement | Largest Bore | Will drive pulley as large as | | Capacity at 100 R. P. M. | Weight |
|---|---|---|---|---|---|---|---|
| | | | | Diameter | Face | | |
| *Inches* | *Inches* | *Inches* | *Inches* | *Inches* | *Inches* | *H. P.* | *lbs.* |
| 5 | 7¾ | 6¼ | 1 11/16 | 10 | 3 | 1¾ | 22 |
| 6 | 9 | 6¾ | 1 15/16 | 12 | 4 | 2½ | 28 |
| 8 | 11 | 6⅞ | 2 7/16 | 18 | 5 | 5 | 46 |
| 10 | 13 | 8¼ | 2 15/16 | 21 | 6 | 7 | 76 |
| 12 | 15½ | 10¼ | 3 15/16 | 27 | 8 | 12 | 142 |
| 14 | 18 | 12½ | 4 7/16 | 32 | 10 | 18 | 215 |
| 16 | 20¼ | 13 | 4 15/16 | 35 | 12 | 25 | 255 |
| 18 | 22¼ | 13¼ | 5 15/16 | 42 | 14 | 34 | 351 |
| 20 | 25½ | 16¼ | 5 15/16 | 50 | 16 | 45 | 544 |
| 24 | 29¾ | 17 | 6 7/17 | 60 | 18 | 65 | 810 |
| 28 | 34 | 19½ | 7 15/16 | 66 | 22 | 85 | 1050 |
| 32 | 39 | 21¾ | 7 15/16 | 72 | 24 | 112 | 1450 |
| 36 | 43 | 23¼ | 8 7/16 | 78 | 26 | 142 | 1800 |
| 42 | 49½ | 25¼ | 9 7/16 | 84 | 30 | 180 | 2500 |
| 48 | 58 | 26½ | 9 15/16 | 96 | 36 | 240 | 3200 |
| Double Disc. 36 | 43½ | 23¾ | 8 7/16 | 108 | 42 | 284 | 2100 |
| Double Disc. 42 | 50 | 26 | 9 7/16 | 120 | 48 | 360 | 3000 |
| Double Disc. 48 | 58 | 27¼ | 9 15/16 | 144 | 54 | 480 | 3700 |

| SIZE OF CLUTCH. | A | B | C | D | E | SIZE OF CLUTCH | A | B | C | D | E |
|---|---|---|---|---|---|---|---|---|---|---|---|
| 5¼ | 3½ | 2¾ | 2⅝ | 2¾ | 7¾ | 24 | 9¾ | 7¼ | 6¼ | 6½ | 29¾ |
| 6½ | 4 | 2¾ | 2⅝ | 2¾ | 9 | 28 | 11¼ | 8¼ | 7¼ | 7½ | 34 |
| 8 | 3⅞ | 3 | 3⅛ | 3¼ | 11 | 32 | 12½ | 9¼ | 7½ | 7¾ | 39 |
| 10 | 4½ | 3¾ | 3¾ | 4 | 13 | 36 | 11½ | 8 | 7½ | 7¾ | 40 |
| 12 | 5⅞ | 4⅜ | 4¾ | 5 | 15½ | 42 | 12 | 9 | 8 | 8¼ | 46½ |
| 14 | 7⅜ | 5⅛ | 5 | 5¼ | 18 | 48 | 13 | 10 | 8½ | 8¾ | 52¾ |
| 16 | 7½ | 5½ | 5 | 5¼ | 20¼ | 36 Double | 12 | 10 | 9 | 9¼ | 40 |
| 18 | 7⅝ | 5½ | 5¾ | 6 | 22¼ | 42 Double | 12½ | 11 | 10 | 10¼ | 46½ |
| 20 | 9¾ | 6½ | 5¾ | 6 | 25½ | 48 Double | 13 | 12 | 10 | 10¼ | 52¾ |

All dimensions in inches.

FIG. 217.




Standard Friction Clutch Cut-off Couplings, with Four- and Six-Arm Clutches.

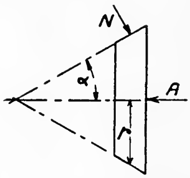
(Fig. 217.)

| Diam. Coupling | H. P. 100 Rev. | Equal to Shaft | SPACE REQUIRED | | | Shipping Weight |
|---|---|---|---|---|---|---|
| | | | C | D | E | |
| 18 | 8 | 1 11/16 | 5 | 4 | 7½ | 196 |
| 20 | 11 | 1 15/16 | 6⅜ | 5 | 7½ | 251 |
| 22 | 15 | 2 3/16 | 6¾ | 5½ | 7½ | 314 |
| 24 | 18 | 2 7/16 | 7 | 6 | 9 | 409 |
| 28 | 29 | 2 11/16 | 7⅞ | 7 | 9 | 554 |
| 32 | 43 | 3 3/16 | 8⅞ | 7½ | 9 | 684 |
| 36 | 57 | 3 11/16 | 9⅝ | 7½ | 10 | 890 |
| 36 | 75 | 4 3/16 | 9⅝ | 7½ | 10 | 1,057 |
| 40 | 90 | 4 7/16 | 10⅜ | 8 | 10½ | 1,180 |
| 40 | 107 | 4 ¾ | 10⅜ | 8 | 10½ | 1,428 |
| 48 | 150 | 5 | 11½ | 9 | 11 | 1,701 |
| 48 | 224 | 5 ¼ | 11½ | 9 | 11 | 2,050 |
| 54 | 275 | 5 ½ | 12¾ | 10½ | 13 | 2,321 |
| 54 | 320 | 6 | 12¾ | 10½ | 13 | 2,857 |
| 60 | 403 | 7 | 12¾ | 17¼ | 15 | 5,018 |
| 72 | 756 | 8 | 14¼ | 29 | ... | 9,300 |
| 84 | 1,200 | 10 | 16 | 31 | ... | 14,800 |

FRICTION DRUM CLUTCHES.—In hoisting engines it is desirable to have the hoisting drum quickly thrown in or out of action, and this is accomplished by a friction clutch. A ratchet wheel is attached to the drum, and this wheel when engaged with its pawl prevents the load running down. As it is not desirable to use the clutch as a lowering brake the lowering of the load is accomplished through a strap brake attached to the drum. To lower the load by this brake it is necessary to throw out the pawl mentioned above.

The theory given for cone brakes applies to this clutch, excepting that the clutch would be thrown into action while the other half of the clutch was in motion, hence the frictional resistance to axial motion of the faces upon each other can be neglected, making the formulæ previously given,

Fig. 218.




$$A = 2N \sin \alpha$$

$$N = \frac{A}{2 \sin \alpha}$$

$$M = Fr = \frac{\mu A r}{2 \sin \alpha}$$

Here A = axial force.
N = Pressure normal to the friction faces.
$\alpha$ = ½ angle of the cone.
M = F.$r$ – turning moment to be transmitted by clutch.
$r$ = radius of friction face (mean).

C. F. Blake, in *Machinery* for January 7, 1907, gives the following proportions: The angle ranges from 5 to 10 degrees. This angle is ½ the angle of the cone, Fig. 219.

Figure 220 shows the application of this type of clutch to a hoisting drum.

| | |
|---|---|
| $A = 4d$ to $8d$. | $e = 0.3d + 0.1$ inch. |
| $f = 0.2d + 0.1$ inch. | $k = 0.2d + 0.3$ inch. |
| $B = A - (2f + 0.257$ inch). | $g = 0.8d$ to $2d$. |
| $h = 2d + 1$ inch. | $D = 1.8d + 0.5$ inch. |
| $c = 0.5d$. | $F = 2d$. |
| $a = 0.3d + 0.3$ inch. | $m = 2d$. |
| $b = 0.4d + 0.4$ inch. | $n = d$ to $1.5d$. |

The shaft M has one portion of the clutch C attached to it and on the outside of this part of the clutch is the gear G, which is in mesh with a pinion receiving its motion from the motor or engine driving the drum. A hand-lever attached to the screw A when turned in the proper direction forces the rod L against the key K; this key in turn pushes against the collar N, which in turn forces the drum D carrying the wooden friction blocks against the other side of the clutch C.

Fig. 219.

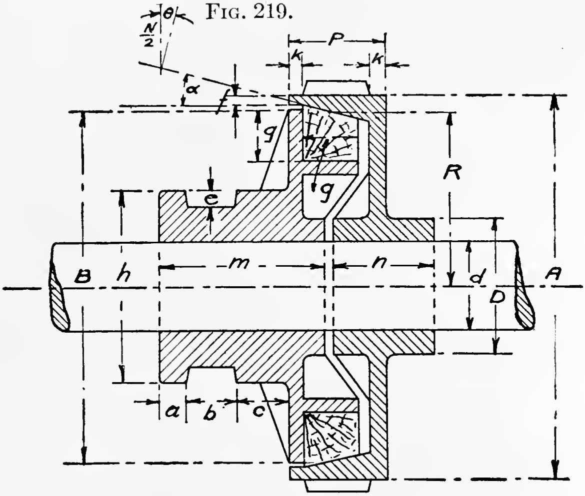
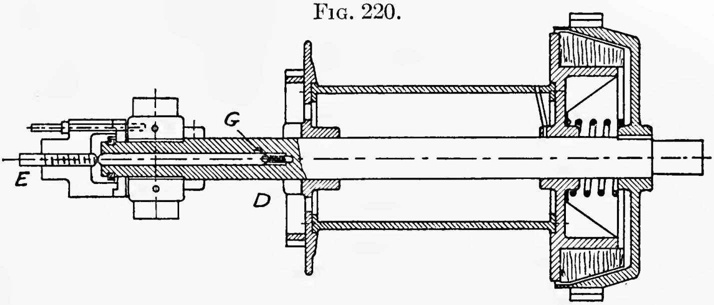



Fig. 220.

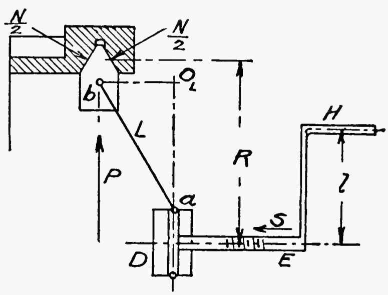



Another type of clutch for hoisting drums is that used by the Lidgerwood Manufacturing Company.

$M = W.r$ = turning moment on the drum due to load. This must be less than $\mu NR$ if this frictional moment is to drive the drum.

Hence $Wr \leq \mu NR = \dfrac{\mu PR}{\sin \alpha}$

Fig. 221.



The easiest way to find the relative velocities of the points $a$ and $b$ is by use of a little kinematics.

Draw $aO_L$ parallel to P and $bO_L$ parallel to screw axis. $O_L$ is then the instant center of the link L and the relative velocity of points in L will be proportional to their respective distances from this center $O_L$, or

$$\frac{\text{Velocity of the point } b}{\text{Velocity of the point } a}=\frac{\overline{bO_L}}{\overline{aO_L}}$$

from which the

$$\frac{\text{Pressure at P}}{\text{Pressure at S}}=\frac{\overline{aO_L}}{\overline{bO_L}} \quad i.e.\ \text{P}=\text{N}\frac{\overline{aO_L}}{\overline{bO_L}}.$$

If the efficiency of the linkage is taken at 90 per cent., then

$$Wr \leqq \frac{\mu R}{\sin \propto}\times\frac{9}{10}\,\frac{N.\overline{aO_L}}{\overline{bO_L}}.$$

If it is still desired to find the forces at the end of the screw lever we have, if

$l$ = length or radius of the hand-lever.

H = force on lever in pounds.

$\beta$ = angle of screw thread.

$r^1$ = pitch radius of screw.

$\phi$ = angle of friction of screw thread = $\text{tang}^{-1}\,\mu$.

$$H.l=Sr^1\tan(\beta+\phi)$$

$$S=\frac{H\times l}{r^1\tan(\beta+\phi)}$$

$$Wr\leqq\frac{\mu R}{\sin\propto}\times\frac{9}{10}\,\frac{\overline{aO_L}}{\overline{bO_L}}\times\frac{H\times l}{r^1\tan(\beta+\phi)}$$

The following assumptions can be made:

$\propto$ = 15 degrees.

$\mu$ = wood on metal. (0.30 to 0.50).

$\mu$ = metal on metal. (0.18 to 0.25).

## Clutches for Hoisting Engines.

The analysis of this clutch is similar to the previous one, excepting for the omission of the toggle levers. Using the same nomenclature, we have

$$Wr \leqq \mu FR = \frac{\mu NR}{\sin \propto}$$

Assuming the efficiency of the mechanism at 90 per cent.,

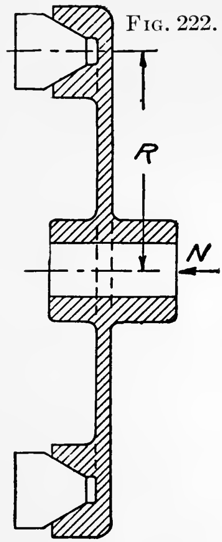
$$Wr \leqq \frac{9}{10} \frac{\mu RN}{\sin \propto}$$

$$N = \frac{QR_1}{r_1 \tan (\beta + \phi)} \text{ and}$$

$$Wr \leqq \frac{9}{10} \frac{\mu R}{\sin \propto} \times \frac{QR_1}{r_1 \tan (\beta + \phi)}$$

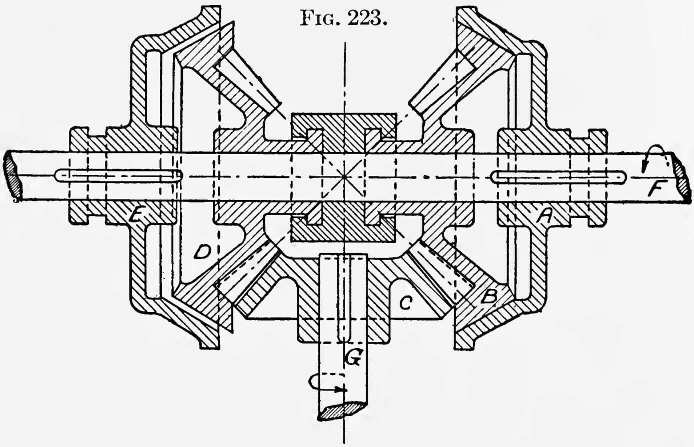
The same general assumptions can be made as in the preceding case.

Fig. 222.




## Reversing Clutches.

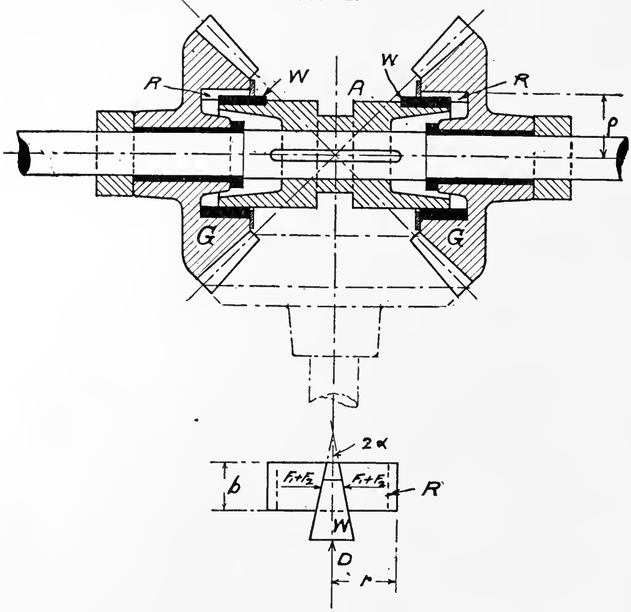
Fig. 223.




The above figure illustrates a common type of reversing clutch. The shaft F turns continuously in the direction shown by the arrow. If the clutch half A which slides on a feather on the shaft is engaged with its companion part B the entire clutch and the gear forming a part of it will rotate with the shaft. Bevel gear C, which meshes with gear B, will then rotate as shown by the arrow on G. Had the clutch ED been thrown into action the gear C and shaft G would have been rotated in the opposite direction to that shown by the arrow.

This clutch is designed simply as a cone clutch, see p. 166.

Fig. 224.




The action of this clutch is similar to the one just described. The sleeve A slides on a feather on the shaft. Attached to this sleeve are 2 wedges W. When the sleeve is moved to one side or the other a wedge W is forced into a corresponding ring R, causing it to expand until sufficient frictional force is produced between this ring and its surrounding cylinder to drive the gear G.

The following theory relates to this clutch:

E = modulus of elasticity of the material of the ring.

$r$ = radius of the ring before being expanded.

$\rho$ = radius when in contact with the cylinder.

I = moment of inertia of the ring section. $=\frac{bd^3}{12}$.

$b$ = width of the ring.

$d$ = depth of the ring.

$p$ = radial pressure per square inch between ring and cylinder.

$\mu$ = coefficient of friction. (0.10.)

C = force at circumference of the ring available for producing turning moment.

$F_1$ = force required to take up clearance between ring and cylinder.

$F_2$ = force required to make the value of C sufficient to give the desired turning moment.

The following formulæ are readily derived:

$$F_1=\frac{(\rho-r)\,EI}{r^3}, \qquad F_2=p.r.b.$$

$$C=2\pi r\mu bp,$$

$$F_2=1.59C, \qquad D=2(F_1+F_2)\tan(\alpha+\phi)$$

If $\alpha=11°$ and $\phi=6°$

$$D \frown U$$

According to Bethman, when the ring is cast-iron the unit pressure should not exceed 140 pounds per square inch. The thickness of the ring can be made $\frac{1}{20}$ of its diameter. If the ring is made of phosphor-bronze this pressure can be increased to 200 pounds per square inch. The outer diameter of the ring is made from $\frac{1}{32}$ inch to $\frac{3}{32}$ inch smaller than the internal diameter of the cylinder.

The fiber stress in the ring can be determined by the following formula. The maximum stress will be compression.

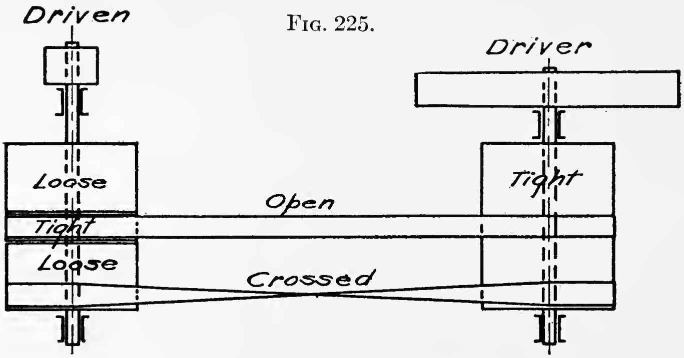
$$p_c=\frac{6F_1r}{bh^2}+\frac{F_2}{bh}.$$

## Open and Crossed Belts for Reversing.

Reverse driving may be accomplished by means of an open and a crossed belt. The driving pulley is the wide pulley and the driven pulley is the narrow one keyed to the other shaft and having a loose pulley upon each side of it. The loose pulleys are each slightly greater than twice the width of the narrow keyed pulley. The belts are readily shifted across the faces of these pulleys so that the driving may be done by either the open or crossed belt.

A belt must be shifted as it runs on a pulley. This arrangement is frequently used where elevators are driven by line-shafting. See Fig. 225.

Fig. 225.




## Power Exerted by Men.

The force a man or an animal can exert will vary with the individual and with the duration of the work, the force being much larger when exerted for only a short time.

The following are average values taken from several sources:

| | Pounds | Velocity ft. per min. |
|---|---|---|
| Man, operating crank | 20 to 40 | 150 |
| Man, operating capstan | 40 to 60 | 160 |
| Man, raising load by pulling rope through block | 40 | 45 |
| Man, max. pull vertically down | 80 | |

Average draught horse working capstan, 100 to 120 pounds; velocity per minute, 215 feet.

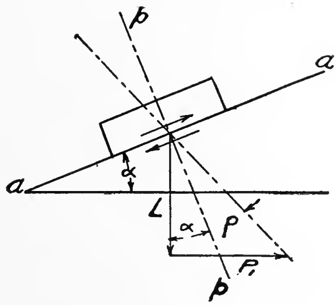
Where the full effect of a man's effort may come upon a machine, as in the machinery of a draw-bridge, it should be taken at 125 pounds per man, or 150 pounds if only 1 man is working.

Cranks.—Cranks should be designed and placed so that men can work them without unnecessary discomfort. The crank radius should not exceed 16 inches, say from 12 inches to 16 inches. The crank handle should have a length of from 10 inches to 14 inches for 1 man and from 16 inches to 20 inches for 2 men.

The crank shaft ordinarily should be placed from 2 feet 9 inches to 3 feet 3 inches above the floor or platform upon which the operators stand.

# PART IV.—WINCHES AND HOISTS.

## Screw Jacks.




$P_1$ = horizontal force required to raise load L.

$P_2$ = horizontal force required to lower load L.

L = load in pounds.

$\alpha$ = angle of screw in degrees.

$\rho$ = angle of friction; tang $\rho = \mu$.

In raising the load the resultant of L and P must make an angle $\rho$ with the perpendicular to the line *aa* upon which sliding occurs. From the figure $P_1 = L$ tang $(\alpha + \rho)$. In lowering the load $P_2 = L$ tang $(\alpha - \rho)$. From the second equation it is evident that if $\rho$ exceeds $\alpha$ the force $P_2$ becomes negative, *i.e.*, it will have to be applied to prevent load from lowering.

If $r$ = mean radius of the screw,

$l$ = length of the lever in inches.

$F_1$ = force in pounds at the end of the lever $l$ raising the load.

Then, raising the load $F_1 l = Lr$ tang $(\alpha + \rho)$

lowering the load $F_2 l = Lr$ tang $(\alpha - \rho)$

Neglecting friction makes $\rho = o$, hence

$F_o l = Lr$ tang $\alpha$, and

Efficiency raising $\eta = \frac{F_o l}{F_1 l} = \frac{\text{tang } \alpha}{\text{tang } (\alpha + \rho)}$

Efficiency lowering $\eta = \frac{F_2 l}{F_o l} = \frac{\text{tang } (\alpha - \rho)}{\text{tang } \alpha}$

Bethman gives as follows,

$$L = \frac{\pi d_1^2}{4} \times p, \text{ from which } p = \frac{4\,L}{\pi d_1^2}$$

and $$M_T = F_1 l = Lr \text{ tang } (\alpha + \rho) = \frac{d_1^3 p_s}{5} \quad p_s = \frac{5M_T}{d_1^3}$$

Since $r=\frac{d_1}{2}$ ; $p_s=\frac{5\,M_T}{d_1^3}=\frac{5\,L\,\text{tang}\,(\alpha+\rho)}{2\,d_1^2}$

The equivalent compressive fiber stress is

$$p_c=0.35\,p_c+0.65\sqrt{p_c^2+4\,B^2\rho_s^2}$$

The allowable stresses are

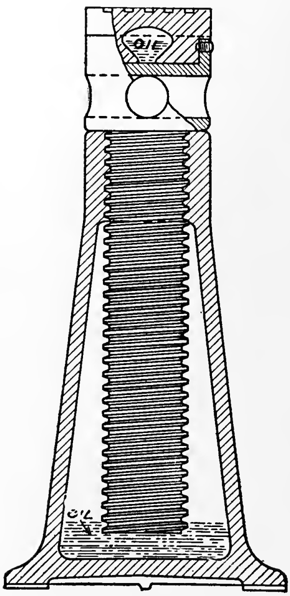
| | $p_c$ | $p_s$ | B |
|---|---|---|---|
| Wrought-iron . . . . . . . . . . . | 8,500 | 5700 | 1.15 |
| Steel . . . . . . . . . . . . . . . . . . . | 13,000 | 8500 | 1.15 |

$$B=\frac{p_c}{1.3p_s}$$

The pressure upon the projected area of the thread should be limited to,

| Material. | Pounds per square inch. |
|---|---|
| Brass nut. . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 1000 |
| Cast-iron nut . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . . | 700 |

Fig. 226.




The thread commonly used on screw-jacks is either the square thread or Acme thread.

| No. of jack | Height over all closed down | Safe rise with load | Height over all screwed up to limit | Diameter of screw. |
|---|---|---|---|---|
| | *Inches* | *Inches* | *Inches* | *Inches* |
| 4 | 4 | 1½ | 5½ | 1 |
| 6 | 6¾ | 2½ | 9¼ | 1½ |
| 8 | 8 | 3¼ | 11¼ | 1¾ |
| 10 | 10 | 4½ | 14½ | 1⅞ |
| 12 | 12 | 6 | 18 | 1⅞ |
| 14 | 14 | 7 | 21 | 2¼ |
| 16 | 16 | 9 | 25 | 2¼ |
| 20 | 20 | 12 | 32 | 2½ |
| 24 | 24 | 16 | 40 | 2½ |
| 28 | 28 | 20 | 48 | 2½ |
| 30 | 30 | 22 | 52 | 2½ |
| 36 | 36 | 28 | 64 | 2½ |

## Design of Winch.

A winch to be capable of exerting a pull of 5300 pounds at the circumference of a drum 12 inches in diameter. The gear reduction to be such that 2 men can operate it not exerting an effort of more than 40 pounds each on the crank. Assume 2 reductions and a probable efficiency of 85 per cent.

The stress in a 9/16 inch plow-steel wire rope of 6 strands with 37 wires each, carrying 5300 pounds and winding upon a 12 inch drum, is

$$p_T = \left(\frac{S}{i \times \frac{\pi \delta^2}{4}}\right) + \left(\frac{3}{8} \times \frac{E}{D}\right) \; ; \; \delta = \frac{0.56}{21} = 0.027 \text{ inch.}$$

$$p_T = \left(\frac{5300}{222 \times \frac{3.14 \times 0.027^2}{4}}\right) + \left(\frac{3}{8} \times \frac{30{,}000{,}000}{12}\right)$$

$$p_T = 41{,}700 + 9360 = 51{,}060 \text{ pounds}$$

$$\text{Factor of safety} = \frac{220{,}000}{51{,}060} = 4.35.$$

The gear ratio required will be

$$\frac{5300 \times 6}{0.85} = 2 \times 40 \times 15 \times x.$$

$$x = \frac{5300 \times 6}{0.85 \times 2 \times 40 \times 15} = 31.2, \text{ say } 31.$$

If it is desired that the 2 reductions be made equal, each reduction will be, $\sqrt{31} = 5.57$.

The twisting moment on pinion No. 1, Fig. 232, is

$$\frac{5300 \times 6}{.95 \times .98 \times 5.5} = 6220 \text{ inch-pounds.}$$

The smallest pinion having 12 teeth that can be used here can be determined by the formula

$$p_c = \sqrt[3]{\frac{2\pi M}{3 \times n \times s\left(0.124 - \frac{0.684}{n}\right)}}$$

As the velocities are low and as these gears will be hand driven the fiber stress can be taken high, 8000 pounds.

$$p_c = \sqrt[3]{\frac{2 \times 3.14 \times 6220}{3 \times 12 \times 8000 \times 0.067}} = 1.265 \text{ inches}$$

The diametral pitch is $\frac{3.1416}{1.265} \approx 2\frac{1}{2}$

The pitch diameter of the pinion with 12 teeth is $\frac{12}{2.5} = 4.8''$.

Making the gear 26 inch pitch diameter would give 65 teeth and make the gear ratio 5.4 :1. The gear formula used assumes the face as 3 times the circular pitch; this makes the face 3.75 inches wide.

If the total reduction is to be 31 :1, the first reduction being

$$5.4:1, \text{ the second must be } \frac{31}{5.4}=5.7:1.$$

The twisting moment on shaft IX is,

$$\frac{5300\times 6}{0.98\times 0.95^2\times 31}=1160 \text{ inch-pounds.}$$

The pitch of the smallest pinion that will be strong enough will be found as before,

$$p_c=\sqrt[3]{\frac{2\pi M}{3\times n\times s\times\left(0.124-\frac{0.684}{n}\right)}}=$$

$$\sqrt[3]{\frac{2\times 3.14\times 1160}{3\times 12\times 8000\times 0.067}}=0.723 \text{ inch.}$$

This corresponds to 4 diametral pitch, and the

$$\text{Pitch diameter of pinion is, } \frac{12}{4}=3 \text{ inches.}$$

To obtain the desired gear ratio the pitch diameter of the gear must be $5.7\times 3=17.1$; 17 inches will be sufficiently close for our purposes, and the gear will then have $17\times 4=68$ teeth.

The face of the gears by this formula being 3 times the circular pitch, will be, $0.723\times 3=2.17$ inches, say 2.25 inches.

To estimate the shafting it will be necessary to know the forces acting at the pitch lines of the gears.

The force between pinion No. 1 and gear No. 2 will equal the twisting moment on shaft VIII divided by the pitch radius of the pinion I.

$$\text{Force on face of pinion I}=\frac{6220}{2.4}=2600 \text{ pounds.}$$

$$\text{Force on face of pinion IV}=\frac{1160}{1.5}=775 \text{ pounds.}$$

## WINCH.

SHAFT VII.—If the gear rim is fastened to the rope-drum there will be no torsion transmitted by the shaft. Combining the rope load and the tooth load graphically and as an approximation assuming that they act in the same plane, *i.e.*, at the end of the drum, the resultant is found to be 6300 pounds. Assuming further that this acts 18 inches from one bearing and that the distance between bearings is 24 inches, the reaction at the left is

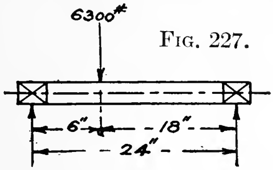


FIG. 227.

$$\frac{6300\times 18}{24}=4700 \text{ pounds}=R_1$$

If the maximum bending is assumed as being at the middle of the gear face, the bending moment is

$$M=4700\times 4=18{,}800 \text{ inch-pounds.}$$

From this the shaft diameter is

$$d=\sqrt[3]{\frac{10M}{f_B}}=\sqrt[3]{\frac{10\times 18{,}800}{9000}}=2.75 \text{ inches.}$$

If the width of the bearing is 4 inches the bearing pressure per square inch of projected area of the bearing will be

$$\frac{4700}{2.75\times 4}=430 \text{ pounds, which is satisfactory.}$$

SHAFT VIII.—Torsional moment, as previously found, 6220 inch-pounds.

$$\text{Approximate bending moment}=\frac{2600\times 20}{24}\times 4=8670$$

$$M_{E.B.}=(0.35\times 8670)+0.65\sqrt{6200^2+8670^2}=10{,}230 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10\times M_{E.B}}{f_B}}=\sqrt[3]{\frac{10\times 10{,}230}{10{,}000}}=2.17 \text{ inches.}$$

Before designing the crank-shaft it will be necessary to at least determine the forces acting in the brake and ratchet wheel. We will therefore design the brake. The turning moment on this shaft has been estimated at 1160 pounds; assuming the diameter of the brake wheel as 10.5 inches.

The value of $k$ for band brakes with an assumed coefficient of friction between leather and iron of 0.30, and an arc of contact of 70 per cent. of a complete circumference, is 3.74. The load P to operate the brake acting with a leverage of 16 inches is found to be,

$$P = \frac{1160 \times 2.5}{5.25 \times 16} \times \frac{1}{(3.74-1)} = 12.6 \text{ pounds.}$$

The total frictional force acting between the brake band and the brake wheel at the circumference is $F = \frac{1160}{5.25} = 221$

$$t = \frac{F}{k-1} = \frac{221}{3.74-1} = 80 \qquad T = 221 + 80 = 301$$

From this $t$ and T are found, $t$ being the tension in the band on the side of the wheel that would leave it if the band were free to move, while T is the tension in the side that would travel on the wheel if this were possible.

Fig. 230.

Fig. 228.

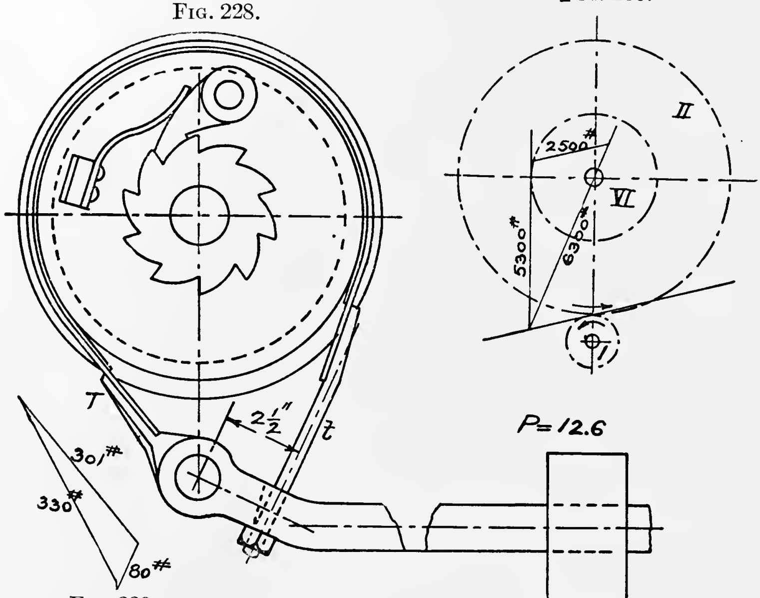



Fig. 229.

Ratchet Gear.—The twisting moment that this wheel opposes is that estimated above for the brake wheel, 1160 inch-pounds. Assuming the width of the ratchet as one inch, the working fiber stress at 2500 pounds on cast-iron, allowing in the case of ratchet wheels for considerable shock. The number of teeth will be taken as 10.

$$p_c=\sqrt{\frac{16\pi M}{f.s.z}}=\sqrt{\frac{16\times 3.14\times 1160}{1.0\times 2500\times 10}}=1.53 \text{ inches.}$$

Taking the circular pitch 1.57 inches we will have 2 teeth per 1 inch of outside diameter, or the outside diameter for 10 teeth will be 5 inches.

Pin Holding Ratchet.—Force on ratchet is $\frac{1160}{2.5}=465$ lbs.

If the distance between the center of the ratchet and the center of support of the pin is taken at 1.5 inches, the bending moment is, $M=465\times 1.5=700$ inch-pounds.

The diameter of the pin is

$$d=\sqrt[3]{\frac{10\times M}{fs}}=\sqrt[3]{\frac{10\times 700}{10,000}}=0.888 \text{ inch, say } \tfrac{7}{8} \text{ inch.}$$

Section of the Brake Band.—$T=301$. Area of the strap $=300/8000=0.0375$ square inch.

If the net section of the strap was made $1.25\times 0.0625=0.078$ square inch, it would be ample. If two ⅜ inch diameter bolts were used in line across the band, the width of the band would become $1.25+.75=2.0$ inches.

Shaft IX.—Since the brake wheel and the pinion will be located close to their respective bearings the greatest bending will be that at the pinion, the force at the pinion being 775 pounds, compared with 330 pounds at the brake wheel.

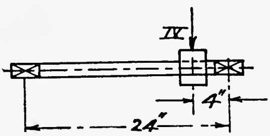
Reaction at the right,

$$\frac{775\times 20}{24}=645 \text{ pounds.}$$

Bending moment

$$M=645\times 4=2580 \text{ inch-pounds.}$$

Fig. 231.




It will now be necessary to see if the bending due to the cranks would likely exceed this.

$$\text{Force on cranks, } F=\frac{775\times 3}{30}=77.5 \text{ pounds.}$$

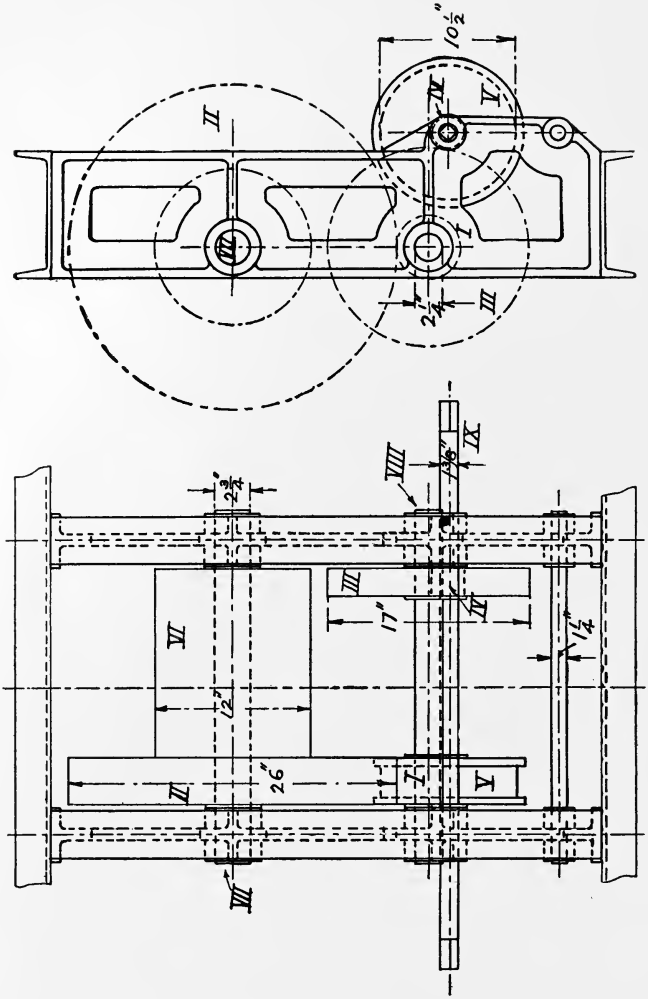



Fig. 232.

If this load were to come entirely on one crank, it would have to be a distance of $\frac{2580}{77.5}=33.2$ inches from the bearing, quite an unlikely dimension; the 2580 inch-pounds will therefore amply cover any bending.

$$M_{E.B.}=0.35M_B+0.65\sqrt{M_B^2+M_T^2}$$
$$=(0.35\times2580)+0.65\sqrt{2580^2+(775\times1.5)^2}$$
$$=900+1840=2740 \text{ inch-pounds}$$
$$d=\sqrt[3]{\frac{10\times M_{E.B.}}{f_B}}=\sqrt{\frac{10\times2740}{10,000}}=1.40 \text{ inches}$$
$$d=1\tfrac{3}{8} \text{ inches approximately.}$$

## Winch for 10-Ton Crane. (Plate I.)

Assuming the load carried on 4 chains, the efficiency of the sheaves is

$$0.98^4=0.92$$

The pull on the chain will be

$$\frac{20,000}{4\times.92}=5440 \text{ pounds.}$$

The chain diameter is given by

$$d=\sqrt{\frac{5440}{14,000}}=0.62 \text{ inch} \backsim \tfrac{5}{8} \text{ inch.}$$

Assuming the winch to have two reductions and to be operated by 2 men with 15-inch cranks.

The twisting moment on the crank shaft will have to overcome the above chain acting on a drum of 22-inch pitch diameter.

$$M_T=\frac{5440\times11}{0.98\times0.92^2\times R}=\frac{72,100}{R}$$

Here R is the reduction between drum and crank shaft. The twisting moment on the cranks due to 2 men each exerting 50 pounds is $2\times50\times15=1500$ inch-pounds.

The reduction between the drum and the crank shaft is

$$R=\frac{72,100}{1500}=48:1$$

This will require 1 reduction of 8 to 1, and 1 reduction of 6 to 1. The twisting moment on the pinion shaft No. 2 is,

$$M_T = \frac{11 \times 5440}{0.98 \times 0.92 \times 8} = 8300 \text{ inch-pounds.}$$

Allowing 12 teeth on the pinion, the smallest pitch is given by

$$p_c = \sqrt{\frac{6.28\ M_T}{n.s.c\left(0.124 - \frac{0.684}{n}\right)}}$$

$$= \sqrt[3]{\frac{6.28 \times 8300}{12 \times 7000 \times 2.5 \times 0.067}} = 1.55 \hookrightarrow 2 \text{ diameter pitch.}$$

Since $c = 2\frac{1}{2}$, the face of the gear is

$$1.55 \times 2.5 = 3.88 \hookrightarrow 4 \text{ inches.}$$

Gear A—48 inches p.d.—2 diameter pitch—96 teeth—4 inches face—cast-iron.
Pinion B—6 inches p.d.—2 diameter pitch—12 teeth—4 inches face—cast-iron.

Twisting moment on the shaft No. 3 (crank shaft).

$$M = 15 \times 100 = 1500 \text{ inch-pounds.}$$

The minimum pinion with 12 teeth will then be

$$p_c = \sqrt[3]{\frac{6.28 \times 1500}{12 \times 7000 \times 2 \times 0.067}} = 0.945 \text{ inch} \hookrightarrow 3 \text{ pitch.}$$

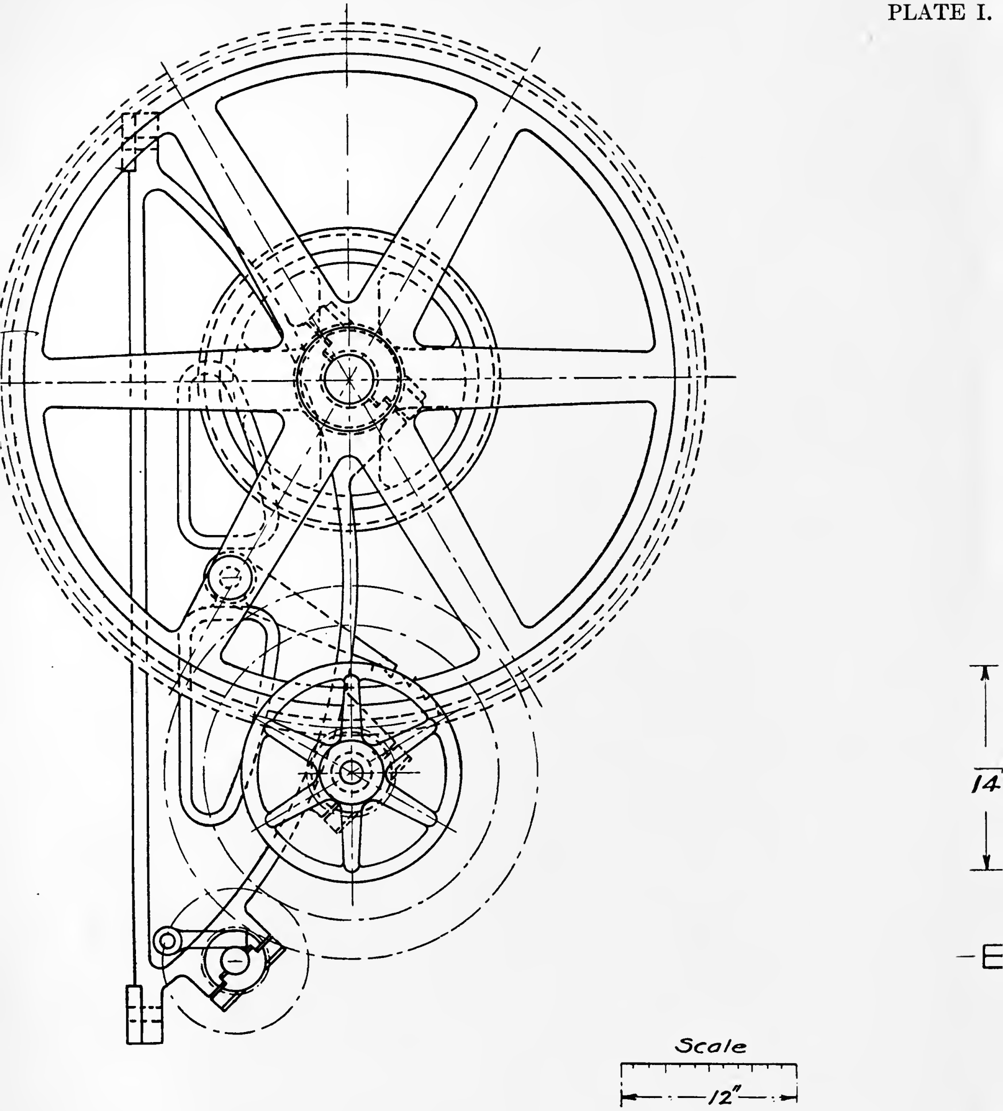
The face of the gear will be

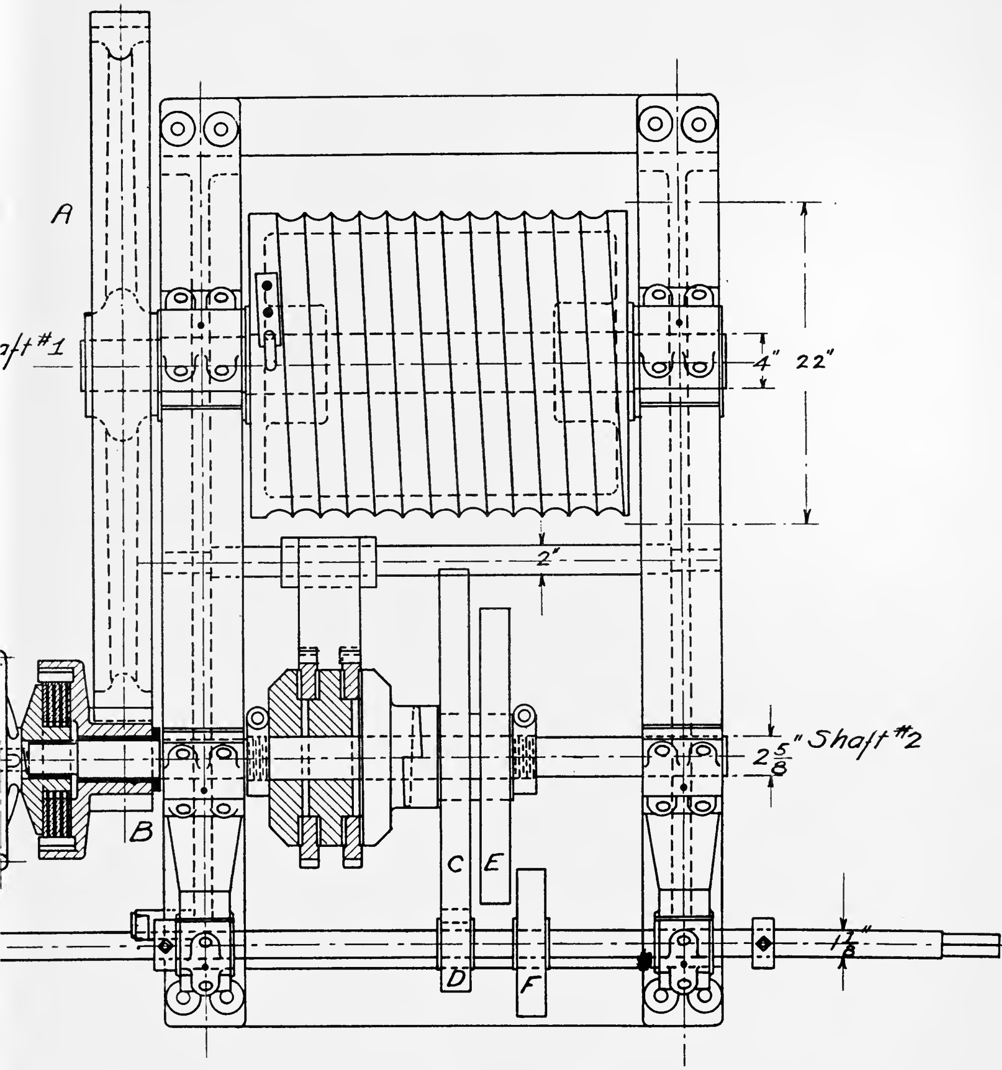
$$p_c \times c = 0.95 \times 2 = 1.90 \text{ inches} \hookrightarrow 2 \text{ inches.}$$

A number 3 diametral pitch with 12 teeth would correspond to a pitch diameter of 4 inches; the shaft being 2 inches diameter would hardly leave sufficient room below the root of the tooth for the key-way; on this account the pinion will be made 4.67 inches pitch diameter with 14 teeth. The gearing on this shaft will be made in accordance with the following list:

Gear C—25.67 inches p.d.; 3 pitch; 77 teeth; 2-inch face; cast-iron.
Pinion D—$4\frac{2}{3}$ inches p.d.; 3 pitch; 14 teeth; 2-inch face; cast-iron.
Gear E—$20\frac{1}{3}$ inches p.d.; 3 pitch; 61 teeth; 2-inch face; cast-iron.
Pinion F—10 inches p.d.; 3 pitch; 30 teeth; 2-inch face; cast-iron.

PLATE I.






WINCH FOR 10 TON CRANE

## Shaft No. 1.

Bending moment on shaft $5450 \times 5.5 = 29{,}920$ inch-pounds.
Twisting moment on shaft $5440 \times 11 = 59{,}840$ inch-pounds.
The equivalent bending moment is

$$M_{B.E.} = 0.35\,M_B + 0.65\sqrt{M_B^2 + M_T^2}$$

$$M_{B.E.} = (0.35 \times 29{,}920) + 0.65\sqrt{29{,}920^2 + 59{,}840^2}$$

$$= 10{,}500 + 43{,}500 = 54{,}000 \text{ inch-pounds.}$$

$$d = \sqrt[3]{\frac{10 \times M_{B.E.}}{9000}} = \sqrt[3]{\frac{10 \times 54{,}000}{9000}} = 3.92 \text{ inches} \backsim 4 \text{ inches.}$$

## Shaft No. 2.

$$\text{Bending moment} = \frac{8300}{3}\,5.5 = 15{,}200 \text{ inch-pounds.}$$

Twisting moment $= 8300$ inch-pounds.
Equivalent bending moment $= 0.35\,M_B + 0.65\sqrt{M_B^2 + M_T^2}$

$$M_{B.E.} = (0.35 \times 15{,}200) + 0.65\sqrt{15{,}200^2 + 8300^2}$$

$$= 5330 + 11{,}250 = 16{,}580 \text{ inch-pounds.}$$

$$d = \sqrt[3]{\frac{10\ M_{B.E}}{p}} = \sqrt{\frac{10 \times 16{,}580}{9000}} = 2.64 \text{ inches} \backsim 2\tfrac{5}{8} \text{ inches.}$$

## Crank Shaft.

Bending moment on shaft due to crank $= 100 \times 26 = 2600$ inch pounds.

Bending moment due to pinion.

The force at the pinion is

$$\frac{M_T}{r} = \frac{1500}{2.33} = 645$$

$$M_B = \frac{WL}{4} = \frac{645 \times 30}{4} = 4830 \text{ inch-pounds.}$$

The equivalent bending is $M_{E.B.} = 0.35M_B + 0.65\sqrt{M_B^2 + M_T^2}$

$$M_{E.B.} = (0.35 \times 4830) + 0.65\sqrt{4830^2 + 1500^2} = 4980 \text{ inch-pounds.}$$

$$d = \sqrt[3]{\frac{10\ M_{E.B.}}{p}} = \sqrt{\frac{10 \times 4980}{9000}} = 1.77 \text{ inches} \backsim 1\tfrac{7}{8} \text{ inches.}$$

Release or dispatch brake. This is placed on shaft No. 2 and as the driving is done through it, the brake must be able to transmit the turning moment required to raise the load; this is 8300 inch-pounds. The following assumptions will now be made:

Mean radius of friction plates, 4 inches.

Coefficient of friction between plates 0.10.

Hand wheel, 14 inches diameter; screw, 1½ inches diameter, with 6 threads per inch. The coefficient of friction between thread and nut will be taken at 0.06. Pitch radius of screw, 0.7 inch. Nine friction faces will be assumed.

The angle of the screw helix is

$$\alpha = \text{tang}^{-1} \frac{0.167}{1.4 \times 3.14} = 2° \, 10'$$

$$\rho = \text{tang}^{-1} .06 = 3° \, 26'$$

$$(\alpha + \rho) = 5° \, 36'$$

*Nomenclature:*

F = force exerted on hand wheel.

$a$ = radius of hand wheel in inches.

$r$ = pitch radius of screw.

$\alpha$ = angle of helix of thread.

$\rho$ = angle of friction = tang$^{-1}$ 0.06 = 3° 23 minutes.

$\mu_1$ = coefficient of friction between nut and disc.

$r_1$ = mean radius of nut, inches.

K = axial force on screw and discs.

$M_T$ = twisting moment on pinion.

$n$ = plate faces in sliding contact.

R = mean radius of friction plates, inches.

$\mu_2$ = coefficient of friction between plates.

$$F.a = Kr \,\text{tang}\,(\alpha + \rho) + \mu_1 K r_1 \qquad (1)$$

$$M_T \leqq n \mu_2 KR \qquad (2)$$

Hence $\quad 8300 \leqq 9.K.4 \times 0.10 \quad K \geqq \frac{8300}{9 \times 4 \times 0.10} = 2300$ pounds.

The force to be exerted upon a 14-inch wheel then is,

$$F=\frac{Kr\tan(\alpha+\rho)+\mu_1 Kr_1}{a}$$

$$F=\frac{(2300\times 0.70\times \tan 5°36')+(0.06\times 2300\times 1.75)}{7}$$

$$F=\frac{158+241}{7}=57 \text{ pounds.}$$

The pressure per square inch on the discs is

$$\frac{2300}{36.3}=63.5 \text{ pounds, which is satisfactory.}$$

Mechanical Brake.—This brake is analogous to the brake discussed on p. 172. To prevent the helix working off under a load

$$Kr\tan(\alpha-\rho)-(\mu_1 KR)\overline{\overline{<}}0$$

or

$$r\tan(\alpha-\rho)\overline{\overline{<}}\mu_1 R_1$$

If we here assume $R_1=2$ inches; $\mu_1=0.10$; $r=3$ inches.

$$\rho=\tan^{-1} 0.10=5°\ 43'$$

$$\tan(\alpha-\rho)\overline{\overline{<}}\frac{\mu_1 R}{r}=\frac{0.10\times 2}{3}=0.067$$

$$\tan^{-1} 0.067=3°\ 50', \text{ hence } \alpha-\rho=3°\ 50'$$

and

$$\alpha=3°\ 50'+5°\ 43'=9°\ 33'$$

The natural tangent of 9° 33′ is 0.1682.
The pitch of the helix will be

$$x=3.14\times 3\times \text{tang } 9°\ 33'=1.584 \text{ inches.}$$

We will make it 1¼ inches, thus making the angle

$$\alpha=\text{tang}^{-1}=\frac{1.25}{3.14\times 3}=7°\ 33'$$

The mean radius of the friction discs is $R_2=5$ inches, $\mu_2=0.10$.

The twisting moment on the shaft due to the load trying to run down

$$M_T = \frac{20{,}000 \times 0.92 \times 11 \times 0.98 \times 0.92}{4 \times 8} = 5700 \text{ inch-pounds.}$$

$$M_T = Kr \tan(\alpha + \rho) + \mu_2 K R_2 n$$

$$K \geq \frac{M_T}{\mu_2 K R_2 n} = \frac{5700}{0.10 \times 5 \times 4} = K \geq \frac{5700}{2} = 2850 \text{ pounds.}$$

Outside diameter of disc, 12 inches area 113.10 square inches.
Inside diameter of disc, 8 inches area 50.27 square inches.
62.83 square inches.

The pressure per square inch on the fiber rings is $\frac{2850}{62.8} = 45.4$ pounds, $M_T = 5700$ inch-pounds as previously found. Assuming the probable diameter of the ratchet wheel as 15 inches (outside diameter), and the width of tooth as 1 inch. Two ratchet wheels will be used, 1 on each friction disc. From the formula, see p. 28,

$$p_c = \sqrt[3]{\frac{6 \times 3.14 \frac{5700}{2}}{3000 \times 1 \times 20}} = 0.96 \text{ inch.}$$

Hence any pitch of 1 inch or over will be satisfactory.

## Hand Hoists.

### DIFFERENTIAL CHAIN BLOCKS.

One of the simplest hoists is the differential pulley block. It has a very small number of parts and is the cheapest hoist on the market. Its drawback, where in constant use, is its low efficiency. On account of this low efficiency, however, it requires no retaining brake.

There are 2 upper sheaves, keyed to the same shaft, and one lower sheave which carries the hook. The chain is continuous, passing around all three sheaves. The hoisting is effected by the upper sheaves differing slightly in pitch diameter, one sheave having 1 more pocket for the chain than the other. Hence if the chain is pulled over the larger sheave, since both sheaves rotate together, 1 more link of the chain will pass on the large sheave than will pass from the small sheave and the load will be raised ½ the length of 1 link.

The efficiency of a differential block is given approximately by

$$\eta=\frac{1-n}{2}\times\frac{1+k}{k^2-n}.$$

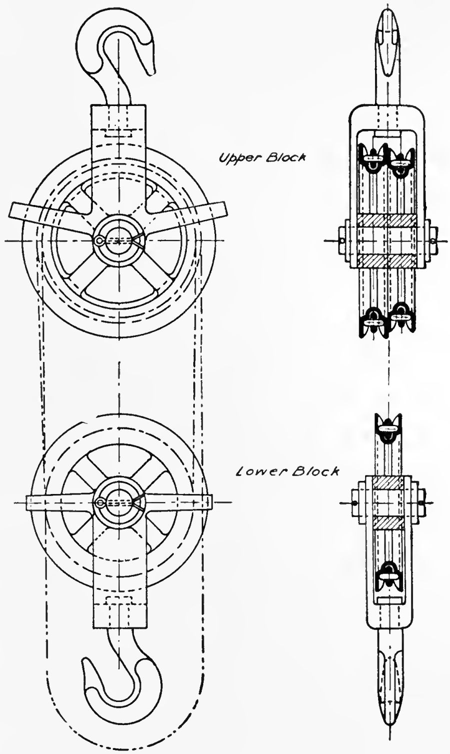
$n$ = ratio of the number of pockets in the smaller sheave to the number of pockets in the larger sheave, *i.e.*, the ratio of pitch diameters.

$k$ = factor previously found for chain sheaves about 1.04.

$\eta$ = efficiency.

A manufacturer's catalogue gives efficiencies ranging from 38 per cent. in a 500 pound block to 28 per cent. in a 6000 pound hoist. The usual sizes range from 500 to 6000 pounds capacity.

Fig. 233.




Differential Pulley Block.

| Capacity pounds | Chain Pull pounds | Chain Pull feet | Efficiency per cent. |
|---|---|---|---|
| 500 | 72 | 18 | 38 |
| 1000 | 122 | 24 | 34 |
| 2000 | 216 | 30 | 31 |
| 3000 | 246 | 36 | 34 |
| 4000 | 308 | 42 | 31 |
| 6000 | 557 | 38 | 28 |

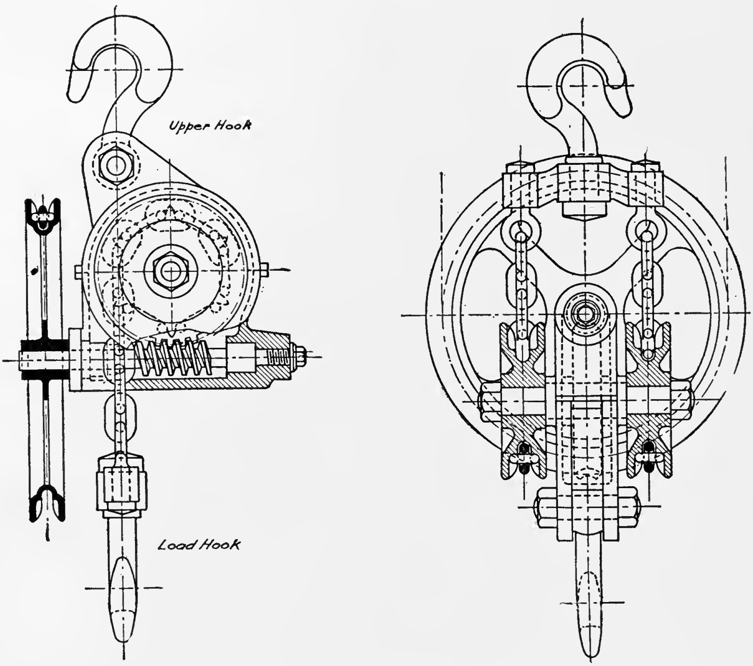
The above table illustrates the capacities of this type of block as made by several manufacturers. The pull given in column 3 is the amount of chain that must pass over the top sheave to lift the load one foot.

## Screw Hoists.

The accompanying figure illu trates a usual arrangement of screw hoist. The worm is steel, running on a bronze worm wheel. The entire mechanism is incased in a housing, thus permitting better lubrication and freedom from dirt.

The efficiency for any particular screw hoist can be estimated as previously shown for worm driving and blocks, the efficiency of the hoist being the product of the efficiency of the worm mechanism and the efficiency of the tackle. Thus in the hoist

Fig. 234.



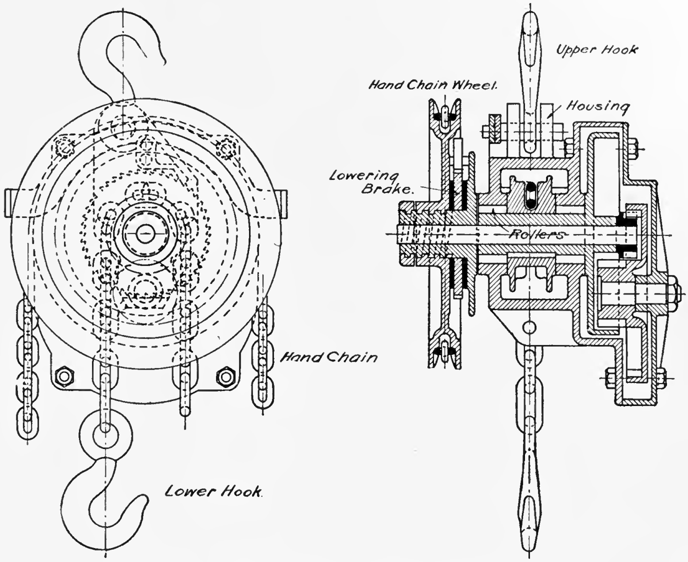

illustrated if the efficiency of the worm and wheel and bearings on the worm shaft is 40 per cent., there being 1 fixed block carrying the load whose efficiency is 96 per cent. and the chain wheel for the hand-chain, efficiency 96 per cent., making a combined efficiency of $.40 \times .96 \times .96 = 37$ per cent.

The efficiency of screw hoists as generally manufactured will range from 25 to 40 per cent., with a possible average of 33 per cent. The capacities run from 500 to 6000 pounds. The smaller hoists are designed to be operated by one man, while the larger hoists require from 2 to 4 men when raising their maximum loads.

Owing to the low efficiency of the 2 types of hoists just described spur geared hoists are made with efficiencies ranging from 70 to 80 per cent. It is necessary to furnish these hoists with a retaining brake, which is commonly of the multiple disc or Weston type.

The accompanying sketch shows the Climax hoist. The hand-chain wheel in raising the load advances on the screw thread on the shaft, thus clamping the discs of the retaining brake, and rotates these discs under the pawl, carrying the drive shaft, at the other

Fig. 235.



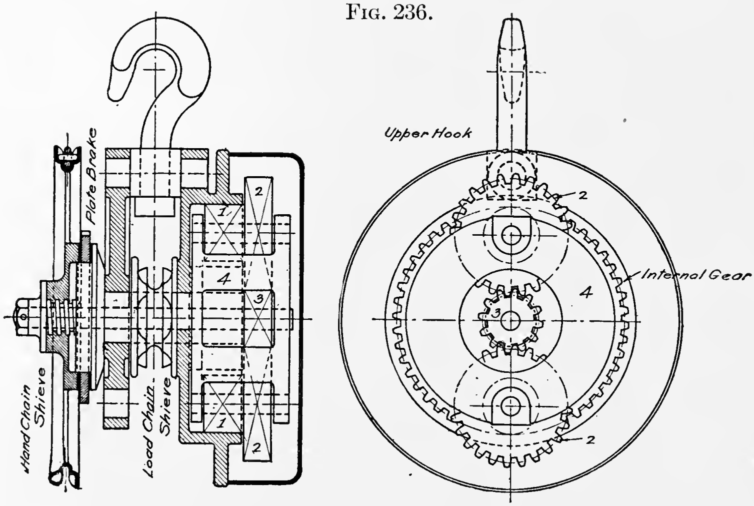

Side and Sectional View of Climax Chain Block

end of which is a pinion meshing internally with the intermediate gear. The intermediate gear carries a pinion, which in turn engages with the load gear, thus making 2 gear reductions. In lowering, the load in attempting to run down engages the pawl with the ratchet disc of the retaining brake, the friction between the discs connected to the shaft and this disc held by the pawl prevents the load descending until the pressure between the discs is relieved by rotating the hand-chain wheel in the opposite direction to that required for hoisting. This backs off this wheel on the screw from the discs, relieving the axle pressure and permitting the load to lower.

In the Peerless hoist made by Harrington, Son & Company, Incorporated, the spur gearing is shown in a case on the right of the block, while on the side with the hand-chain wheel is shown the ratchet wheel forming one of the retaining wheel brake discs. The hand-chain wheel is screwed upon the hub of the second disc and the friction between these discs is increased by a leather washer. In other respects the brake action resembles that already explained for the Climax hoist.

## Geared Hoists.

Fig. 236.




The triplex chain block, made by the Yale & Towne Manufacturing Company, differs from the others in using an epicyclic gear train. The reduction in this train is calculated as follows:

R = radius of internal gear. This gear is fixed.

$r_1$ = radius of pinion No. 1.

$r_2$ = radius of gear No. 2.

$r_3$ = radius of pinion No. 3.

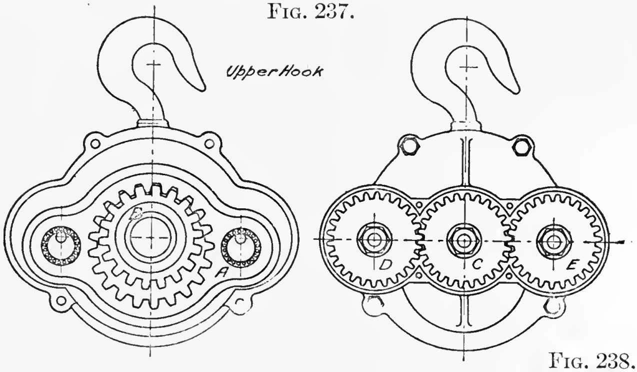
$n_4$ = revolution per minute of the cage No. 4 carrying the chain sprocket wheel.

$n_3$ = revolution per minute of the pinion No. 3 attached to the hand-chain wheel.

$$n_3 = n_4\left[\left(\frac{R \times r_2}{r_1 \times r_3}\right) + 1\right]$$

In this formula the number of teeth can be substituted for the radii of the several pinions and gears.

FIG. 237.




Chisholm & Moore Hand Hoist.

T = number of teeth in the internal gear.
$t_1$ = number of teeth in pinion No. 1.
$t_2$ = number of teeth in gear No. 2.
$t_3$ = number of teeth in pinion No. 3.

$$n_3 = n_4\left[\left(\frac{T \times t_2}{t_1 \times t_3}\right)+1\right].$$

This hoist requires a brake of the disc type.

### CHISHOLM & MOORE HAND HOIST.

The cuts illustrate the cyclone hoist manufactured by Chisholm & Moore Manufacturing Company. The sizes range from ½ to 20 tons and the efficiency claimed for the hoist is about 80 per cent.

The hoist uses the principle of the Eade's gear train; the internal gear is given a motion of circular translation by the 2 eccentrics.

The sheave carrying the load chain is fastened to the gear B. The eccentrics giving the required motion to the external gear A are driven by the gears D and E, which mesh with the central gear C; this is driven directly by the hand-chain sheave.

FIG. 238.

Chisholm & Moore Hand Hoist.

The gear reduction is found as follows:

$r_1$ = pitch radius of external gear. A

$r_2$ = pitch radius of internal gear. B

$n_2$ = revolutions per minute of the eccentrics.

$n_1$ = revolutions per minute of the external gear.

Then $$n_1 = n_2\left(\frac{r_1 - r_2}{r_1}\right)$$

This result being negative indicates that the rotation is in the opposite sense to that of the eccentrics.

The number of teeth can be substituted for the respective radii of the gears in the above formula.

$t_1$ = number of teeth in the external gear. A

$t_2$ = number of teeth in the internal gear. B

$$n_1 = n_2\left(\frac{t_1 - t_2}{t_1}\right).$$

The accompanying illustrations show several methods of carrying hoists along runways. In some cases the trolley is dragged

Fig. 239.

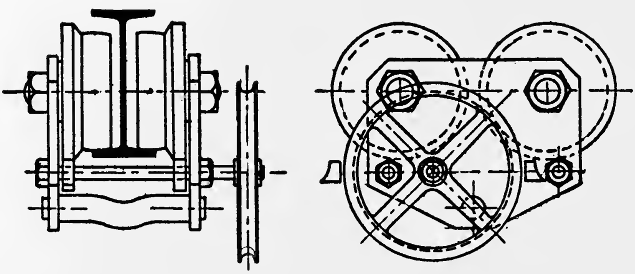
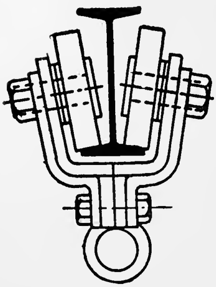


Fig. 240.



along by the hand-chain, while in others the motion is produced by using a hand-chain that operates the trolley wheels through gearings.

The relative merits of the 3 types of hoists is well illustrated by the following table published in a catalogue of the Yale & Towne Manufacturing Company.

## Chain Block Hoisting Speeds and Specifications

| Capacity in Tons | Pull in Pounds required on Hand-Chain to Lift Full Loads | | | Feet of Hand-Chain to be Pulled by Operator to Lift Load One Foot High | | | Hoisting Speeds. Feet per Minute Attainable and No. of Men Required for Hoisting Full Loads without Pulling over 80 Lbs. | | | | | | | | Load one Man can Handle without Pulling over 80 Lbs. | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | | | | | | | Triplex | | | | Duplex | | Differential | | | | |
| | Triplex | Duplex | Differential | Triplex | Duplex | Differential | Full Load | Half Load | Quarter Load | † No. of Men | Full Load | † No. of Men | Full Load | † No. of Men | Triplex | Duplex | Differenial |
| ¼ | ... | ... | 72 | ... | ... | 18 | ... | .... | .... | .. | .... | .. | 6.00 | 1 | ..... | ..... | 500 |
| ½ | 62 | 68 | 122 | 21 | 40 | 24 | 8.0 | 16.0 | 24.0 | 1 | 4.00 | 1 | 6.00 | 2 | 1000 | 1000 | 600 |
| 1 | 82 | 87 | 216 | 31 | 59 | 30 | 4.0 | 8.0 | 12.0 | 1 | 2.00 | 1 | 3.70 | 3 | 2000 | 1700 | 800 |
| 1½ | 110 | 94 | 246 | 35 | 80 | 36 | 4.8 | 9.6 | 14.4 | 2 | 2.40 | 2 | 2.50 | 3 | 2300 | 2500 | 1000 |
| 2 | 120 | 115 | 308 | 42 | 93 | 42 | 3.6 | 7.2 | 10.8 | 2 | 1.80 | 2 | 2.30 | 4 | 2600 | 2700 | 1100 |
| 3 | 114 | 132 | 557 | 69 | 126 | 38 | 2.3 | 4.6 | 6.9 | 2 | 1.10 | 2 | 2.30 | 7 | 4000 | 3300 | 1000 |
| 4 | 124 | 142 | ... | 84 | 155 | .. | 1.7 | 3.5 | 5.2 | 2 | .80 | 2 | .... | .. | 5000 | 4600 | .... |
| 5 | 110 | 145 | ... | 126 | 195 | .. | 1.3 | 2.6 | 3.9 | 2 | .65 | 2 | .... | .. | 6500 | 5300 | .... |
| 6 | 130 | 145 | ... | 126 | 252 | .. | 1.1 | 2.2 | 3.3 | 2 | .50 | 2 | .... | .. | 7000 | 6500 | .... |
| 8 | 135 | 160 | ... | 168 | 310 | .. | .8 | 1.6 | 2.4 | 2 | .35 | 2 | .... | .. | 9000 | 7800 | .... |
| 10 | 140 | 160 | ... | 210 | 390 | .. | .6 | 1.2 | 1.8 | 2 | .30 | 2 | .... | .. | 11,000 | 10,000 | .... |
| 12 | 130* | ... | ... | 126* | ... | .. | 1.1 | 2.2 | 3.3 | 4 | .... | .. | .... | .. | 13,000 | ..... | .... |
| 16 | 135* | ... | ... | 168* | ... | .. | .8 | 1.6 | 2.4 | 4 | .... | .. | .... | .. | 17,000 | ..... | .... |
| 20 | 140* | ... | ... | 210* | ... | .. | .6 | 1.2 | 1.8 | 4 | .... | .. | .... | .. | 20,000 | ..... | .... |

* On each of the two hand-chains.

† The number of men is based on each man pulling not over 80 lbs. One man pulling 160 lbs. or less, as given in the first two columns, can lift the full capacity of any Triplex or Duplex Block.

## Electrically Driven Hoists.

The accompanying cuts illustrate several types of electrically driven hoists.

Fig. 241.

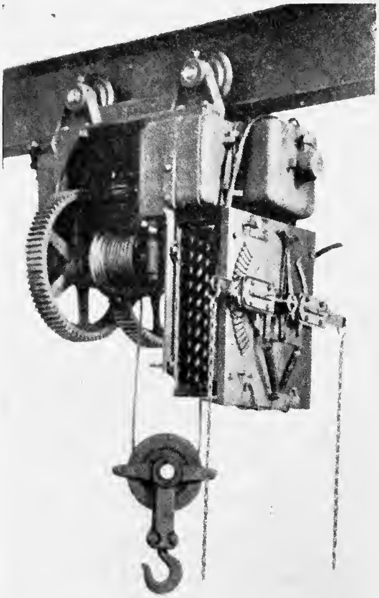


Fig. 242.

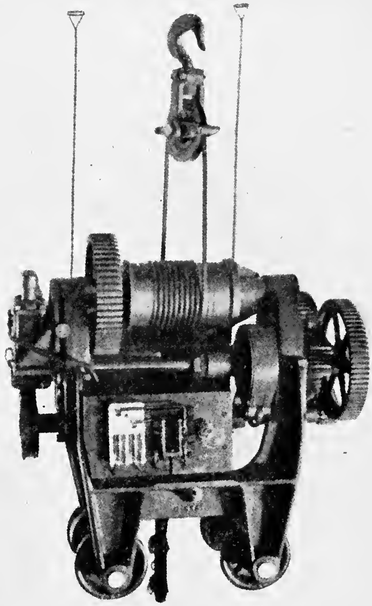
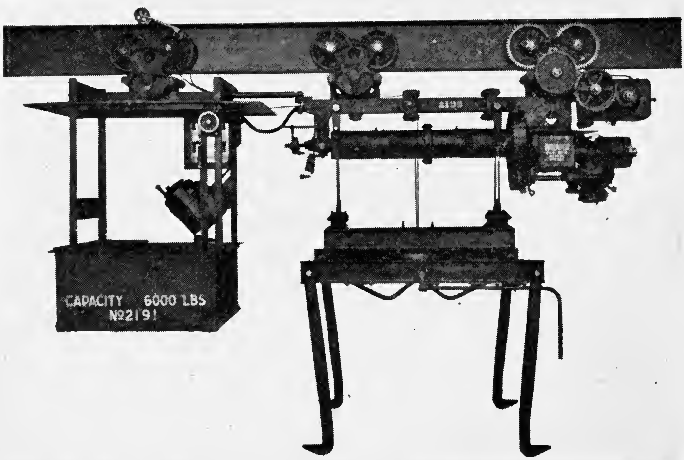


Fig. 243.




Two-Ton (4000 pounds) Assembling Hoist.

For convenience in operation, *i.e.*, decreasing the number of men required and increasing the speeds, hoists are designed electrically driven. The hoist may also be carried on a trolley which is electrically driven.

The electrical control may be effected in one of three ways.

1. The controller is attached to the hoist and operated by chains brought within easy reach of the operator standing on the floor. The operator walks with the hoist as it moves on its track.

2. The controller and operator may be located at a point from which the operator can see all the hoist movements without travelling with the hoist.

3. The controller and operator may be in a cage attached to the hoist and travel with it. This is especially desirable where the trolleys carrying the hoists are electrically driven.

As the lift is less than is usual in the ordinary travelling crane the hoisting speeds are generally kept low.

| Capacity of Hoist Tons. (2000 Lbs.) | Hoisting speed.—Feet per minute. Empty hook. | Full load. |
|---|---|---|
| 1 | 25 | 20 |
| 15 | 9 | 5 |

## Two-Ton (4000 Pounds) Assembling Hoist.

The hoist shown (see Plate II) is intended to be carried under roof-trusses placed 16 feet *cc.* A lifting beam carried under the hoist will extend under the space beyond each truss, this lifting beam being 30 feet long. The maximum load which it is possible to lift at any point along the 30-foot lifting beam plus the weight of the tackle and part of the weight of the lifting beam, etc., that will come upon either block can be assumed at 5200 pounds.

Assuming the load carried by four ½ inch diameter ropes, the load per rope is $\frac{5200}{4} = 1300$ pounds.

Determining the stress when run upon a drum 12 inch pitch diameter.

$$f_T = \left(\frac{S}{i \times \frac{\pi\delta^2}{4}}\right) + \left(3/8 \frac{E \times \delta}{D}\right)$$

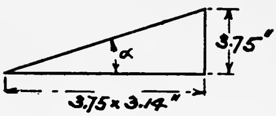
$$= \left(\frac{1300}{0.0995}\right) + \left(3/8 \times \frac{30{,}000{,}000 \times 0.033}{12}\right)$$

$$= 13{,}050 + 31{,}200 = 44{,}250 \text{ pounds.}$$

Using crucible steel rope the factor of safety is $\frac{180{,}000}{44{,}250} = 4.07$

Trying a worm wheel $17\frac{29}{32}$ inches pitch diameter—45 teeth triple threaded worm $1\frac{1}{4}$ inches circular pitch, reduction 15 to 1.

The angle of the thread measured on the pitch line




$$\text{tang } \alpha = \frac{3.75}{3.75 \times 3.1416} = 17° \, 40'$$

The axial pressure on the worm teeth is

$$\frac{2600 \times 12}{17.90} = 1740$$

Owing to the intermittent character of the work we can allow a temperature rise of 100°F. The idea here is that if the work were continuous this temperature would be attained, but as the work is not only not continuous but also the average load does probably not exceed $\frac{1}{3}$ of the maximum it is possible that not more than $\frac{1}{3}$ of this rise in temperature will be reached.

According to the formulæ of Bach & Roser (see p. 34),

$K = c.f.p.$

$c = 490$, where $t_1 - t_2 = 100$ and $v = 150$

$K = 490 \times 3.25 \times 1.25 = 1990$ pounds.

As this is considerably above the 1740 pounds coming on the worm it should prove of ample capacity. Use a hardened steel worm on a phosphor-bronze wheel.

The estimated efficiency of the worm, assuming a coefficient of friction of 0.06, will be,

$$\text{Approximate efficiency } \eta = \frac{\tan \alpha}{1.10 \tan (\alpha + \varphi)}$$

The angle corresponding to the natural tangent 0.06 is 3 degrees 20 minutes.

$$\alpha = 17° \, 40' \text{ and } \varphi = 3° \, 20'$$

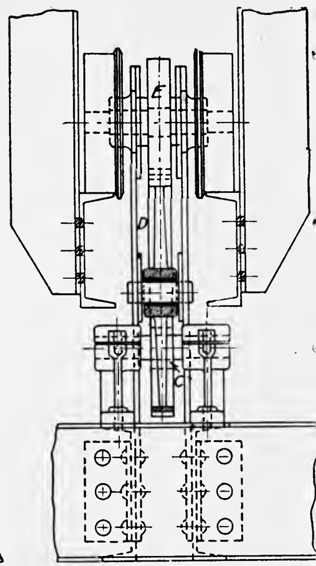
$$\eta = \frac{\text{tang } 17° \, 40'}{1.10 \text{ tang } (17° \, 40' + 3° \, 20')} = \frac{\text{tang } 17° \, 40'}{1.10 \text{ tang } 21°} = \frac{0.319}{1.10 \times 0.384}$$

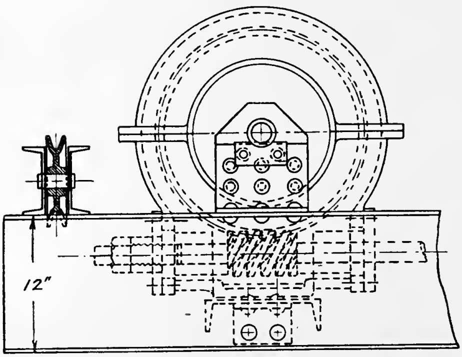
$= 75$ per cent.

The efficiency of the complete hoist can now be estimated.

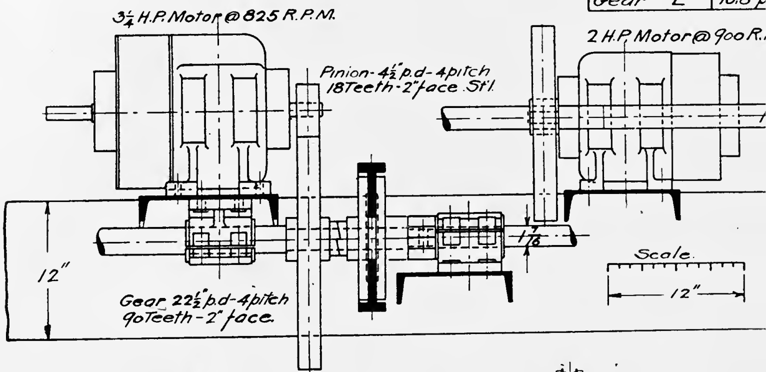
Efficiency of the floating block..........97 per cent.
Efficiency of rope winding on drum.....97 per cent.
Efficiency of drum shaft bearings ......98 per cent.
Efficiency of worm shaft and worm.....75 per cent.
Efficiency of gearing at motor and brake
mechanism 90 per cent.

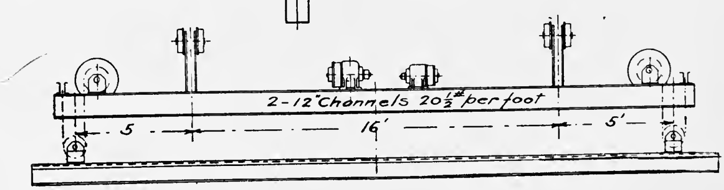
# 2 TON ASSEMBLING HOIST.
(4000#)

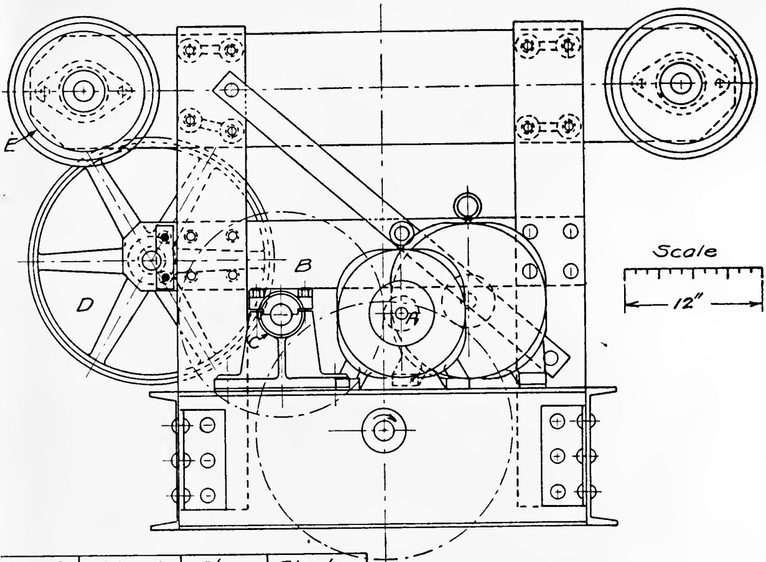


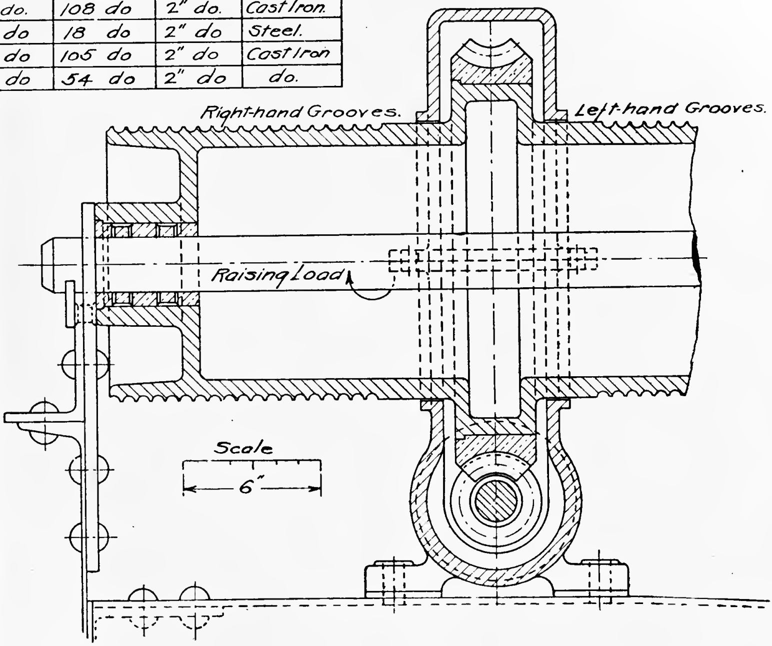

| | |
|---|---|
| Pinion A | 3" p. |
| Gear B | 18" p. |
| Pinion C | 3.6" p. |
| Idler D | 21" p. |
| Gear E | 10.8" p |






| | | | |
|---|---|---|---|
| 6 pitch | 18 teeth | 2" face | Steel. |
| 6 do. | 108 do | 2" do. | Cast Iron. |
| 5 do | 18 do | 2" do | Steel. |
| 5 do | 105 do | 2" do | Cast Iron |
| 5 do | 54 do | 2" do | do. |



Combined efficiency $.97^2 \times .98 \times .75 \times .90 = .62$, say 60 per cent. The horse power of the motor will be

$$\frac{(\text{Total load raised}) \times (\text{lifting speed feet per minute})}{33{,}000 \times \text{efficiency}} = \text{horse power.}$$

$$\frac{5300 \times 15}{33{,}000 \times .60} = 4.00 \text{ H.P.}$$

A motor of nominally 3¼ horse power was selected having a speed of 700 revolutions per minute at 4 horse power. The speeds and reductions between load and motor will be
Hoisting speed 15 feet per minute.
Velocity of ropes running on drum $15 \times 2 = 30$ feet.

Revolution per minute of drum $\frac{30}{\pi D} = \frac{30}{3.14 \times 1} \doteq 9.55$ R.p.m.

Velocity of worm shaft

$$\frac{\text{R.p.m. of drum} \times \text{teeth in worm wheel}}{\text{Threads on worm}} = \frac{9.55 \times 45}{3} = 143.$$

Reduction at motor $\frac{700}{143} = 4.88$, approximately 5 to 1.

Diameter of worm shaft

$$d = 68.5\sqrt[3]{\frac{\text{H.P.}}{f_s \times n}} = 68.5\sqrt[3]{\frac{4.00}{6000 \times 143}} = 1.14 \text{ inches, use } 1\tfrac{7}{16} \text{ inches.}$$

Step-Bearing.—When one drum is raising the full load the step-bearing at the end of its worm will be called upon to resist the full axial pressure of the worm, *i.e.*, 1740 pounds.

$$d \geqq \frac{Pn}{w} = \frac{1740 \times 143}{220{,}000} = 1.14 \text{ inches,}$$

so that 1$\frac{7}{16}$ inches should allow for turning down for couplings, and provide strength enough at gearing for any bending that may be introduced.

Gearing at motor reduction 5 to 1. Assuming a velocity of 1000 feet per minute, the allowable fiber stress is for steel

$$S = 20{,}000\left(\frac{600}{600 + 1000}\right) = 7500 \text{ pounds.}$$

The motor is called upon to develop 4 horse power. Using the formula for the smallest pinion with 15 teeth, we have

$$p=\sqrt[3]{\frac{396{,}000\times 4}{15\times 7500\times 2.5\times 700\left(0.124-\frac{0.684}{15}\right)}}=0.468 \text{ inch},$$

use 4 diametral pitch.

These gears will be made

Pinion—18 teeth—4 diametral pitch—Pitch diameter $4\frac{1}{2}$ inches.

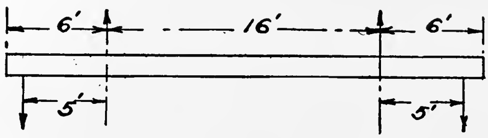
Gear—90 teeth—4 diametral pitch—Pitch diameter $22\frac{1}{2}$ inches.

The faces of both gear and pinion to be 2 inches.

The diameter of the motor shaft upon which the pinion is placed is $1\frac{3}{8}$ inches, leaving ample metal in a $4\frac{1}{2}$ inch pitch diameter pinion.

## Frame.

Fig. 244.




The trusses carrying the hoist are assumed as 16 feet *c–c*, this being a very common dimension for truss spacing. An overhang of 6 feet will make the total length 28 feet, leaving 4 feet between adjacent hoists, should similar hoists be placed in alternate panels.

The bending at CD will be that due to the cantilevered load; this will have to be roughly estimated. It is as follows:

| | |
|---|---|
| Maximum load at 1 end (previously estimated) | 5200 pounds. |
| Structural work, machinery, etc., at end, ...... | 600 pounds. |
| Total ...................................... | 5800 pounds. |

The bending moment $M=W\times L$.

$M=5800\times 5\times 12=348{,}000$

$M=pI/e=348{,}000=11{,}000\times I/e$

The section modulus $I/e=348{,}000\div 11{,}000=31.7$.

This being the section modulus of 2 channels, the section modulus for 1 channel is 16. According to manufacturers' handbooks, the lightest channels that approximate this are

10-inch channel 20 pounds per foot section modulus 15.7

12-inch channel $20\frac{1}{2}$ pounds per foot section modulus 21.4

The difference in weight being so slight the 12 inch channel $20\frac{1}{2}$ pounds will be the better selection on account of its additional stiffness. Before this section is finally decided upon the bending effect upon the frame of the motors must be determined. The motor sizes will be assumed as one 5 horse-power and one 3 horse-power motor, together weighing 700 pounds.

| | |
|---|---|
| Weight of frame $2\times16\times20.5$ | 655 |
| Weight of shafting, bracing, etc | 345 |
| Total | 1000 |

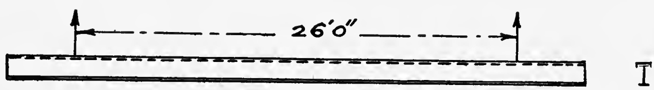
The above loadings are equivalent to the following central load:

| | |
|---|---|
| Motors | 700 |
| $\frac{1}{2}$ Uniform load | 500 |
| Total | 1200 |

$$\text{Bending } M=\frac{WL}{4}=\frac{1200\times16\times12}{4}=57{,}600 \text{ inch-pounds.}$$

As this bending is in the opposite direction to that due to the overhung load, it is evident that the 348,000 inch-pounds already found is a maximum and the 12-inch channels selected are ample. It is better practice to place light rails upon the channel flanges instead of running the wheels directly upon the channels. We can now make a rough estimate of the total weight of the hoist and then determine the horse power required for travel.

First finding the section for a lifting beam capable of holding 4000 pounds at any point.




$$\text{Load in the middle } M=\frac{W_1L}{4}+\frac{W_2L}{8}=\left(\frac{W_1}{4}+\frac{W_2}{8}\right)L$$

$W_1=4000$

$W_2=26\times40=1040$

$$M=\left(\frac{4000}{4}+\frac{1040}{8}\right)\times26\times12=352{,}560 \text{ inch-pounds.}$$

This span being altogether too long for an ordianry rolled section to be used without lateral support, a section will be built up of a channel and an I beam. This will have to be done by trial. Using first a 10 inch I beam 25 pounds per foot and a 7 inch channel 9¾ pounds per foot on top of it we have the radius of gyration of 7 inch channels referred to its principal axis is 2.72.

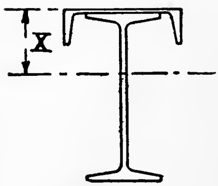
$$\text{hence } \frac{l}{r}=\frac{26\times12}{2.72}=115.$$

The compressive fiber stress permitted (see p. 111) is 83 per cent. of the maximum compressive fiber stress.

$$12{,}000\times0.83=9{,}960 \text{ pounds per square inch.}$$

Now finding the center of gravity and the moment of inertia of the trial section

FIG. 245.



| Name of Section | Area | Stat. Mo. |
|---|---|---|
| 7-inch channel 9.75 pounds. | 2.85× .55.. | 1.57 |
| 10-inch I 25 pounds ...... | 7.37×5.21.. | 38.40 |
| | 10.22 | 39.97 |

X = statical moment ÷ area

X = 39.97/10.22 = 3.92 inches.

The moment of inertia of the trial section is found as follows:

| Name of Section | Inertia |
|---|---|
| 7-inch channel 9.75 pounds. Inertia referred to its own axis...................... | 0.98 |
| $A.h^2=2.85\,(3.92-0.55)^2=$ | 32.37 |
| 10-inch I beam 25 pounds. Inertia referred to its own axis..................... | 122.10 |
| $A.h^2=7.37\,(5.21-3.92)^2=$ | 9.50 |
| | 164.95 |

The compressive fiber stress can now be found $M=pI/e$

$e$ = distance center of gravity of section to extreme fibers; in this case the fibers in compression = 3.92

$$p_c=\frac{Me}{I}=\frac{352{,}560\times3.92}{164.95}=8380 \text{ pounds.}$$

In a similar way the maximum tensile strength will be

$$p_t=\frac{Me}{I}=\frac{352{,}560(10.21-3.92)}{164.95}=13{,}400 \text{ pounds.}$$

The tensile fiber stress is in excess of that desired, although the compressive fiber stress is about as close to that allowed, 9960 pounds, as could be attained. To reduce this tensile stress we will try a 10-inch I 30 pounds instead of the 25 pound I beam.

Making the calculation as in the case just tried

X = 4.08 inches, the distance from back of channel to the center of gravity of the built-up section.

The moment of inertia = 180.7

The tensile stress $p_t = \frac{M_e}{I} = \frac{352{,}560(10.21-4.08)}{180.7} = 12{,}000$ pounds.

This section is satisfactory.

The total weight of the hoist, lifting beam, etc., can now be estimated.

| | Pounds |
|---|---|
| Load | 4,000 |
| Lifting beam | 1,200 |
| Tackle | 200 |
| Frame | 1,700 |
| Gearing, suspension, etc. | 500 |
| Motors | 700 |
| Total | 8,300 |

Assuming a travel of 150 feet per minute and as there will be very considerable tendency to twist on the track, ample allowance must be made for this; in the following formula make C = 2.5.

$$\text{H. P.} = C \times \frac{(f+\mu r)\ (W_1+W_2)\times S}{R\times 33{,}000}$$

Making the following assumptions

$$f = 0.003 \text{ and } \mu = 0.08$$

$r = 1$ inch, radius of the axle

R = 6 inches, radius of the wheel.

$$\text{H. P.} = 2.5\ \frac{[0.003+(0.08\times 1)]\times 8300\times 150}{6\times 33{,}000} = 1.3.$$

Use a 2-horse-power 220 volt direct current series motor at 850 revolutions per minute. Using a 2-horse-power instead of a 3-horse-power motor would reduce the weight of the motors somewhat, but not sufficiently to make it worth while to recalculate the frame sections.

GEARING.—2-horse-power motor to driving wheels.

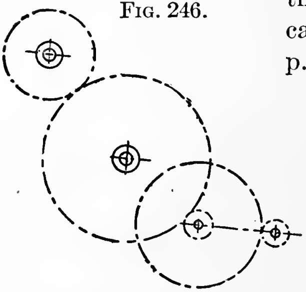
The driving wheels being 12 inches in diameter, to travel 150 feet per minute they will have to make $\frac{150}{1\times 3.14}=47.8$ revolutions per minute.

The reduction from the motor to the driving wheels then is

$$\frac{850}{47.8}=\frac{17.8}{1}$$

Assuming a 2-horse-power motor at 850 revolutions per minute, the smallest pinion having 18 teeth that can be used is given by the formula (see p. 19).

FIG. 246.



$$p=\sqrt[3]{\frac{396{,}000\times 1.30}{18\times 6000\times 3\times 850\times 0.086}}=0.279$$

To provide for wear we will make this pitch 6 diametral pitch, or 0.524 inch circular pitch. This will make the pinion 3 inches pitch diameter with 18 teeth, while the gear will be 18 inches pitch diameter with 108 teeth.

The second reduction will have to be $\frac{17.8}{6} \approx 3$

The minimum pinion here will be

$$p=\sqrt[3]{\frac{396{,}000\times 1.30\times 0.95}{18\times 8000\times 3\times 142\times 0.086}}=0.453 \text{ inch } c.p.$$

The gear at the driving wheel must now be determined to see what pitch it must be if made of cast-iron, and how that will agree with the pitch found above, *i.e.*, 0.453 inch circular pitch.

$$p=\sqrt[3]{\frac{396{,}000\times 1.30\times 0.95^2}{72\times 4000\times 3\times 47.8\times 0.114}}=0.462 \text{ inch.}$$

Here we will use 5 diametral pitch, making the pinion rolled steel 3.6-inch pitch diameter with 18 teeth and the gear on the driving wheel axle 10.8-inch pitch diameter with 54 teeth, this gear being cast-iron. The intermediate or idler gear will be cast-iron and can be made any diameter to suit the frame, so that a bearing can be readily located.

The shaft running along the top of the frame to drive the driving wheels should be heavy to resist twisting so, as far as possible, to prevent the frame from twisting. Designing this shaft for twisting and allowing 6,000 pounds shearing stress,

$$d=\sqrt[3]{\frac{321,000\times 2}{6000\times 142}}=0.910\text{ inch} \backsim 1 \text{ inch.}$$

To allow for keys, necking, etc., we will use 1$\frac{3}{16}$ inch diameter shafting.

As the calculated efficiency of the hoisting mechanism is 61 per cent. there is danger of the load being dropped unless a mechanical or safety brake is placed upon the hoist. This will be placed upon the shaft next to the motor, which when operating at its rated load makes 143 revolutions per minute. The shaft is 1$\frac{7}{16}$ inches in diameter. It is necessary to determine the twisting moment on this shaft both in raising and lowering the load.

The efficiency of the worm in raising has already been found to be 75 per cent. In a similar way the efficiency in lowering is given by

$$\eta=\frac{\text{tang }(\alpha-\varphi)}{1.10\text{ tang }\alpha}=\frac{\text{tang }(17^\circ\,40'-3^\circ\,20')}{1.10\text{ tang }17^\circ\,40'}=73\text{ per cent.}$$

From this it is seen that the efficiencies in raising and lowering the load are practically equal. The efficiency to the brake, therefore, can be taken at 60 per cent.

The twisting moment on the shaft upon which the brake is to be placed must now be found both for lowering and raising the load. First finding the twisting moment on the worm shaft neglecting friction, we have

$$\frac{\text{Load}\times\text{radius of drum}}{\text{Gear reduction, drum to worm wheel}}=\frac{5200\times 6}{19}=1650\text{ inch-pounds}$$

Considering friction we have

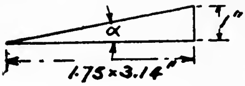
In raising the load, twisting moment $M_T=\frac{1650}{0.62}=2650$ inch-pounds.

In lowering the load, twisting moment $M_T=1650\times .60$

$=985$ inch-pounds.

In raising the load the brake must be able to transmit a twisting moment of 2650 inch-pounds, while in lowering the load 985 inch-pounds will be the frictional resistance offered by the brake.

As the shaft is $1\frac{7}{16}$ inches in diameter it is not desired to cut away much of this section, so that the pitch diameter of the screw will be made $1\frac{3}{4}$ inches and a square thread of 1 inch pitch will be tried. The angle of this helix will be




$$\alpha = \text{tang}^{-1}\frac{1}{1.75\times 3.14} = 10^\circ\ 20'.$$

For the theory of brake design see p. 176.
To retain the load

$$r\ \text{tang}\ (\alpha+\varphi) \leqq \mu_1 R$$

From which $$R \geqq \frac{r\ \text{tang}\ (\alpha+\varphi)}{\mu_1}$$

Assuming $\varphi = 4^\circ$ $\mu_1 = 0.06$

$$R \geqq \frac{0.875\ \text{tang}\ (10^\circ\ 20' + 4^\circ\ 0')}{0.06} = 3.73 \text{ inches.}$$

Assuming $R = 3\frac{3}{4}$ inches, we must now find the axial pressure.

$$Pa - \mu r_1 P = Kr\ \text{tang}\ (\alpha+\varphi) + \mu_1 KR$$

Neglecting $\mu r_1 P$ we have the axial pressure K

$$K = \frac{Pa}{r\ \text{tang}\ (\alpha+\varphi) + \mu_1 R} = \frac{985}{(0.875\times 0.256)+(0.06\times 3.75)}$$
$$= 2200 \text{ pounds.}$$

If 3000 foot-pounds of work per minute are to be allowed per square inch of brake surface, we have
Work to be absorbed per minute

$$= \text{Load}\times\text{lowering speed}\times\text{efficiency.}$$
$$= 5200\times 15\times 0.60 = 46{,}800 \text{ foot-pounds.}$$

$$\text{Braking surface} = \frac{46{,}800}{3{,}000} = 15.6 \text{ square inches.}$$

There being 2 faces, each face must have 7.8 square inches.

If the mean diameter is $2\times 3.75 = 7.50$ and the plates are made 1 inch wide, the net area of the plates is

| | |
|---|---|
| Outside diameter. . $8\frac{1}{2}$ inches | Area. . 56.75 sq. ins. |
| Inside diameter. . $6\frac{1}{2}$ inches | Area. . 33.18 sq. ins. |
| | Net Area. . 23.57 |

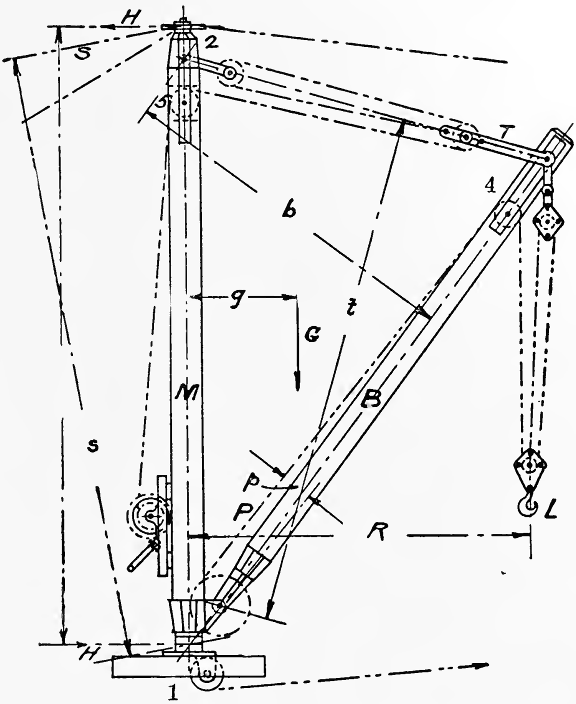
$$\text{Axial pressure per square inch of plate} \frac{2200}{23.6} = 93 \text{ pounds.}$$

# PART V.—PILLAR CRANES.

## Derricks.

Derricks in their simplest form consist of a mast M carried upon a step-bearing at 1 and usually held vertically by "guy-ropes" S running from its top bearing.

Fig. 247.

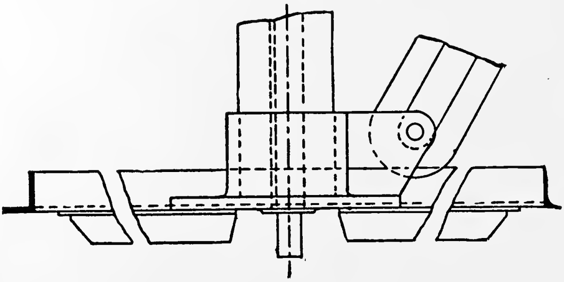



The boom or diagonal piece B is held in position or moved into any desired angle by the ropes T. The load L is raised, lowered or held in position by the hoisting rope P. The mast and boom can be rotated about the vertical bearings. When the guy-ropes clear the boom the derrick can be rotated 360°. In a stiff-leg derrick the guy-ropes are replaced by 2 struts, and these are commonly placed to permit a rotation of the derrick of 270°

Any or all of these motions may be produced by hand, horse, steam, electric or other power. The commonest way of obtaining the desired motions is by means of a hoisting engine with 1, 2 or 3 drums, according to the number of movements of the derrick to be obtained from the engine. The smaller sized derricks are timber, while the larger sizes may be either trussed timbers or structural material.

When the rotation or slewing is done by power a bull wheel is fastened to the bottom of the mast; a wire rope wound around this wheel has its ends secured to the slewing drums of the hoisting engine, so that one end wraps on the drum while the other runs from it. The wheels range in diameter from 8 to 16 feet.

Fig. 248.



They should be braced to the mast and secured to the end of the jib by tension rods, thus relieving the foot of the mast of torsion.

To permit the rotation of the derrick, it is necessary to take the ropes for raising the load and the boom, where both are raised by the engine, down through the center of the lower end of the mast and step-bearing as shown in Fig. 247.

Timber masts range in size from 10 inches to 22 inches square, while the corresponding booms run from 8 inches to 16 inches square.

The accompanying cuts illustrate fittings for timber members:

Fig. 249.—Trussed boom.

Fig. 250.—Mast top.

Fig. 253.—Mast and boom bottom.

Fig. 251.—Boom point.

Fig. 252.—Guy cap. This cap is placed on the gudgeon pin on the mast top, Fig. 252.

## Four Rod Truss Boom.

Fig. 249.

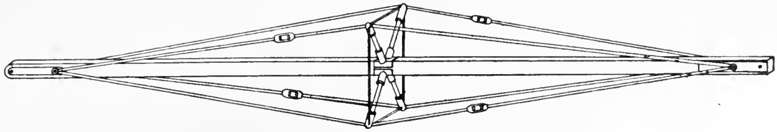


Fig. 250.

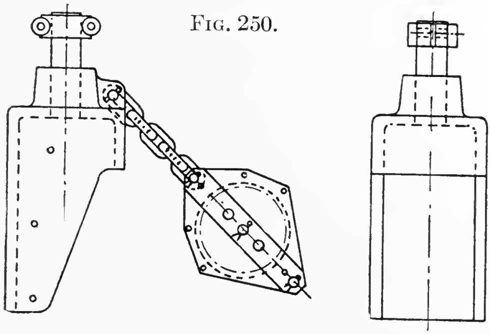


Fig. 251.

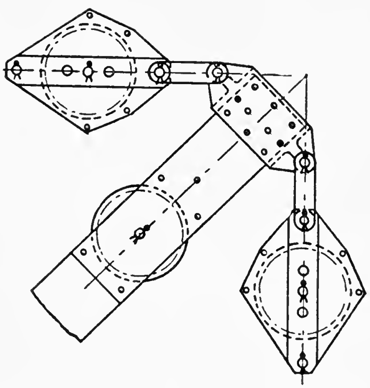


Fig. 252.

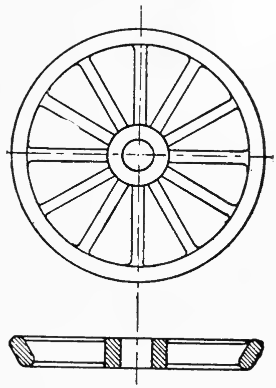
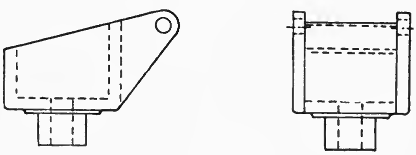


Fig. 253.



P = pull in hoisting rope, pounds.

T = tension holding boom, pounds.

G = weight in pounds of boom, mast and tackle.

B = compression in boom, pounds.

$p$, $t$, $g$, etc., are the respective lever arms of these several forces from the fulcrum 1.

N = number of ropes or chains carrying L.

L = load in pounds plus weight of the lower block and hook.

$\eta$ = efficiency of blocks, in this case 1 floating and 2 fixed blocks.

$$P=\frac{L}{N.\eta}$$

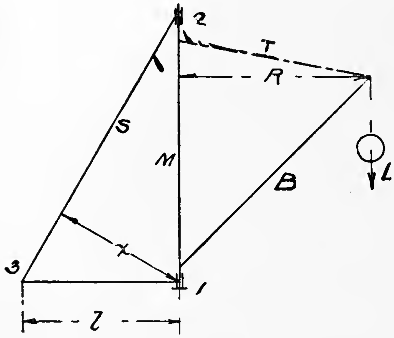
$$T=\frac{(L.R)+(G.g-(P.p)}{t} \qquad \text{(Tension).}$$

$$B=\frac{(L.R)+(G.g)+(P.c)}{b} \qquad \text{(Compression).}$$

$$S=\frac{(L.R)+(G.g)}{s} \qquad \text{(Tension).}$$

The accompanying figure illustrates a stiff-leg derrick or a derrick in which the guy is secured close to the mast. The stress in the mast is found by taking moments about 3, cutting the pieces S, M and B, and equating the internal and external moments.

Fig. 254.




$L(R+l)=Bx+Ml$ from which

$$M=\frac{L(R+l)-Bx}{l} \qquad \text{(Compression)}$$

The pull K to raise the boom where

$n$ = number of ropes carrying load and

$\eta$ = efficiency of blocks

is $$K=\frac{T}{n.\eta}$$

In addition to these direct stresses, bending may be induced in the members by the attachment of the ropes for raising the boom and the load; thus in Fig. 247 the stresses in the rope running over sheave 4 produce bending in the boom B, and in the same way the ropes running over sheave 5 produce bending in the mast.

$\overline{AB}$ is the boom having the hoisting rope run over the sheave S which is attached to it. $P_1$ is found from $P_1 = \frac{L}{N.\eta}$, the direction of both $P_1$ and $P_2$ being known, and the fact that the reaction of $P_1$ and $P_2$ must pass through the sheave pin and the intersection of $P_1$ and $P_2$, the direction of R is also known. Now since the direction of $P_1$, $P_2$ and R, together with the magnitude of $P_1$, is known, the magnitudes of $P_2$ and R can be found by completing

Fig. 255.

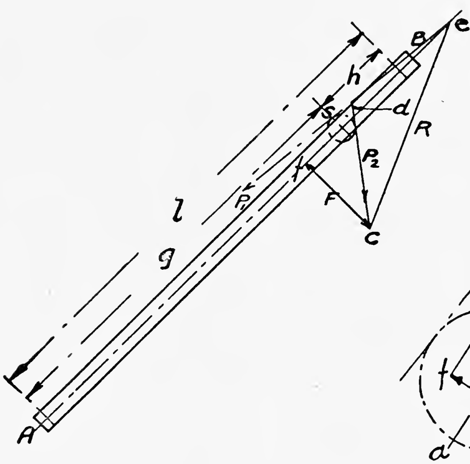
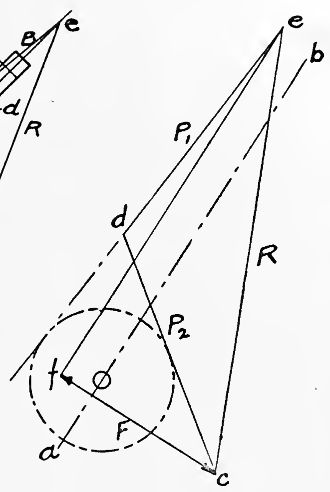



Fig. 256.




the force triangle *cde*. If *cf* is the component of R normal to the boom axis *ab*, its magnitude will be used in calculating the bending on the boom due to the forces at the sheave S. If this normal force *cf* equals F we have the bending moment at the sheave as

$$M = \frac{F}{l} \times g \times h.$$

If F is in pounds, and $g$, $h$ and $l$ in inches, M will be in inch-pounds.

This bending should be kept as low as possible by keeping the sheave S close to the boom point B.

## Rotating a Derrick.

The turning moment required to rotate a derrick neglecting acceleration can be estimated as follows:

$r_1$ = radius of gudgeon at top of mast.
$r_2$ = radius of gudgeon at bottom of mast.
H = horizontal force acting at top and bottom bearing of mast.
$\mu$ = coefficient of friction.
M = turning moment in inch-pounds to rotate derrick.
R = maximum radius of derrick.
E = pull required at the circumference of the bull wheel to turn derrick.
V = vertical pressure on step-bearing.
V = L + G + (vertical component of S).

$M = \mu H r_1 + \frac{2}{3} V r_2 \mu + \mu H r_2 + \mu E r_2$ and

$$H = \frac{(L \times R) + (G \times g)}{h}$$

**Problem.**—What force acting on a bull wheel 8 feet in diameter will be required to rotate a 5-ton derrick? Make the following assumptions:

Mast 25 feet, boom 30 feet, maximum radius 30 feet, weight of derrick 3200 pounds. Center of gravity 8 feet from center line of mast. Gudgeon diameters 3½ inches, $\mu = \frac{1}{10}$.

$$H = \frac{[(10{,}000 + 500) \times 30] + (3200 \times 8)}{25} = 13.625 \text{ pounds.}$$

V = L + G + (vertical component of S)

This last has been assumed at ⅓ H

$$V = 10{,}500 + 3500 + \frac{13{,}625}{3} = 18{,}540 \text{ pounds.}$$

$$M = \mu H r_1 + \tfrac{2}{3} \mu V r_2 + \mu H r_2 + \mu E r_2.$$

$$M = (\tfrac{1}{10} \times 13{,}625 \times 1.75) + \tfrac{2}{3} \times \tfrac{1}{10} \times 1.75 \times 18{,}540) + \tfrac{1}{10} \times 13{,}625 \times 1.75)$$

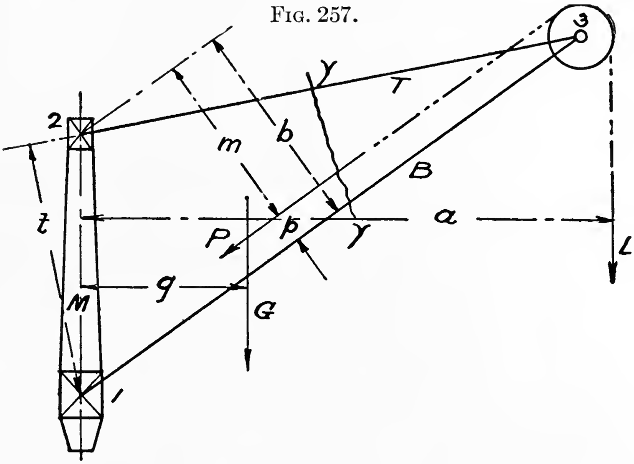
$$M = 2384 + 2163 + 2384 = 6931 \text{ inch-pounds.}$$

The bull wheel being 8 feet in diameter, 48 inches radius, the pull E at its circumference is $E = \frac{6931}{48} = 145$ pounds.

In the equation for M the portion $\mu E r_2$ was neglected and E has been found so small that it is unnecessary to revise the calculation of M.

**Pillar Crane.**

FIG. 257.




L = live load plus weight of block and tackle, pounds.

G = weight of frame, pounds.

P = chain pull in pounds.

T = tension in tie, in pounds.

B = compression in boom, in pounds.

$\eta$ = efficiency.

$$P=\frac{L}{\eta}.$$

Taking moments about 1 and considering the frame cut in *yy* we have

$$L.a+G.g-T.t-P.p=0$$

$$T=\frac{L.a+G.g-P.p}{t} \qquad \text{(Tension)}$$

In the same way taking moments about 2

$$L.a+G.g+P.m-B.b=0$$

$$B=\frac{L.a+G.g+P.m}{b} \qquad \text{(Compression)}$$

When the member T is made of eye-bars they may be designed as follows:

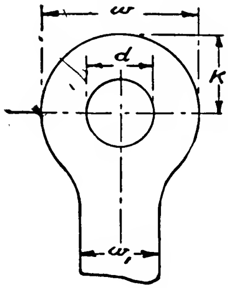
Let A = area of cross section of 1 eye-bar $A = w_1.c.$

$w_1$ = width of bar.

$c$ = thickness of bar.

$p$ = allowable working tensile fiber stress.

FIG. 258.




$$T = 2Ap = 2w_1cp \quad \therefore \quad A = \frac{T}{2p}$$

$p_1$ can be taken 7000 to 8000 pounds per square inch.

According to Hütte

$$K = \frac{c}{2} + \frac{7d}{6} \text{ and } w = c + \frac{5d}{3}$$

If the boom is a tube the radius of gyration will vary with the ratio of inside to outside diameter. $d$ = inside diameter. D = outside diameter. $\rho$ = radius of gyration.

| $d/D$ | $\rho$ |
|---|---|
| 1 | 0.353 D |
| 0.95 | 0.345 D |
| 0.90 | 0.336 D |
| 0.80 | 0.320 D |
| 0.70 | 0.305 D |
| 0.60 | 0.292 D |
| 0.50 | 0.280 D |

Limiting the value of $\frac{l}{\rho}$ to from 100 to 140 would make $D = 0.025l$ to $D = 0.04l$. For the usual values of $d/D_1$ $\rho$ approximates 0.35D and having assumed an outside diameter, $\rho$ can be approximated and the allowable unit fiber stress determined from Ritter's formula,

$$p_1 = \frac{p}{1 + \frac{F}{m\pi^2 E} \times \left(\frac{l}{\rho}\right)^2} \quad \text{(see page 106).}$$

This for soft steel reduces to

$$p_1 = \frac{p}{1 + \frac{1}{10{,}000} \times \left(\frac{l}{\rho}\right)^2}$$

The thickness and dimensions of rolled steel plates can be taken from manufacturers' hand-books.

**Example**.—The strut of a pillar crane resists a force of 50,000 pounds, its length is 25 feet. Design it as a hollow tube, the maximum fiber stress for soft steel not to exceed 10,000 pounds per square inch.

**Solution.**—Since $D = 0.03l$ to $0.04l$,

$$D = 25 \times 12 \times \begin{cases} 0.03 = 9 \text{ inches} \\ 0.04 = 12 \text{ inches} \end{cases}$$

Trying an outside diameter of 10 inches, the radius of gyration will be approximately,

$$\rho = 0.35 \times 10 = 3.50$$

$$\frac{l}{\rho} = \frac{25 \times 12}{3.5} = 85.7$$

$$p_1 = \frac{10{,}000}{1 + \frac{1}{10{,}000} \times (85.7)^2} = 5750 \text{ pounds.}$$

External area . . . . . . . . . . . . . . . . 78.54 square inches.

Tube area $= \frac{50{,}000}{5750} =$ 8.70 square inches.

Internal area . . . . . . . . . . . . . . . . . 69.84 square inches.

The inside diameter corresponding most closely to this area is 9⅜ inches area 69.03 square inches.

$$\text{Thickness of tube } \frac{10 - 9.375}{2} = \tfrac{5}{16} \text{ inch.}$$

Although a circular column section is the theoretically ideal one, in practice preference is given to the latticed channels, because of its cheaper construction. The channels are usually separated a greater distance at the foot of the pillar than at the other end of the strut. This stiffens the strut by increasing its radius of gyration about the vertical axis and especially strengthens it to resist the accelerating forces acting on it when the crane is rotated by power.

The previously designed strut when made of channels would be calculated as follows. Trying two 10-inch channels 15 pounds per foot $\rho=3.87$

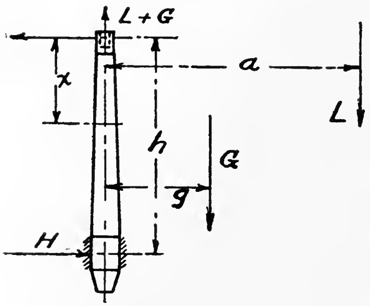
$$\frac{l}{\rho}=\frac{25\times12}{3.87}=77.$$

$$p_1=\frac{10{,}000}{1+\frac{1}{10{,}000}\times(77)^2}=\frac{10{,}000}{1.61}=6200 \text{ pounds}$$

Two 10-inch channels area 4.46 square inches will carry a load of $2\times4.46\times6200=55{,}300$ pounds, which is ample. According to Manufacturer's hand-book the channels must be separated at least 6¼ inches to make the $\frac{l}{\rho}$ values about the 2 axes at least equal and must be latticed at intervals not exceeding 55.8 inches.

$$\frac{L}{R}=\frac{l}{\rho}\quad\frac{25\times12}{3.87}=\frac{l}{0.72}\quad\therefore l=55.8 \text{ inches.}$$

Fig. 259.




Post.

L = Live load plus weight of block, tackle, etc.

G = weight of crane frame.

$a$ and $g$ = moments arms of corresponding weights.

If the weight of the frame and the load rests upon the top of the post there will be a direct compressive stress in the post equal to L+G.

If A = area of cross section in square inches.

$p_1$ = direct fiber stress due to compression.

$$p_1=\frac{L+G}{A} \qquad (1)$$

$$H=\frac{L.a+G.g}{h}$$

The bending moment on the post a distance $x$ from the top is $M=H.x$.

At the center of the lower bearing where

$$x=h,\quad M_1=H.h=L.a+G.g$$

If the resistance of this section is $I/e$

$I$ = inertia of the section.

$e$ = distances from neutral axis to extreme fibers.

The maximum fiber stress due to bending is

$$p_2=\frac{(L.a+G.g)e}{I}.$$

The combined fiber stress is

$$p=p_1+p_2=\frac{L+G}{A}+\frac{(L.a+G.g)e}{I}.$$

The post may be either solid or hollow. When hollow the inside diameter will generally not be less than ½ the external diameter. For the solid post

$$M \curvearrowright \frac{D^3 p_2}{10}$$

from which

$$D=\sqrt[3]{\frac{10\,M}{p_2}}$$

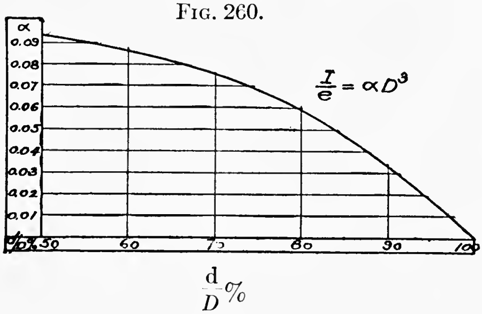
For a ring section $M \curvearrowright \dfrac{D^4-d^4}{10\,D}\times p_2.$

The resistance $\dfrac{I}{e}$ of any ring section can be approximated with sufficient accuracy by means of the accompanying curve; thus the resistance of a ring 10 inches outside diameter and 6.5 inches inside diameter will be found as follows:

$$\alpha=0.082,\ \frac{I}{e}=0.082\times 10^3=82.0$$

RING SECTION BEAMS.

FIG. 260.




Another example will illustrate still further the use of the curve. A post is subjected to a bending moment of 2,400,000 inch-pounds. Assume that the maximum fiber stress is to be 11,000 pounds, leaving 10,000 pounds fiber stress for bending; assume further that the inside diameter is 65 per cent. of the outside diameter.

The required resistance is $\frac{2,400,000}{10,000}=240.$

$$\frac{I}{e}=\propto D^3, \quad \propto =0.082$$

$$\frac{I}{e}=\propto D^3=240=0.082\,D^3$$

$$D=\sqrt[3]{\frac{240}{0.082}}=14.3 \text{ inches.}$$

$$d=0.65\times 14.3=9.3 \text{ inches.}$$

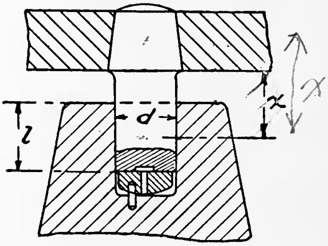
Making $D=14\frac{1}{2}$ inches and $d=9\frac{1}{2}$ inches.

The following working fiber stresses can be used in posts of this type:

| | Pounds per square inch |
|---|---|
| Hollow cast-iron .................. | 5 000 to 6,000 |
| Wrought-iron and steel castings.... | 10,500 to 11,500 |
| Forged or rolled steel .. .......... | 14,000 to 15,000 |

## Upper Step-Bearing.

Fig. 261.



$d$ = diameter of journal.

$l$ = length of bearing.

$x$ = lever arm of force H.

M = bending moment on journal.

$$M=H.x=\frac{d^3\times p_1}{10} \therefore p_1=\frac{10\,Hx}{d^3}.$$

Generally $l=d$ and assuming $x \backsim d$

we have $p_1=\frac{10\,H}{d^2}.$

For the direct compression $L+G=\frac{\pi d^2}{4}\times p_2 \backsim 0.8d^2p_2$

from which $p_2=\frac{L+G}{0.8d^2}$ hence

$$p_{max.}=p_1+p_2=\frac{10\,H}{d^2}+\frac{L+G}{0.8d^2}$$

and $d=1.1\sqrt{\frac{8\,H+(L+G)}{p_{max.}}}.$

The maximum fiber stress can have the following values:

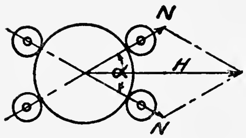
| Material of pin | Pounds per square inch |
|---|---|
| Rolled steel (hand driven) | 14,000 to 17,000 |
| Rolled steel (power driven) | 11,000 |

The side pressure on the journal must be limited to $H = p_1 \times l \times d_1$. According to Ernst,

$p_1 = 1850$ pounds per square inch for hand driving.

$p_1 = 1400$ pounds per square inch for power driving.

Fig. 262.




Lower Bearing.—If this is a neck journal its design will be similar to the one just described. To reduce the frictional resistance at this point, due to the large diameter of the bearing, rollers are frequently used.

N = normal pressure between roller and post.

$\alpha$ = angle subtended by rollers.

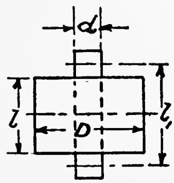
$$N = \frac{H}{2 \cos \frac{\alpha}{2}}.$$

## Rollers and Pins.

To resist the bending moment on the pin its diameter must be

$$d = \sqrt[3]{\frac{10N\left(\frac{l_1}{4} - \frac{l}{8}\right)}{p}}.$$

Fig. 263.




For steel $p$ can be taken at 14,000 pounds per square inch.

The unit pressure on the projected area of the pin in the roller should be limited to 1800 to 2000 pounds per square inch.

According to Bethman $D \sim 5d$, and the pressure on the roller should not exceed $N = p \times l \times D$, where $p$ has the following values:

| Material | $p$ |
|---|---|
| Cast-iron | 350 |
| Steel | 850 |

## Base-Plate, Foundation and Foundation Bolts.

L = live load plus portion of frame weight at end of jib.
$a$ = arm of weight L from center line of post.
$n$ = number of foundation bolts.
$r$ = radius of foundation bolt circle.
$l$ = distance from center of foundation bolt to hub holding post.
W = resistance of arm section at hub.
$p$ = allowable working fiber stress 2500–3500 pounds per square inch for cast-iron.
R = reaction at foundation bolt.

$$W \geqq L\left(\frac{2a}{nr}+\frac{1}{n}\right)\frac{l}{p}.$$

Fig. 264.

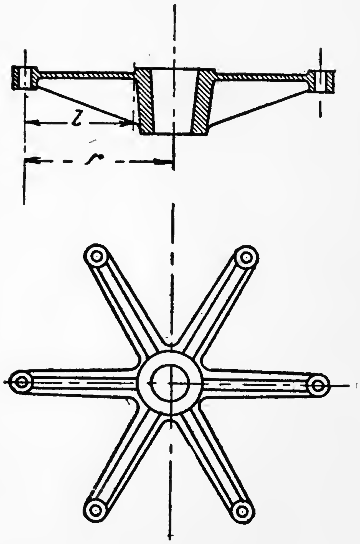
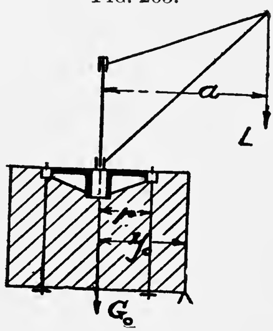



Fig. 265.




From this the reaction at the foundation bolt is found to be

$$R=L\left(\frac{2a}{nr}+\frac{1}{n}\right).$$

The diameter of the foundation bolt can now be found by selecting a bolt whose area at the root of the thread shall be

Area $=\frac{R}{p_t}$, $p_t$ can be assumed at 8000 pounds per square inch.

## Foundation.

P = weight of base-plate and post.

F = weight of foundation (minimum).

$$(F+P)y_0=L(a-y_0),$$

$$F=\frac{La}{y_o}-(L+P).$$

To insure stability the foundation is usually made from 2 to 2½ times F. The foundation when made of concrete can be assumed as weighing from 100 to 125 pounds per cubic foot.

HOISTING MECHANISM.—The load will ordinarily be raised by hand, the machinery not differing materially from the ordinary jib crane. Air hoists are also sometimes used with these cranes.

ROTATING.—The turning moment required to rotate the loaded crane can be estimated as follows:

$\rho_1$ = radius of journal at step-bearing.
$\rho_2$ = radius of journal at second bearing.
$\rho_3$ = radius of roller axles.
$\rho_4$ = radius of post at roller bearing.
$\rho_5$ = radius of rollers.
H = horizontal reaction at bearings in pounds.
L = live load in pounds.
G = weight of crane frame in pounds.
N = pressure between roller and post in pounds.
$\mu$ = coefficient of sliding friction.
$f$ = coefficient of rolling friction.
M = turning moment, inch-pounds.

In the case of the crane without a roller bearing

$$M=\frac{2\mu(L+G)\rho_1}{3}+\mu H\rho_1+\mu H\rho_2$$

With the roller bearing we have

$$M=\mu H\rho_1+\frac{2\mu(L+G)\rho_1}{3}+\frac{2\mu N\rho_3}{\rho_5}\times\rho_4+\frac{2fN}{\rho_5}\times\rho_4.$$

### GRAPHICAL DETERMINATION OF FRAME STRESS.

In these diagrams the chain or rope pull has been neglected. (See page 116 )

G = weight of crane frame, not including post.
$g$ = lever arm of G. Distance from center of gravity of frame to center line of post.
L = live load.

The equivalent part of G necessary to be concentrated at 3 to produce the same turning of the frame about 1 and 2 is $\frac{Gg}{a}$, the remainder of the weight $G\frac{a-g}{a}$ must act at the center line of the post.

N = load on post normal to rollers.

The force P will lie in a line joining points 1 and 2. The diagrams follow as shown. In Diagram II, V is the vertical component of the normal pressure N.

Diagram I.

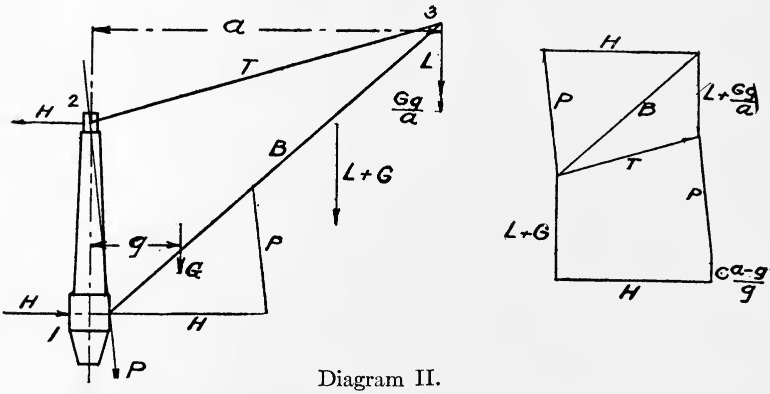
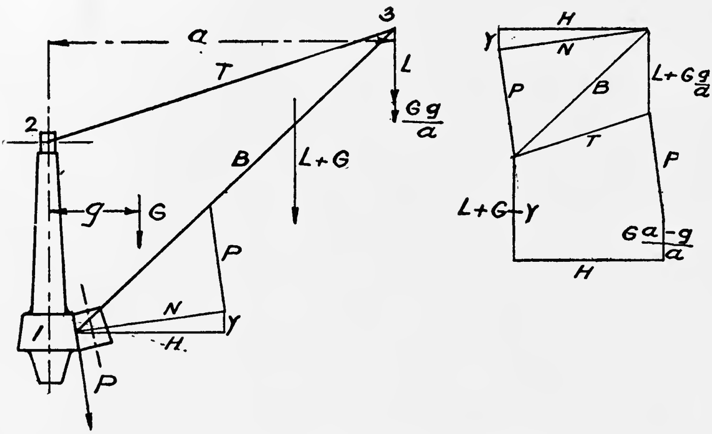



Diagram II.



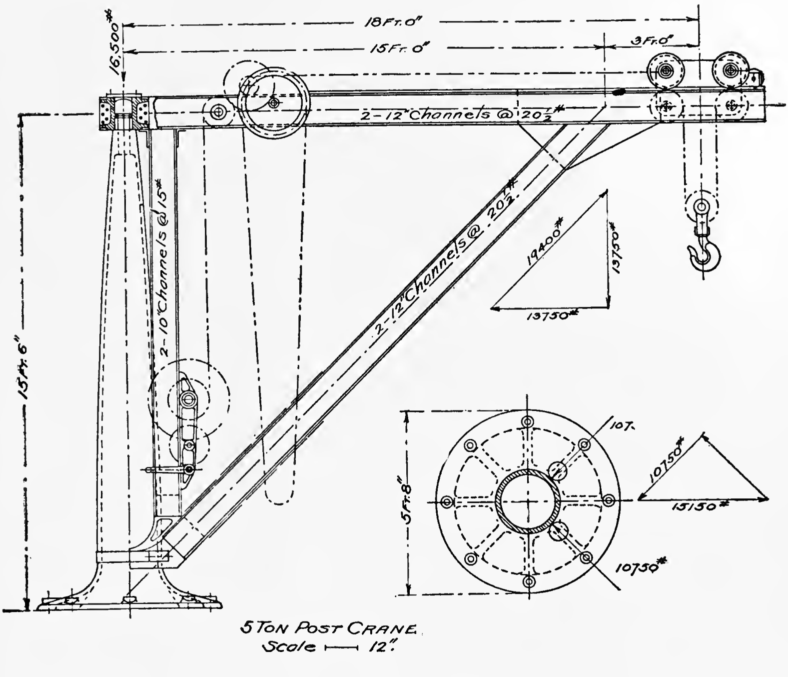

FIVE-TON POST CRANE.

The load on the trolley wheels will be estimated as, live load, 10,000 pounds; hook, block, trolley, chain, etc., 500 pounds; making a total of 10,500 pounds. The extreme position of this load being 18 feet from the post center, the equivalent load concentrated 15 feet from the post will be

$$\text{Equivalent load} = \frac{10{,}500 \times 18}{15} = 12{,}600 \text{ pounds.}$$

To this must be added the portion of the crane frame weight estimated as coming at the juncture of the jib and the strut, which will be taken at 1150 pounds. The total equivalent apex load at

Fig. 266.




this point then is 12,600+1150=13,750 pounds. Drawing the force triangle gives the stress in the strut as 19,400 pounds and that in the jib as 13,750 pounds, the former producing compression, the latter tension.

The step-bearing at the top of the post takes the entire weight of the crane, excepting the post, together with the live load; this total is estimated at 16,500 pounds. This bearing is frequently designed so that the vertical load is taken by balls, while the horizontal thrust is carried by a roller bearing. The rollers at foot of the post are also frequently run on roller bearings.

Design of the strut. This strut can be placed a short distance from the foot of the post; the remaining distance is taken as 11 feet.

Trying 12-inch channels, the radius of gyration about their minor axis is 0.81; this gives an $\frac{l}{r}$ value of $\frac{11\times 12}{0.81}=163$.

This value is high and can only be used with a very low fiber stress.

The allowable fiber stress according to Ritter's formula is

$$p=\frac{12,000}{1+\frac{1}{10,000}\times\left(\frac{l}{r}\right)^2}=\frac{12,000}{1+\frac{1}{10,000}\times 163^2}=3300 \text{ pounds.}$$

The permissible load is $3300\times 6.02=19\,800$ pounds. This is about double that coming on it, so it will be taken.

The horizontal force on the post is

$$\frac{13,750\times 15}{13.6}=15,150 \text{ pounds.}$$

If the weight of the hook, block and chain is assumed as 200 pounds, the pull in the chain over the jib is

$$\text{Chain pull}=\frac{10,200}{2\times 0.97\times 0.98}=5350 \text{ pounds}$$

The direct stress in the jib is $13,750-5350=8400$ pounds.

The cantilevered extension of the jib is 42 inches, the supported span is 13 feet 4 inches.

The bending on the cantilever is

$$M=(10,500\times 42)-(5350\times 6)=408,900 \text{ inch-pounds.}$$

Bending on the supported span is found by placing one of the trolley wheels 6 inches from the center of the span. The reactions then are

$$R_1=\frac{(5250\times 74)+(5250\times 98)}{160}=5640 \text{ pounds.}$$

$$R_2=10,500-5640=4860 \text{ pounds.}$$

Bending moment $=4860\times 74=359,640$ inch-pounds.

Bending due to chain pull $(5350\times 6)=32,100$ inch-pounds.

Total bending $=359,640+32,100=391,740$ inch-pounds.

Allowing a maximum fiber stress of 12,000 pounds per square inch and assuming that this span has a section unsupported laterally of 90 inches, this is about 30 flange widths, and reduces the allowable working fiber stress to about 85 per cent. of the maximum desired for this span.

Allowable fiber stress $12{,}000 \times 0.85 = 10{,}200$ pounds.

Trying two 12-inch channels at $20\frac{1}{2}$ pounds per foot we have, Direct tension $= 8400 \div$ Area of 2 channels $= 2 \times 6.03 = 700$ pounds per square inch.

Allowable unit compression $10{,}200 + 700 = 10{,}900$ pounds.

$$\frac{I}{e} = \frac{391{,}740}{2 \times 10{,}900} = 17.95.$$

Cantilevered span. Direct compression $5350 \div (2 \times 6.03) = 445$. Total allowable compression $10{,}200 - 4\ \ 0 = 9750$ pounds.

$$\frac{I}{e} = \frac{408{,}900}{2 \times 9{,}750} = 21.$$

The section modulus of 12-inch channels $20\frac{1}{2}$ pounds per foot being 21.4, this section will do.

Post.—The horizontal force on the post is 13,750 pounds. Max'mum bending on post $13{,}750 \times 15 \times 12 = 2{,}475\ 000$ inch-pounds.

Assuming the outside diameter of the post at this section at 24 inches and the direct load as 16,500 pounds, we must first estimate the direct compression due to this load of 16 500 pounds. If the post were only $1\frac{1}{4}$ inches thick the area of the post sect on would be $452 - 363 = 89$ square inches. The unit compression corresponding to this is $16{,}500 \div 89 = 185$ pounds. It is therefore safe to assume that this is probably under 200 pounds per square inch. Allowing a total fiber stress of 4,000 pounds per square inch, that available as flexural stress is $4000 - 200 = 3800$.

The section modulus then is $\frac{I}{e} = \frac{M}{p} = 2{,}475{,}000 \div 3800 = 650$.

Referring to the curve p. 235, $\frac{I}{e} = \propto D^3$.

$$\propto = \frac{I}{e \times D^3} = \frac{650}{24^3} = 0.047.$$

This corresponds to an inside diameter of 86 per cent. of the outside diameter. Inside diameter $= 24 \times 0.86 = 20.6$ inches, say $20\frac{1}{2}$ inches. Bolts for post. Diameter of base 5 feet 8 inches. The turning moment on the post is 2,475,000 inch-pounds.

Force acting at the center of the post to prevent overturning,

$$F = 2{,}475{,}000 \div \text{radius of bolt circle.}$$

$$F = 2{,}475{,}000 \div 30 = 82{,}500 \text{ pounds.}$$

From this must be deducted the estimated weight of the post and that part of the crane frame assumed as acting at the post center, say 12,500 pounds.

Allowing 8 bolts and assuming that the bolt farthest from the point about which the post is assumed as turning receives twice the average load upon the bolts, we have the load upon this extreme bolt

$$\frac{(82{,}500 - 12{,}500) \times 2}{8} = 17{,}500 \text{ pounds.}$$

Allowing a fiber stress of 10,000 pounds per square inch at the root of the bolt threads, this area is found to be $17{,}500 \div 10{,}000 = 1.75$ square inches, which corresponds to a bolt diameter of $1\frac{3}{4}$ inches.

## Design of 5-Ton Post Crane.—Plate III.

The load of 5 tons (10,000 pounds) will be assumed as carried by 2 chains.

Efficiency of 1 floating and 1 fixed sheave $.95 \times .97 = 93$ per cent.

Load on 1 chain $\dfrac{10{,}000}{2 \times 0.93} = 5400$ pounds.

According to the formula for chain strength, $L = 14{,}000.d^2$,

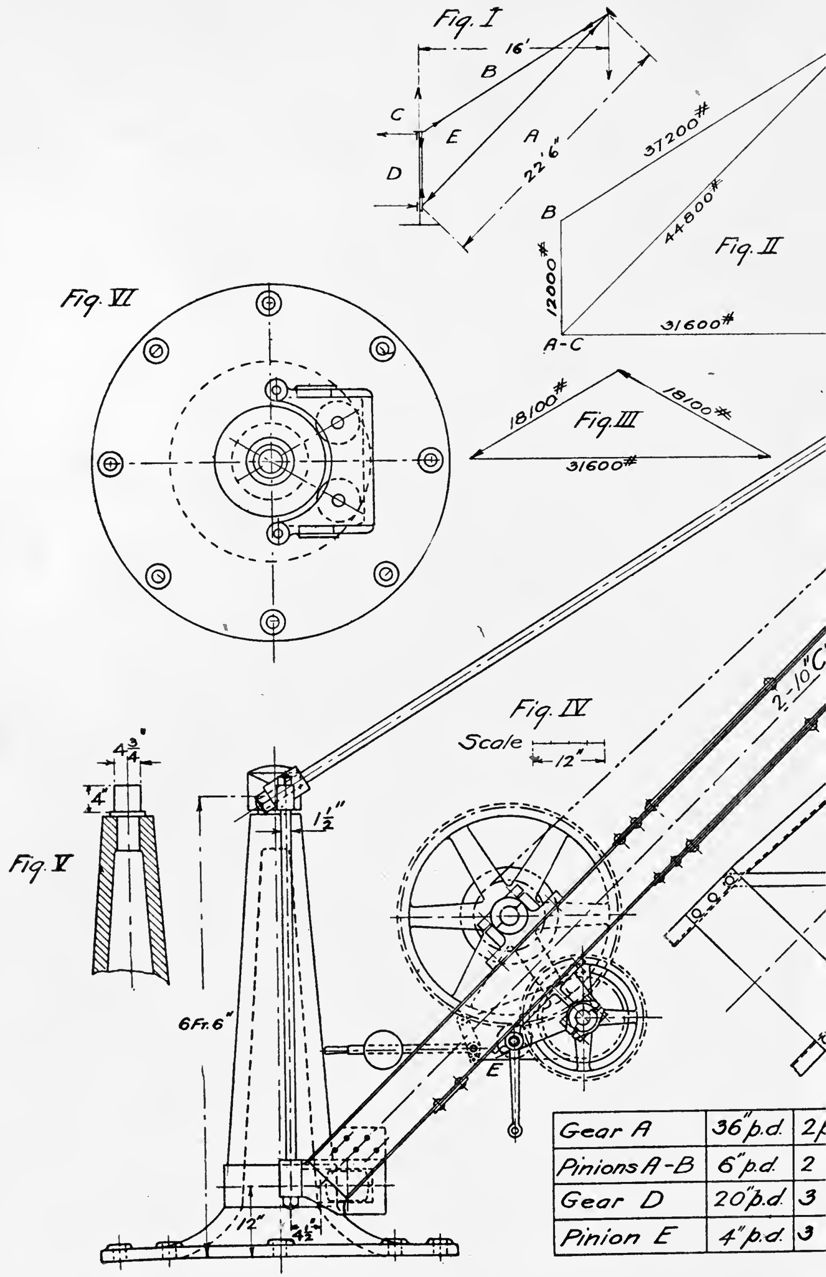
hence $\qquad d = \sqrt{\dfrac{5400}{14{,}000}} = 0.62 \text{ inch} \backsim \frac{5}{8} \text{ inch.}$

## Design of Post and Frame.

The skeleton of the crane with the external forces acting upon it is shown in Fig. I, Plate III. The stress diagram is given in Fig. II.

The member EB is in tension, due to a force of 37,200 pounds; a unit tensile stress of 12,000 pounds per square inch would demand a net section of $\dfrac{37{,}200}{12{,}000} = 3.10$ square inches.

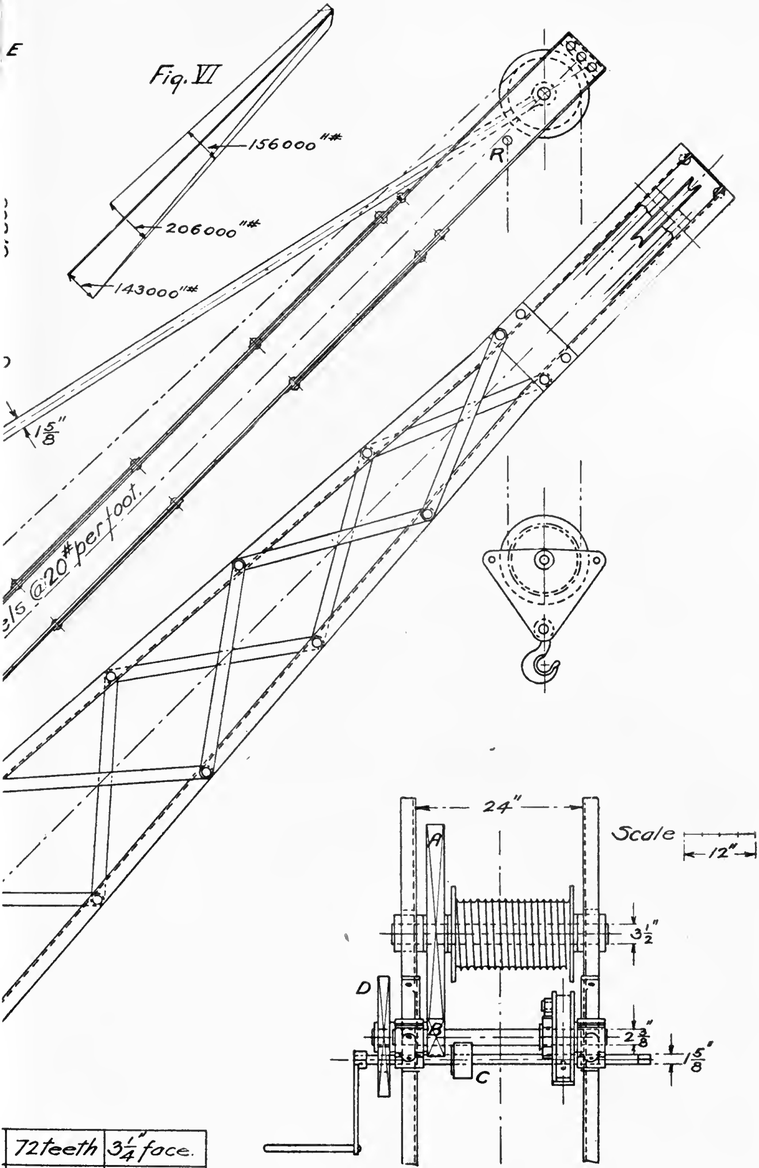
Using 2 rounds, the area at the root of the threads would be $\dfrac{3.10}{2} = 1.55$ square inches and would require bolts $1\frac{5}{8}$ inches in diameter.




| | | |
|---|---|---|
| Gear A | 36" p.d. | 2 |
| Pinions A-B | 6" p.d. | 2 |
| Gear D | 20" p.d. | 3 |
| Pinion E | 4" p.d. | 3 |

I.

E




| | |
|---|---|
| 72 teeth | 3¼" face. |
| 12 do. | 3¼" do. |
| 60 do. | 2" do. |
| 12 do. | 2" do. |

5 TON POST CRANE.

Member DE is subjected to a tension of 31,500 pounds and upon the same basis would require

$$\frac{31,500}{12,000}=2.63 \text{ square inches,}$$

or two 1½ inch diameter rods.

Member EA is subjected to combined compression and bending. The total direct stress equals 44,800 pounds plus the rope pull of 5400 pounds, or 44,800+5400=50,200 pounds.

The maximum bending due to the chain pull is M=5400×15 =81,000 inch-pounds. There is bending induced in this piece by the fact that the forces acting at its lower end do not intersect at a common point; the vertical tie rods are 4½ inches to the left of the intersection of the axis of the channels and the axis of the rollers. This bending moment is M=31,800×4.5=143,100 inch-pounds.

There is also bending due to one end of the chain being fastened to the channels at R; this bending amounts to 3700×10= 37,000 inch-pounds. The distribution of these moments upon the piece AE is represented in Fig. VI. As the bending moments have all been carefully estimated, it is allowable to use a higher fiber stress, say 15,000 pounds per square inch.

The length of the piece is approximately 21 feet. The maximum bending is 206,000 inch-pounds. The bending at the middle of the member is approximately 156,000 inch-pounds. Trying two 10-inch channels at 20 pounds per foot, and using the bending at the middle, we have since $\frac{l}{r}=\frac{21\times 12}{3.66}\backsim 70.$

Allowable fiber stress, as a column,

$$p=\frac{15,000}{1+\frac{1}{10,000}\left(\frac{l}{r}\right)^2}=\frac{15,000}{1.49}\backsim 10,000 \text{ pounds.}$$

Direct compression $p_c=\frac{50,200}{2\times 5.88}=4250$ pounds.

Flexural compression $p_b=\frac{156,000}{2\times 15.7}=5000$ pounds.

Combined compression, extreme fibers, $p_c+p_b=4250+5000=$ 9250 pounds.

These 10-inch channels should therefore be satisfactory.

To prevent these struts from failing about their minor axes the 2 channels must be latticed so that the intervals between the lacing divided by the least radius of gyration of the channel shall not exceed the length of the strut divided by its radius of gyration about its principal axis. It was found above that $l/r=70$, hence $l^1=70\times r^1=70\times .70=49.0$ inches.

According to the manufacturers' hand-books, these channels should not be placed closer than 6⅜ inches back to back.

POST.—The post is assumed 6 feet 6 inches high. It is made of cast-iron, and a fiber stress of 4500 pounds per square inch will be allowed upon a ring section.

It has been shown (see p. 235) that when the inside diameter can be assumed as some percentage of the outside diameter the value of $\frac{I}{e}$ can be taken as approximately $\frac{I}{e} = \propto D^3$. Taking $d=0.65\,D$, $\propto=0.082$. Calculating a section 12 inches from the top, the bending moment is $M=31{,}600\times 12=379{,}200$ inch-pounds.

$$\frac{I}{e}=\frac{M}{p}=\frac{379{,}200}{4500}=\propto D^3=0.082\,D^3$$

$$D=\sqrt[3]{\frac{379{,}200}{4500\times 0.082}}=10.15 \text{ inches.}$$

$$d=10.15\times 0.65=6.6, \text{ say } 6\tfrac{1}{2} \text{ inches.}$$

The area of the ring is $(80-33)=47$ square inches.

The direct compression is $\frac{12{,}000}{47}=256$ pounds per square inch.

Section 2–2 taken 36 inches from the top.

$$M=31{,}600\times 36=1{,}137{,}600 \text{ inch-pounds.}$$

$$\frac{I}{e}=\frac{M}{p}=\frac{1{,}137{,}600}{4{,}500}=\propto D^3=0.082\,D^3$$

$$D=\sqrt[3]{\frac{1{,}137{,}000}{4500\times 0.082}}=14.6 \text{ inches.}$$

$$d=14.6\times 0.65=9.5 \text{ inches.}$$

Section 3–3 taken 60 inches from the top.

$$M = 31{,}600 \times 60 = 1{,}896{,}000 \text{ inch-pounds.}$$

$$\frac{I}{e} = \frac{M}{p} = \frac{1{,}896{,}000}{4500} = 0.082\,D^3$$

$$D = \sqrt[3]{\frac{1{,}896{,}000}{4500 \times 0.082}} = 17.3 \text{ inches.}$$

$$d = 17.3 \times 0.65 = 11.25 \text{ inches.}$$

In the above calculations the top of the post is assumed as the middle of the pin.

### Step-Bearing.

$$d = 1.1\sqrt{\frac{(8\times H)+(L+G)}{p_b}} = 1.1\sqrt{\frac{(8\times 31{,}600)+(10{,}000+3000)}{14{,}000}}$$

$$d = 1.1\sqrt{\frac{265{,}000}{14{,}000}} = 4.79 \text{ inches} \backsim 4\tfrac{3}{4} \text{ inches.}$$

$$k.l.d. \geqq H.$$

$$l \geqq \frac{H}{k.d} = \frac{31{,}600}{1800 \times 4.75} = 3.75 \text{ inches, say 4 inches.}$$

Vertical pressure on step-bearing,

$$\frac{4(L+G)}{\pi d^2} = \frac{4(10{,}000+3000)}{3.14 \times 4.75^2} = 735 \text{ pounds per square inch.}$$

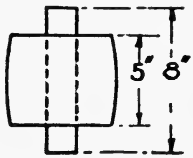
This is satisfactory.

### Rollers.

For normal pressure see Fig. III, $N = 18{,}100$ pounds.

$$N = p.l.D \therefore l.D \geqq \frac{N}{p} = \frac{18{,}100}{850} = 21.3.$$

$$d = \sqrt[3]{10\,N\left(\frac{l_1}{4} - \frac{l}{8}\right)} = \sqrt[3]{\frac{10 \times 18{,}100\left(\frac{8}{4} - \frac{5}{8}\right)}{14{,}000}}$$




$$= 2.56 \text{ inches} \backsim 2\tfrac{9}{16} \text{ inches.}$$

The pressure per square inch of wheel on axle is

$$\frac{18{,}100}{2.56 \times 5} = 1410 \text{ pounds.}$$

This pressure is satisfactory.

## Hoisting Mechanism.

The twisting moment on the shaft carrying pinion B is

$$M_T = \frac{5400 \times 6}{6 \times .92 \times .98} = 6000 \text{ inch-pounds.}$$

Assuming that pinion B has 12 teeth and that it is made of cast-iron, we have

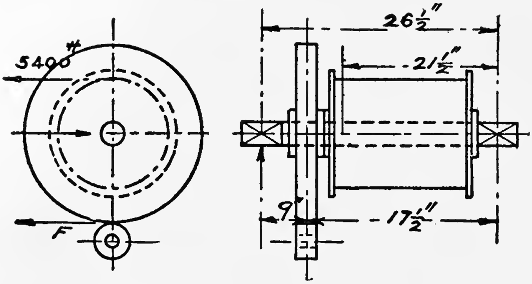
$$p = \sqrt[3]{\frac{6.28\ M_T}{n.s.c.\left(0.124 - \frac{0.684}{n}\right)}} \quad \text{(See page 19.)}$$

$$p = \sqrt[3]{\frac{6.28 \times 6000}{12 \times 6000 \times 2 \times 0.067}} = 1.58 \text{ inches.}$$

2 diametral pitch with 3¼ inch face will be used.

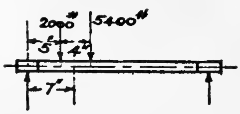
## Drum Shaft.

Fig. 267.




$$F = \frac{5400 \times 6}{18 \times .98 \times .92} = 2000.$$

Reaction, $$R = \frac{2{,}000 \times 21.5}{26.5} + \frac{5{,}400 \times 17.5}{26.5} = 5200.$$




$M = (5200 \times 7) - (2000 \times 2) = 32{,}400$ inch-pounds.

Designing the shaft for both twisting and bending, we have

$$M_{B.E.} = 0.35\ M_B + 0.65\sqrt{M_B^2 + M_T^2}$$
$$= (0.35 \times 32{,}400) + 0.65\sqrt{32{,}400^2 + 32{,}400^2}$$
$$= 40{,}500 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10.2\,M_{B.E.}}{p_b}}=\sqrt[3]{\frac{10.2\times40{,}500}{9{,}000}}=3.58 \text{ inches.}$$

$d=3.58$ inches, say $3\frac{1}{2}$ inches.

If the bearing adjoining the gear is 5 inches long the pressure per square inch of projected area is

$$\frac{5{,}200}{5\times3.5}=315 \text{ pounds.}$$

This, considering the character of the work, is very moderate.
Shaft carrying pinion B:

The bending moment $M_B = 2000\times4.5=9000$ inch-pounds.

The twisting moment $M_T=2000\times3=6000$ inch-pounds.

The equivalent bending moment is

$$M_{B.E.}=0.35\,M_B+0.65\sqrt{M_B^2+M_T^2}$$

$$M_{B.E.}=(0.35\times9{,}000)+0.65\sqrt{9{,}000^2+6000^2}=10{,}170 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10.2\,M_{B.E.}}{p_b}}=\sqrt[3]{\frac{10.2\times10{,}170}{9{,}000}}=2.26, \text{ say } 2\tfrac{3}{8} \text{ inches.}$$

The driving pinion E will be made with 12 teeth of cast-iron.
The turning moment on the crank shaft is

$$M_T=\frac{6{,}000}{5\times.92}=1300 \text{ inch-pounds.}$$

$$p=\sqrt{\frac{6.28\,M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times1{,}300}{12\times6{,}000\times2\times0.067}}=0.946''$$

Use 3 diametral pitch.

Pinion E 4-inch pitch diameter, 12 teeth, 2-inch face.

Gear D 20-inch pitch diameter, 60 teeth, 2-inch face.

Crank Shaft.—The cranks will be 15 inches long. The force required at the crank for operation at full load is,

$$F=\frac{1300}{15}=87 \text{ pounds.}$$

If 2 men were assumed at the crank, each exerting 50 pounds on it, the maximum bending under the left bearing is

$$M_B=(100\times8.5)+\left(\frac{100\times15}{2}\times4\right)=3850 \text{ inch-pounds.}$$

The diameter is then found to be

$$d=\sqrt[3]{\frac{10.2\times 3{,}850}{9{,}000}}=1.62, \text{ say } 1\tfrac{5}{8} \text{ inches.}$$

**Brake.**—The brake will be the strap-brake type. See p. 167 for discussion of the brake. The following assumptions will be made, $a=5$ inches; $b=2$ inches; $l=15$ inches. The turning moment on the shaft is

$$M_T=\frac{5400\times 6\times 0.98\times 0.92}{6}=4860 \text{ inch-pounds.}$$

$$P=\frac{F}{l}\left(\frac{a-bk}{k-1}\right); \quad F=\frac{4{,}860}{7}$$

From the table page 167 for an arc of contact between strap and brake wheel of 70 per cent. and $\mu=0.18$, $k=2.21$.

$$P=\frac{4{,}860}{7\times 15}\left[\frac{5-(2\times 2.21)}{2.21-1}\right]=22 \text{ pounds.}$$

**Ratchet Wheel for Brake.**—Assuming the wheel has 10 teeth, and that it is made of cast-iron, we have

$$p_c=\sqrt{\frac{16\times 3.14\times 4{,}860}{1.5\times 2500\times 10}}=2.55 \text{ inches.}$$

This is approximately $1\frac{1}{4}$ per inch and requires an 8-inch diameter wheel.

**Foundation Bolts:**

$n$ = number of bolts.

$a$ = area of 1 bolt, measured at the root of the thread, square inches.

$p$ = allowable fiber stress, tension, pounds per square inch.

R = radius of bolt circle, in inches.

M = moment tending to overturn the crane, due to the dead load of the crane and the live load. Inch-pounds.

$$a=\frac{2\,M}{n.p.R}, \quad M=(10{,}000\times 16\times 12)+\frac{3{,}000\times 16\times 12}{4}$$

$$a=\frac{2\times 2{,}064{,}000}{10\times 12{,}000\times 27}=1.28 \text{ square inches.}$$

This corresponds to $1\frac{1}{2}$ inches diameter bolts.

### Foundations.

1. The size and weight of the foundation must be sufficient to prevent overturning of the crane.

2. The maximum pressure on the edge of the foundation must

not exceed that allowed on the soil when the crane carries its maximum load.

The size can be calculated by assuming a size for the foundation and then taking moments about the corner *d*.

$$L(l-f)=G(f-g)+F_1 f$$

from which
$$F_1=\frac{L(l-f)-G(f-g)}{f}$$

In this case $F_1$ is the weight required to just balance the crane; according to Bethman, the foundation is usually made from 2 to 3 times this weight.

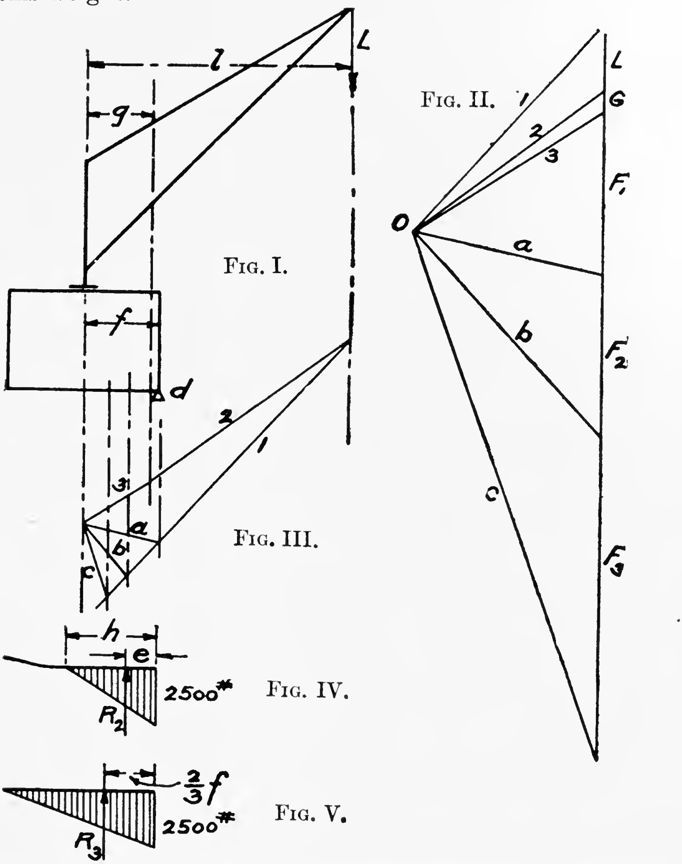



FIG. I. FIG. II. FIG. III. FIG. IV. FIG. V.

The accompanying sketches illustrate a graphical solution of the same problem.

Fig. I is the skeleton of the crane with the assumed foundation of width 2.*f*. G is the weight of the crane and *g* the distance

Fig. 268.

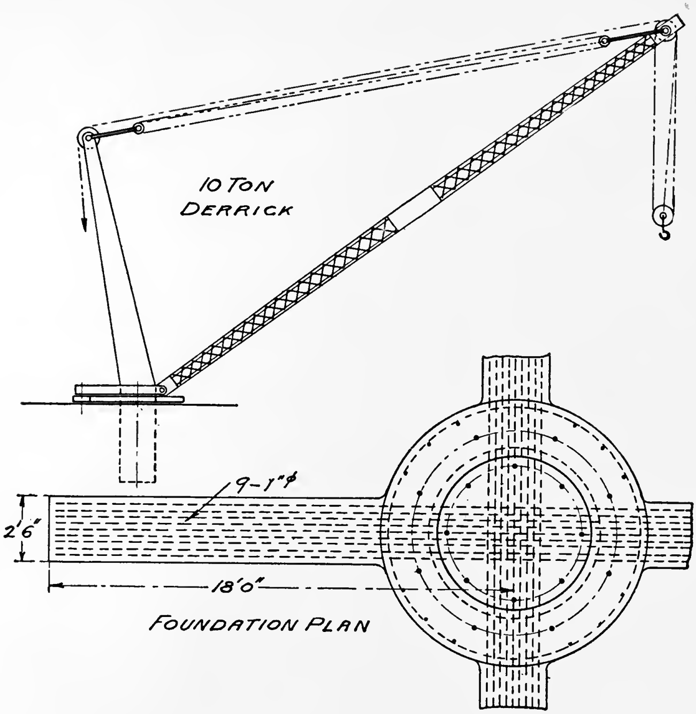
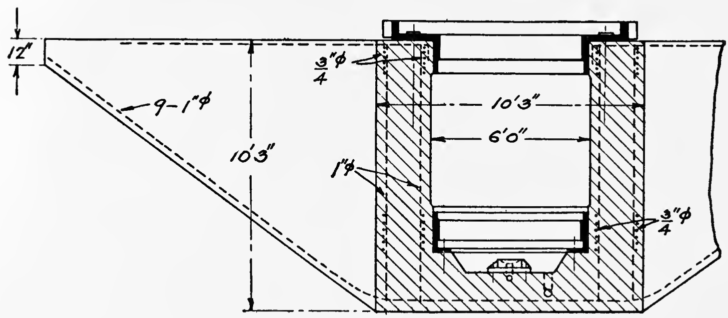



Fig. 269.



from the center line of the crane-post to the center of gravity of the crane-frame. Fig. II is a force polygon having a pole O to which the rays 1, 2, and 3 are drawn. The rays 1 and 2 hold force L in equilibrium, so that in the equilibrium polygon, Fig. III, 1 and 2 intersect in a point on L, 2 and 3 intersect on G, and 3 is terminated by the line representing the weight of the foundation, or the center line of the post. Ray 1 is terminated by the line in which the resultant is to be assumed as acting, so that joining the extremities of 1 and 3 gives the closing side of the equilibrium polygon *a*, *b*, or *c*, as the case may be, and when the rays are drawn in Fig. II parallel to these lines in Fig. III the intercepts between 3 and *a*, *b*, or *c* give the weights of the foundations for the line of action of the resultant in each case. Thus if it is desired to know what weight will just balance the crane, from the corner *d* drop a line parallel to the forces L and G. Where this line intersects string 1 in Fig. III gives the terminal of the string *a*, draw *a* and in Fig. II draw the ray *a* parallel to it. The intercept between the two rays 3 and *a* on the load line gives the weight of the foundation to just balance the load. The theoretical maximum unit pressure on the edge of the foundation under these circumstances becomes infinite.

Should it be desired that the weight of the foundation should be double that required to just balance the crane, in Fig. II lay-off $F_2=F_1$, making the intercept between 3 and $b_1=2\,F_1$. In Fig. III draw *b* through the extremity of string 3 until it intersects string 1; the resultant must go through this point of intersection. Project this point upon the foundation by a line parallel to the load lines L and G. If this resultant acts a distance *e* to the left of the corner *d* the earth pressure will be distributed over a distance *h* of the foundation, so that $h=3e$ and the maximum pressure at the corner of the foundation is given by

$$p=\frac{R_2}{h\times f}$$

In this case

$L=10{,}000$ pounds; $l=16$ feet; $G=3{,}000$ pounds; $g=4$ feet; $f=4.5$ feet, and $R_2$ was found from Fig. II to be

$$R_2=F_1+F_2+G+L=24{,}500+24{,}500+3000\times 10{,}000=62{,}000 \text{ pounds}$$

and from this $p=\dfrac{R_2}{h\times f}=\dfrac{62{,}000}{5.5\times 4.5}=2{,}500$ pounds.

Making the foundation sufficiently heavy to bring the resultant $R_3$ within the middle third would increase the stability of the crane, and distribute pressure over the entire base of the foundation, but would not decrease the extreme pressure, as is seen in Fig. V.

The depth of foundation required in the second case could be determined as follows:

Each foot of depth of foundation, if of concrete, would weigh

$$9\times 9\times 1\times 125=10{,}125 \text{ pounds.}$$

Hence if the foundation according to Fig. II is to weigh $F_1+F_2=$ 49,000 pounds, its depth must be

$$d=\frac{49{,}000}{10{,}125}\approx 5 \text{ feet.}$$

Under any circumstances, the foundation must be carried to proper soil or bottom and below frost line.

A novel foundation for a derrick which greatly decreases the required foundation weight is illustrated in the *Engineering News* for February 2, 1911. The derrick has a capacity of 10 tons and a boom length of between 65 and 70 feet. A sketch of the derrick frame and drawings of the foundation are shown in Figs. 268 and 269. By using reinforced concrete a shallow foundation is spread sufficiently to maintain the derrick upright.

The graphical treatment just shown is easily applied to the design of this foundation.

# PART VI.—JIB CRANES.

## Five-Ton Jib Crane.

FRAME.—The load is

| | Pounds |
|---|---|
| Live Load | 10,000 |
| Block, etc | 200 |
| Total | 10,200 |

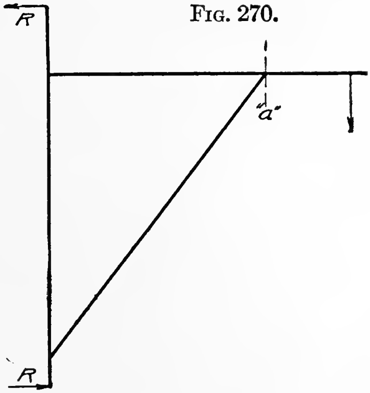



FIG. 270.

The equivalent load at the apex $a$ is

$$\frac{10{,}200\times 210}{170}=12{,}600 \text{ pounds.}$$

The horizontal reactions at the top and bottom of the mast are found to be

$$R=\frac{10{,}200\times 210}{234}=9160 \text{ pounds.}$$

Bending on the jib. On the cantilever portion we have

$$\text{Due to load } M_1 = 10{,}200\times 42 = +428{,}400 \text{ inch-pounds.}$$

$$\text{Due to rope } M_2 = \frac{10{,}200}{2\times 0.98^2}\times 6 = -\ \ 31{,}800 \text{ inch-pounds.}$$

$$M = 396{,}600 \text{ inch-pounds.}$$

Using a maximum allowable stress of 14,000 pounds per square inch and reducing this on account of the ratio of flange width to span, we find the allowable stress due to bending to be

$$p=14{,}500\times 0.90=12{,}600 \text{ pounds per square inch.}$$

Trying two 12-inch channels 20½ pounds per foot, section modulus 21.4, we find, since $M=\frac{pI}{e}$,

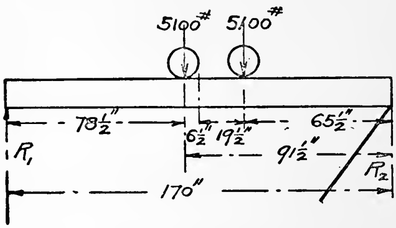
$$p=\frac{Me}{I}=\frac{396{,}600}{2\times 21.4}=9260 \text{ pounds.}$$

Area 2 channels $2\times 6.03=12.06$ square inches.

$$\text{Direct stress} = \frac{12{,}800}{12.06} \approx 1000 \text{ pounds.}$$

| | | |
|---|---|---|
| Max. tensile stress | 10,260 lbs. per □″ | On cantilever portion. |
| Max. compressive stress | 8,260 lbs. per □″ | |

FIG. 271.




Now considering the supported span between the section *a* and the mast, we find the maximum bending to occur when one wheel is 6½ inches from the center of the span.

$$R_1 = \frac{(5100 \times 65.5) + (5100 \times 91.5)}{170} = 4700 \text{ pounds.}$$

$$M = 4700 \times 78.5 = 368{,}950 \text{ inch-pounds.}$$

The bending from the rope pull is $\frac{31{,}800}{2} = \underline{15{,}900}$

$$M = 384{,}850$$

Since $M = \frac{pI}{e}$ $\qquad p = \frac{Me}{I}$ $\qquad p = \frac{384{,}850}{2 \times 21.4} = 9000.$

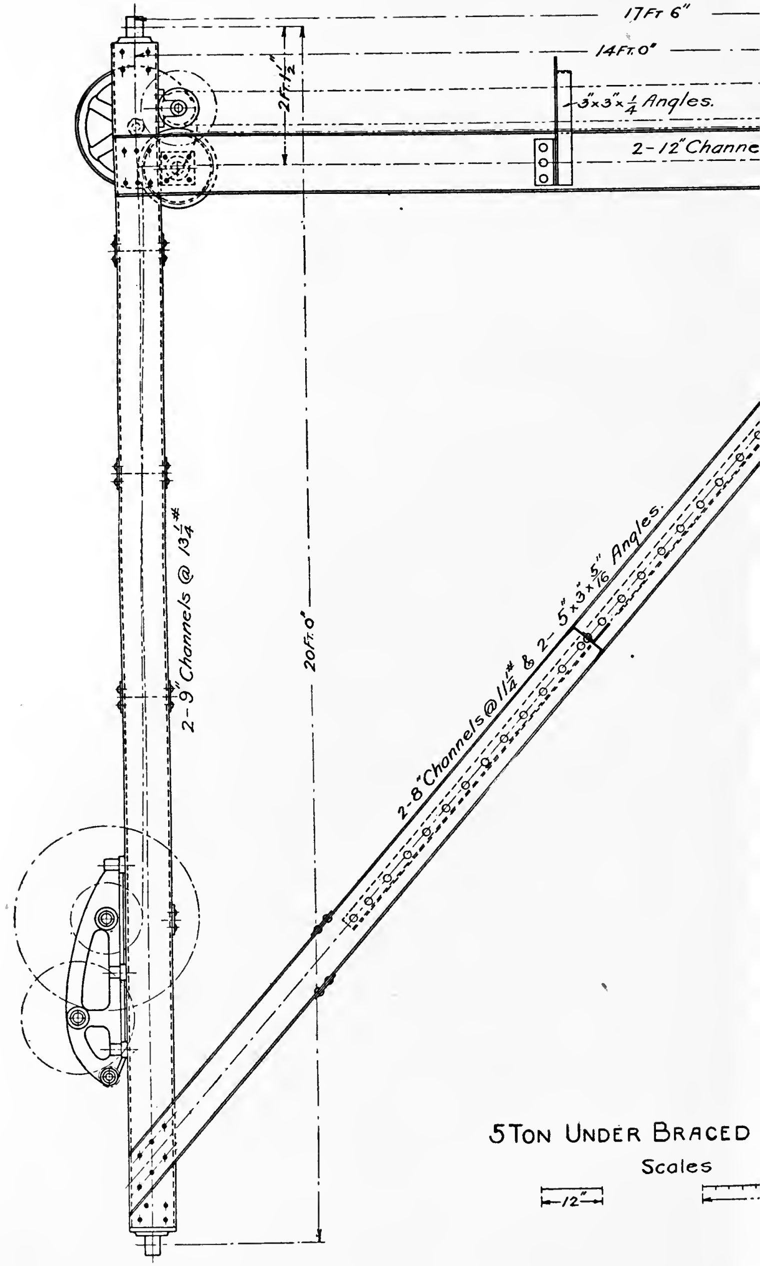
The reaction at *a* is $10{,}200 - 4700 = 5500$ pounds.

The direct stress in the jib between the section *a* and the mast is found from the diagram to be 4,800 pounds, making the unit stress $\frac{4800}{2 \times 6.03} = 380$ pounds.

This gives the following maximum stresses at this section:

Tension.......$9000 - 380 = 8620$ pounds per square inch.
Compression ..$9000 + 380 = 9380$ pounds per square inch.

Owing to the extreme length of this span relative to the flange width, the working fiber stress must be cut down very materially from the maximum allowable, *i.e.*, 14,000. The bracing at the middle of the span undoubtedly stiffens the beams very much, but nevertheless it is an indeterminate amount. Referring to p. 111, the curves indicate an allowable working stress of about 65 per cent. of the maximum for a supported beam with the load at the middle. This would suggest a maximum working stress of $14{,}000 \times 0.65 = 9100$ pounds per square inch. This agrees fairly well with the fiber stresses found above.

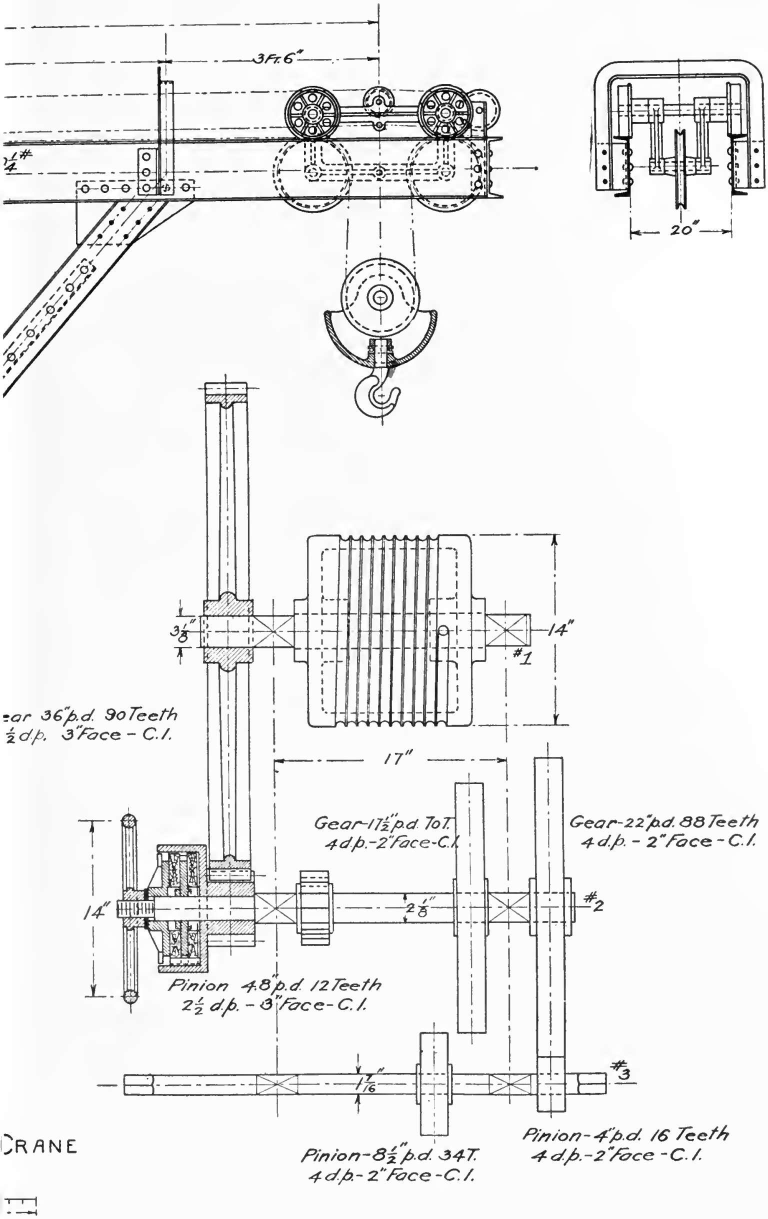



5 TON UNDER BRACED

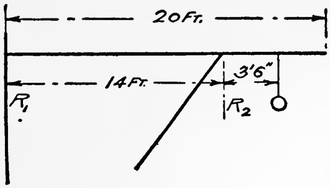
Scales

12"

IV.


3Ft.6"
20"
3 1/8"
14"
#1
17"
ear 36"p.d. 90 Teeth
1/2 d.p. 3"Face - C.I.
Gear-17 1/2"p.d. 70 T.
4 d.p.-2"Face-C.I.
Gear-22"p.d. 88 Teeth
4 d.p. - 2"Face - C.I.
14"
2 1/8"
#2
Pinion 4.8"p.d. 12 Teeth
2 1/2 d.p. - 3"Face-C.I.
1 7/16"
#3
Pinion-4"p.d. 16 Teeth
4 d.p.-2"Face - C.I.
Pinion-8 1/2"p.d. 34 T.
4 d.p.-2"Face-C.I.
RANE

Mast.—The mast will have to be considered both as a beam and as a column. The reaction at the mast due to the weight of the jib will first be found.




Weight of jib $20 \times 40.5 = 810$ pounds.

$$R_2 = \frac{810 \times 10}{14} = 580.$$

$R_1 = 810 - R_2 = 810 - 580 = 230$ lbs.

The reaction at R due to the live load

$$R_1 = \frac{10{,}200 \times 3.5}{14} = 2550 \text{ pounds.}$$

This is of the opposite sense to the reaction due to the dead load, so that $R_1 = 2550 - 230 = 2320$ pounds.

The maximum bending on the mast is that due to the maximum horizontal reaction multiplied by the lever arm (25½ inches).

$$M = 10{,}700 \times 25.5 = 272{,}850 \text{ inch-pounds.}$$

The allowable fiber stress being given at 14,000 pounds per square inch, the required section modulus is, since $M = \frac{pI}{e}$,

$$\frac{I}{e} = \frac{M}{p} = \frac{272{,}850}{2 \times 14{,}000} = 9.75$$

This corresponds to the section modulus of a 9-inch channel at 13¼ pounds, area 3.89 square inches.

The direct stress per square inch is $\frac{2320}{2 \times 3.89} = 298$ pounds $\approx 300$.

This makes the allowable stress in bending $14{,}000 - 300 = 13{,}700$ pounds.

The corrected section modulus is

$$\frac{I}{e} = \frac{M}{p} = \frac{272{,}850}{2 \times 13{,}700} \approx 10.$$

It is evident from this that two 9-inch channels at 13¼ pounds per foot are still ample.

When the load is close to the mast it acts more as a column than a beam, so that it should be investigated from this point of view. Under these circumstances the load on the column approximates 10,200 pounds. The radius of gyration of 9-inch channels about their principal axes is R=3.49. The length of the column is 18 feet (216 inches).

$$\frac{L}{R}=\frac{216}{3.49}=62$$

According to the approximate form of Ritter's formula,

$$p=\frac{p^1}{1+\frac{1}{10{,}000}\left(\frac{L}{R}\right)^2}=\frac{14{,}000}{1+\frac{3844}{10{,}000}}=10{,}100\text{ pounds.}$$

As a column, therefore, the two 9-inch channels at 13¼ pounds per foot will carry 2×3.89×10,100=78,500 pounds.

Hence they are more than ample to carry the load of 10,200 pounds.

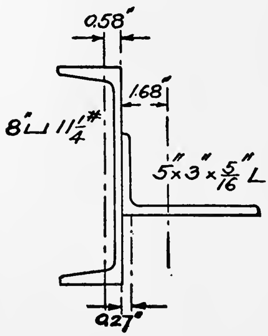
In order that the $\frac{l}{r}$ values about the two axes shall be equal

$$\frac{L}{R}=\frac{l}{r}\text{ or }l=\frac{L\times r}{R}=\frac{216\times 0.67}{3.49}=41\tfrac{1}{2}\text{ inches.}$$

Hence the 2 channels must be fastened together at intervals not exceeding 41½ inches. This is accomplished by plates 6 inches×¼ inch thick. Although this method is not as stiff as lacing, it is commonly done this way in light cranes, and when the stress is low, as in this case, it should prove ample.

STRUT.—The direct stress in the strut is 19,500 pounds. Owing to the length of this strut and the fact that the two channels or other sections used for it cannot be braced together, it will be necessary to use a built-up section if the value of $\frac{l}{r}$ is to be kept down.

FIG. 272.




Having decided to try the following section, it is necessary to determine its center of gravity and moment of inertia.

CENTER OF GRAVITY.

| Area sq. ins. | | Arm. | Moment. |
|---|---|---|---|
| 3.35 | × | 0. | 0. |
| 2.41 | × | 2.26 | 5.45 |
| Total 5.76 | | | 5.45 |

$$x=\frac{5.45}{5.76}=0.95\text{ inch.}$$

INERTIA:

$Ah^2$ (channel) . . . . . . . . . . . . . . . . . . $3.35 \times 0.95^2 = 3.01$
Inertia of channel . . . . . . . . . . . . . . . (own axis) = 1.33
Inertia of angle . . . . . . . . . . . . . . . . . (own axis) = 6.26
$Ah^2$ (angle) . . . . . . . . . . . . . . . . . . . $2.41 \times 1.31^2 = 4.13$

14.73

From these the radius of gyration is found to be

$$r = \sqrt{\frac{I}{A}} = \sqrt{\frac{14.7}{5.76}} = 1.6. \qquad \frac{l}{r} = \frac{200}{1.6} = 125.$$

This value of $\frac{l}{r}$ of 125 is amply stiff, so that unless it is desired to lighten the section it will do.

## MACHINERY.

The pull on the chain running on the drum is

$$\frac{10{,}200}{2 \times 0.98^4} = 5540 \text{ pounds.}$$

The size of the chain to carry this load is given by

$$d = \sqrt{\frac{L}{14{,}000}} = \sqrt{\frac{5540}{14{,}000}} = 0.625 \text{ inch.}$$

GEARING.—Making the first reduction 7.5 to 1, and assuming a 12-tooth pinion on shaft No. 2, we first find the twisting moment on the shaft to be

$$M_T = \frac{5540 \times 7}{7.5 \times 0.92} = 5620 \text{ inch-pounds.}$$

The pinion, being part of the brake casing, is reinforced on one side. It will be assumed as having 12 teeth and made of cast-iron. Its working fiber stress can be taken high, since it is hand driven. The formula for the circular pitch of the smallest pinion having 12 teeth that will carry the given twisting moment is,

$$p_c = \sqrt[3]{\frac{6.28\ M_t}{n.s.c\left(0.124 - \frac{0.684}{n}\right)}}$$

$$= \sqrt[3]{\frac{6.28 \times 5620}{12 \times 8000 \times 2.5 \times 0.067 \times 1.5}} = 1.20 \text{ inches.}$$

This corresponds to 2½ diametral pitch. The factor 1.5 is inserted on account of the pinion being partially shrouded.

Gear, 36 inches p. d. cast-iron, 2½ diametral pitch—3-inch face, 90 teeth.

Pinion, 4.8 inches p. d. cast-iron, 2½ diametral pitch—3-inch face, 12 teeth.

Pinion on crank shaft.—Assuming the reduction 5½ to 1, the twisting moment is found to be

$$M_t = \frac{5620}{5.5 \times 0.92} = 1100 \text{ inch-pounds.}$$

The force on two 15-inch cranks is

$$F = \frac{1,100}{2 \times 15} = 36.7 \text{ pounds on each crank.}$$

This being only for full loads is within the capacity of a man.

The pinion being assumed as having 16 teeth, the circular pitch of the smallest pinion is found to be

$$p_c = \sqrt[3]{\frac{6.28 \times 1,100}{16 \times 8000 \times 2 \times 0.081}} = 0.695 \text{ inch, say 4 diametral pitch.}$$

To increase the speed at which lighter loads can be handled an additional pinion will be placed on the crank shaft, meshing with an extra gear on the brake shaft, the sizes as follows:

Gear 22 inches p. d. cast-iron, 4 pitch, 2-inch face, 88 teeth.

Pinion 4 inches p. d. cast-iron, 4 pitch, 2-inch face, 16 teeth.

Gear 17½ inches p. d. cast-iron, 4 pitch, 2-inch face, 70 teeth.

Pinion 8½ inch p. d. cast-iron, 4 pitch, 2-inch face, 34 teeth.

Shafting. Shaft No. 1.—The bending moment on this shaft can be approximated as follows:

The force acting on the face of the pinion must be found

$$F = \frac{M_t}{R} = \frac{5540 \times 7}{.92 \times 18} = 2340 \text{ pounds.}$$

The twisting moment on the drum shaft is

$$M_t = \frac{5540 \times 7}{0.92} = 42,200 \text{ inch-pounds.}$$

From this the bending moment is found to be

$$M_B = F \times a = 2340 \times 3.25 = 7600 \text{ inch-pounds.}$$

The equivalent bending moment due to the twisting and bending that the shaft is subjected to is given by

$$M_{E.B.} = 0.35\ M_B + 0.65\sqrt{M_B^2 + M_T^2}$$
$$= (0.35 \times 7600) + 0.65\sqrt{7600^2 + 42{,}200^2}$$
$$= 2660 + 27{,}900 = 30{,}560 \text{ inch-pounds.}$$

The shaft diameter is now found from the formula

$$d = \sqrt[3]{\frac{10\ M_{E.B.}}{f}} = \sqrt[3]{\frac{10 \times 30{,}560}{10{,}000}} = 3.12 \text{ inches.}$$

Shaft No. 2.—The twisting moment is 5620 inch-pounds as previously found.

The bending moment is $M_B = 2340 \times 3.25 = 7600$ inch-pounds.

The equivalent bending is

$$M_{E.B.} = (0.35 \times M_B) + 0.65\sqrt{M_B^2 + M_T^2}$$

$$M_{E.B.} = (0.35 \times 7600) + 0.65\sqrt{7600^2 + 5620^2} = 8900 \text{ inch-pounds.}$$

From these the shaft diameter is determined to be,

$$d = \sqrt[3]{\frac{10\ M_{E.B.}}{p}} = \sqrt[3]{\frac{10 \times 8900}{9000}} = 2.15 \text{ inches, say } 2\tfrac{1}{8} \text{ inches.}$$

Ratchet Wheel.—The twisting moment on the brake shaft due to the load attempting to run down is

$$M_t = \frac{10{,}200 \times 0.98^4 \times 7}{2} \times \frac{12 \times 0.92}{90} = 4020 \text{ inch-pounds.}$$

The smallest ratchet wheel to carry this twisting moment will have a circular pitch of

$$p_c = \sqrt{\frac{16\pi\ M_t}{fsz}} = \sqrt{\frac{16 \times 3.14 \times 4020}{2 \times 3000 \times 12}} = 1.68 \text{ inches.}$$

If the wheel is made $6\tfrac{1}{2}$ inches outside diameter with 12 teeth it will have a pitch of $\frac{6.5 \times 3.14}{12} = 1.702$ inches.

Brake.—This brake is really a release or dispatch brake. The pinion runs loose on the shaft, and carries the casing of the brake with it. The cast-iron discs in the casing are prevented from rotating on the shaft by a feather. The wooden discs are fastened to the outer casing. Tightening the hand wheel on the screw clamps these discs, causing the discs secured to the shaft by the feather to drive the pinion through the casing and the discs attached to it.

The mean radius of friction discs is $\frac{8.25+3.25}{4}=2\frac{7}{8}$ inches.

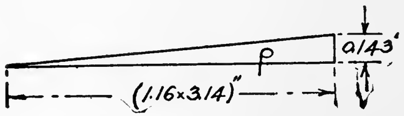
The twisting moment on the shaft in raising the load is 5620 inch-pounds as previously found.

Assuming the coefficient of friction between the cast-iron and wooden disc faces as 0.40. The numbers of faces in sliding contact being 3, the total frictional coefficient is $3\times 0.40=1.20$.

This will require a pressure between the blocks of

$$F=\frac{5620}{2.88}\times\frac{1}{1.2}=1620 \text{ pounds.}$$

The hand wheel on the brake will turn on a $1\frac{1}{4}$ inch screw, having 7 threads per inch. The pitch diameter of the screw thread is $\frac{1.25+1.065}{2}=1.157$ inches.




$$\rho=\tan^{-1}\frac{1}{1.16\times 3.14\times 7}=\text{ton}^{-1}\frac{1}{25.5}=\tan^{-1}0.0392=2^{\circ}\ 20'.$$

The coefficient of friction between oiled surfaces will be assumed at 0.06.

The general formula for screw or worm motion is given on p. 32.

$$Fa=Kr \text{ tang } (\rho+\theta)+\mu K r_1$$

Here $F$ = pull on hand wheel in pounds.

$a$ = radius of hand wheel in inches.

$\rho$ = angle of thread of screw.

$\theta$ = angle whose tangent is $0.06=3^{\circ}\ 30'$.

$r$ = pitch radius of screw, inches.

$r_1$ = mean radius of washer under hand wheel hub.

$$F\times 7=1650\times\frac{1.16}{2}\text{tang } (2^{\circ}20'+3^{\circ}30')+(0.06\times 1650\times 1.06)$$

$$F=\frac{(1650\times 0.58\times 0.10)+(0.06\times 1650\propto 1.06)}{7}=28.7 \text{ pounds.}$$

This being easily within the capacity of 1 man, the brake will be satisfactory, provided the unit pressure on the friction discs is not too great.

Outside diameter of discs 8 inches. Area = 50.3 square inches.
Inside diameter of discs 3 inches. Area = 7.1 square inches.

Net area 43.2 square inches.

Pressure per square inch $\frac{1650}{43.2}=38.2$ pounds, which should prove satisfactory.

## Six-Thousand-Pound Top-Braced Jib Crane.

This crane frame is the same discussed on p. 131. The stresses in the jib and the mast are increased by the manner of locating the hoisting gear and tackle, and the calculations for these will therefore have to be modified.

The stress diagrams on the crane drawing are made to include approximately the weight of the crane frame. The weight of the frame has been assumed at 3000 pounds and the center of gravity taken at ¼ the crane radius. The equivalent load at a radius of 25 feet is $\frac{3000}{4}=750$ pounds. The weight of the block will be assumed at 75 pounds and the trolley at 175 pounds.

The equivalent load at $a$ is $\frac{25(6000+750+75+175)}{23}=7600$ pounds.

The horizontal reaction at the top of the crane is $\frac{25\times7000}{25}=$ 7000 pounds.

Now checking the jib previously chosen, the direct stress from this diagram is 12,500 pounds.

Bending due to trolley $M=\frac{WL}{4}=\frac{6250\times11.5\times12}{4}=215,625$ inch-pounds.

Bending due to rope pull $M=3160\times15.5=48,980$ inch-pounds.

Total bending $=215,625+48,980=264,605$ inch-pounds.

Trying the two 12-inch channels 20.5 pounds per foot previously used we have, since the section modulus $\frac{I}{e}$ is $2\times21.4=42.8$ and $p=\frac{Me}{I}$, $p=\frac{264,605}{42.8}=6180$ pounds per square inch.

The area of the 2 channels being $2\times6.03=12.06$ square inches, the direct compression is $\frac{12,500}{12.06}=1040$ pounds per square inch.

The total maximum compression is 6,180+1040=7,220 pounds per square inch. The allowable fiber stress as previously determined (see p. 134) was 7350 pounds per square inch, so that the fiber stress of 7220 pounds per square inch is permissible, and the section will do.

Now investigating the mast, the bending due to the rope pull is this rope pull times the distance between the rope center and the mast center, or

$$M=\frac{3160}{.98}\times 6.5=21{,}000 \text{ inch-pounds.}$$

The section modulus, $\frac{I}{e}$, of two 9-inch channels 13.25 pounds per foot is $\frac{I}{e}=2\times 10.5=21.0$.

Since $p=\frac{Me}{I}$, $p=\frac{21{,}000}{21}=1000$ pounds.

$$\text{Direct stress}=\frac{34{,}600}{2\times 3.89}=4450 \text{ pounds per square inch.}$$

The total maximum compression is 4450+1000=5450 pounds per square inch. As the allowable unit stress previously found for this column (see p. 135) was 5,250 pounds per square inch, this is sufficiently close.

Step-Bearing.—Vertical pressure = Live load + total dead load = 6000+250+3000 = 9250 pounds.

The horizontal reaction previously found was 7000 pounds.

Designing the pin so that it shall not only be strong enough, but that its pressure per square inch shall not exceed 1800 pounds on either its vertical or horizontal projection, we have,

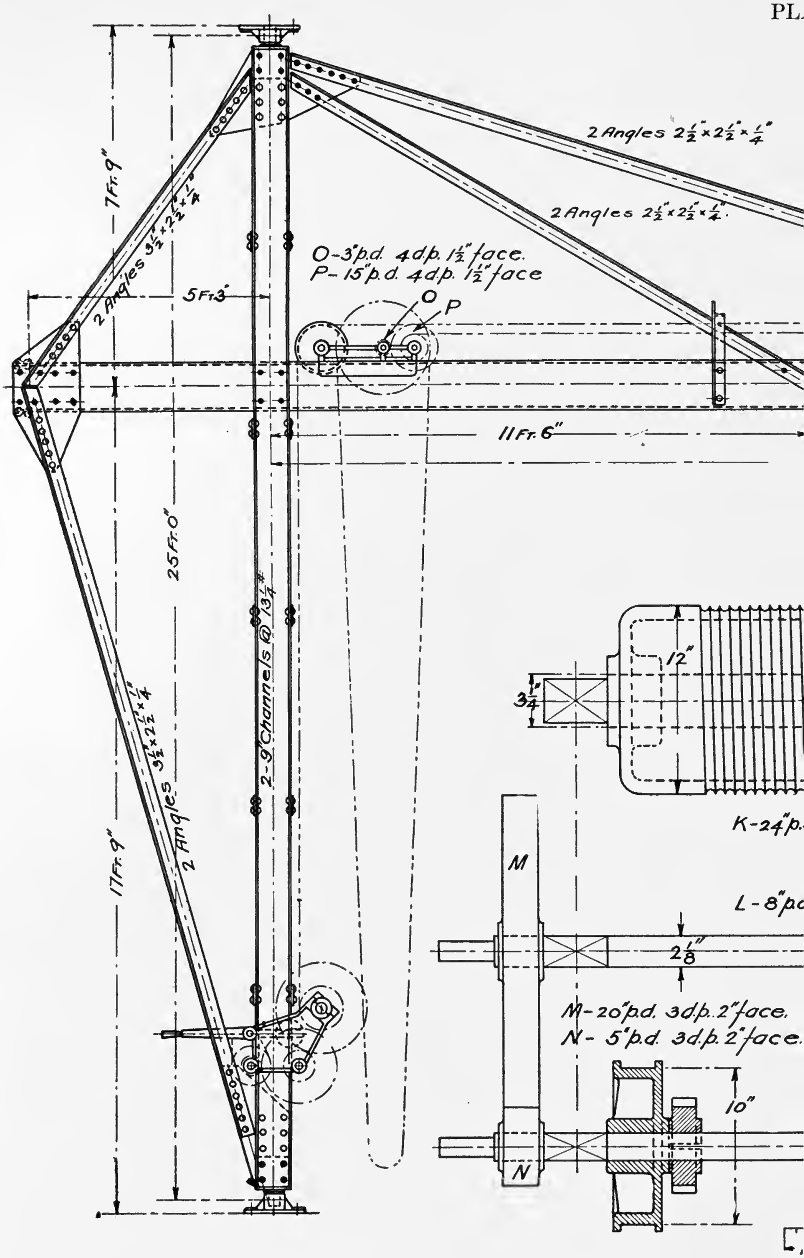
$$\text{Horizontal area}=\frac{9250}{1800}=5.15 \text{ square inches.}$$

This corresponds to a diameter of 2⅝ inches approximately.

The vertical projection is $d\times l=\frac{7000}{1800}=3.89$ square inches,

hence $$l\geqq\frac{3.89}{2.62}=1.48 \text{ inches.}$$

Now the diameter will have to be examined for strength.


2 Angles 2½"×2½"×¼"
2 Angles 2½"×2½"×¼"
7 Ft. 9"
2 Angles 3½"×2½"×¼"
O-3"p.d. 4d.p. 1½" face.
P-15"p.d. 4d.p. 1½" face
O
P
5 Ft. 3"
11 Ft. 6"
25 Ft. 0"
2-9" Channels @ 13¼#
12"
3¼"
K-24"p.
M
L-8"p.
2⅛"
M-20"p.d. 3d.p. 2" face.
N-5"p.d. 3d.p. 2" face.
2 Angles 3½"×2½"×¼"
17 Ft. 9"
10"
N

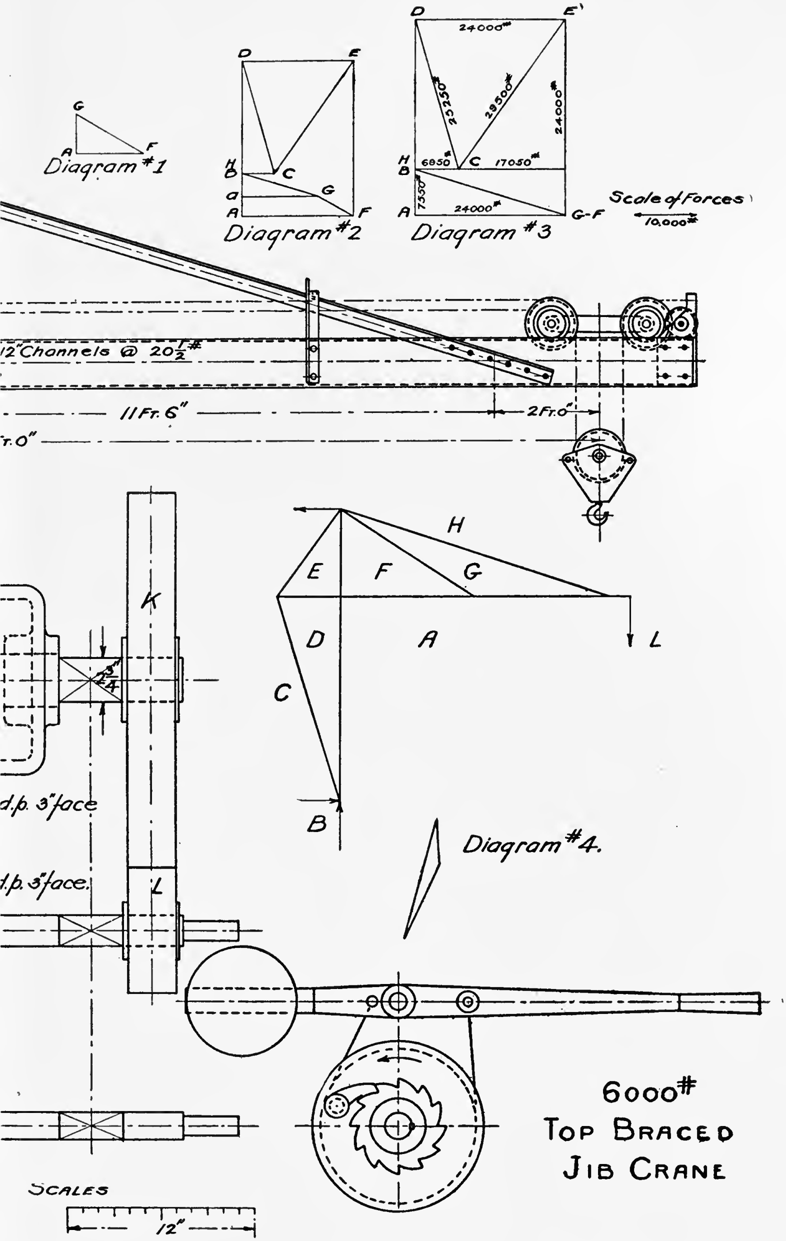


Diagram #1
Diagram #2
Diagram #3
24000#
25250#
29500#
24000#
6850
17050#
7550#
24000#
Scale of Forces
10,000#
12" Channels @ 20 1/2#
11 Ft. 6"
2 Ft. 0"
Diagram #4.
d.p. 3" face
d.p. 3" face.
Scales
12"
6000#
Top Braced
Jib Crane

Take $l$ equal to 4 inches, the bending moment on the pin will be $$M = \text{Horizontal reaction} \times \frac{l}{2}.$$

$$M = 7000 \times \frac{4}{2} = 14{,}000 \text{ inch-pounds}.$$

$$d = \sqrt[3]{\frac{10\,M}{p}}.$$

If the desired total stress, combined flexure and direct compression is to be limited to 9000 pounds per square inch, the value of $p$ in the above formula is $9000 - 1800 = 7200$ pounds per square inch, hence

$$d = \sqrt[3]{\frac{10 \times 14{,}000}{7200}} = 2.69, \text{ say } 2\tfrac{3}{4} \text{ inches}.$$

HOISTING MECHANISM.—Efficiencies are:

| | |
|---|---|
| Floating block | .97 per cent. |
| Fixed blocks | .98 per cent. |
| Drum | .98 per cent. |
| Gearing, each reduction | .92 per cent. |

The force at the drum is $\frac{6000+150}{2 \times .97 \times .98^2} = 3300$ pounds.

The twisting moment on the drum is $M_T = 3300 \times 6 = 19{,}800$ inch-pounds.

The efficiency from the drum to crank shaft is $\Sigma = .98 \times 92^2 =$ 83 per cent.

Assuming 2 cranks each with 2 men exerting 35 pounds pressure on 15-inch cranks, the twisting moment upon the cranks will be

$$M_T = 2 \times 2 \times 35 \times 15 = 2100 \text{ inch-pounds}.$$

The required reduction between drum and crank shaft will be

$$\text{Reduction} = \frac{\text{Twisting moment on drum shaft}}{\text{Twisting moment on crank shaft} \times \text{efficiency}}$$

$$= \frac{19{,}800}{2100 \times 0.83} = 11.35, \text{ say 12 to 1}.$$

This can be had by 1 reduction of 4 to 1 and 1 reduction of 3 to 1.

HOISTING ROPE.—Trying a ½ inch extra flexible plow steel rope, made up of 8 strands of 19 wires each.

$$\delta = \frac{0.50}{18} = 0.028 \text{ inch}.$$

Trying this on a 12-inch diameter drum, the stress is found as follows:

The pull on the rope running on the drum is 3300 pounds. The estimated total fiber stress in the rope then is,

$$p_T=\left(\frac{S}{i\times\frac{\pi d^2}{4}}\right)+\left(\frac{3}{8}\quad\frac{E\times\delta}{D}\right)$$

$$p_T=\left(\frac{3300}{152\times\frac{3.14\times0.028^2}{4}}\right)+\left(\frac{3}{8}\times\frac{30,000,000\times0.028}{12}\right)=61,500\text{ lbs.}$$

The factor of safety is, $\frac{220,000}{61,500}=3.58.$

Twisting moment on the crank shaft

$$M_T=35\times4\times15=2100\text{ inch-pounds.}$$

The minimum 15-tooth pinion that can be used on this shaft will be,

$$p=\sqrt[3]{\frac{6.28\ M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times2100}{15\times6000\times2\times0.079}}=0.976\text{ in.}$$

Use 3 diametral pitch, making the pinion 5 inches in diameter and the gear 20 inches in diameter, both faces 2 inches.

Second Shaft.—Twisting moment $=2100\times4\times.92=7720$ inch-pounds.

The minimum pinion having 16 teeth will be,

$$p=\sqrt[3]{\frac{6.28\ M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times7720}{16\times6000\times2\times.081}}=1.47''$$

This corresponds to 2 diametral pitch.

Pinion 8-inch pitch diameter, 16 teeth, 3-inch face.

Gear 24 inches pitch diameter, 48 teeth, 3-inch face.

Drum Shaft.—Torsional moment $=19,800$ inch-pounds.

Bending moment $=3300\times2=6600$ inch-pounds.

Equivalent bending moment

$$=(0.35\times6600)+0.65\sqrt{6600^2+19800^2}$$
$$=2310+13,690=15,900\text{ inch-pounds.}$$

From this the diameter of the shaft is found to be

$$d=\sqrt[3]{\frac{10\ \mathrm{M_{E.B.}}}{p}}=\sqrt[3]{\frac{10\times 15{,}900}{9000}}=2.62 \text{ inches, say } 2\tfrac{3}{4} \text{ inches.}$$

BRAKE.—The strap type of brake will be used.

The twisting moment at the crank shaft due to the load running down is

$$\mathrm{M_T}=\frac{6150\times .97\times .98^2\times 6}{2}\times\frac{.83}{12}=1190 \text{ inch-pounds.}$$

See p. 167, referring to strap brakes.

The diameter of the brake wheel will be assumed as 10 inches, the strap will be considered as making contact with 70 per cent. of the circumference of the brake wheel The coefficient of friction between the strap and the wheel will be taken as 0.18; this makes $k=2.21$. The following additional assumptions will be made:

$a=4.5$ inches; $b=1.5$ inches; $l=10$ inches.

$$\mathrm{P}=\frac{\mathrm{F}}{l}\left(\frac{a-bk}{k-1}\right),\quad \mathrm{F}=\frac{1190}{5}=238 \text{ pounds.}$$

$$\mathrm{P}=\frac{238}{10}\left[\frac{4.5-(1.5\times 2.21)}{2.21-1.0}\right]=\frac{238}{10}\left(\frac{1.18}{1.21}\right)=23.2 \text{ pounds, say}$$

25 pounds.

If the weight is made 3 inches thick the area of its circular face will be, $\text{Area}=\dfrac{25}{3\times 0.26}=32$ square inches.

The weight of a cubic inch of cast-iron being taken at 0.26 pound, the diameter corresponding to this area of 32 square inches is 6.39 inches, say 6½ inches.

The strap must be made thin so it will wrap properly around the 10-inch diameter cylinder, say $\frac{1}{16}$ inch.

The tension in the two ends of the strap will be,

$$\mathrm{T}=\frac{\mathrm{F}\times k}{k-1}=\frac{238\times 2.21}{1.21}=435 \text{ pounds.}$$

$$t=\frac{\mathrm{F}}{k-1}=\frac{238}{1.21}=197 \text{ pounds.}$$

Knowing T, the width of the strap is found to be,

$$b=\frac{435}{4000\times t}=\frac{435}{4000\times\frac{1}{16}}=1.74 \text{ inches} \backsim 2 \text{ inches.}$$

Diagram No. 4, Plate V, is the force polygon of the forces acting on the brake lever, and from it the pressure on the fulcrum pin is found to be 660 pounds.

The bolt length being assumed at 2½ inches, its diameter to resist the bending is found to be

$$d=\sqrt[3]{\frac{10\times M_B}{p}}=\sqrt[3]{\frac{10\times 660\times 2.5}{8\times 9000}}=0.613 \text{ inch} \curvearrowright \tfrac{5}{8} \text{ inch.}$$

The ratchet wheel will now have to be designed (see p. 28). Assuming it as having 12 teeth and made of cast-iron, we have the minimum pitch to be,

$$p_c=\sqrt{\frac{16\pi M_T}{fsn}}=\sqrt{\frac{16\times 3.14\times 1190}{1.5\times 2500\times 12}}=1.15 \text{ inches,}$$

say 2 per inch of diameter.

Trolley Movement.—It will be first necessary to find the size of the trolley wheel axles. The axles will be assumed as 16 inches long and the sheave load will be taken as distributed over 4 inches at the middle of the axle. The load on each axle will be approximately

$$R=\frac{L}{2}\times 1.4=\frac{6000}{2}\times 1.4=4200 \text{ pounds.}$$

The reaction $=2100$ pounds.

$M=(2100\times 8)-(2100\times 1)=14{,}700$ inch-pounds.

$$d=\sqrt[3]{\frac{10\,M}{p}}=\sqrt[3]{\frac{10\times 14{,}700}{9000}}=2.54 \text{ inches} \curvearrowright 2\tfrac{1}{2} \text{ inches.}$$

For the discussion of the force required to move the trolley see p. 83.

$$P_1=\frac{6150}{2}=3075 \text{ lbs.} \qquad P_3=\frac{3075}{0.97\times 0.96}=3300 \text{ lbs.}$$

$$P_2=\frac{3075}{0.97}=3170 \text{ lbs.} \qquad P_4=\frac{3075}{0.97\times 0.96\times 0.97}=3400 \text{ lbs.}$$

$$F=\left[(3400-3075)+1.5\left(\frac{(0.003\times 6150)+(0.10\times 1.25\times 6150)}{4}\right)\right]$$

$F=325+(1.5\times 198)=622$ pounds. S will be neglected.

If the trolley is to be operated by the pull of 40 pounds on a hand chain and the efficiency of this mechanism is taken at 80 per cent., we have the following reduction required:

$$\text{Reduction}=\frac{622\times 3}{.80\times 12\times 40}=4.9 \text{ to } 1, \text{ say } 5 \text{ to } 1.$$

This assumes a 6-inch diameter sprocket wheel that the chain fastened to the trolley runs over, and a 24-inch diameter sprocket wheel that the hand chain runs on, both chains being ¼ inch chains. The twisting moment on the 24-inch sprocket wheel shaft is $M=12\times 40\times 0.97=466$ inch-pounds.

The smallest pinion having 12 teeth that can be used for this twisting moment is

$$p_c=\sqrt[3]{\frac{6.28\times M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times 466}{12\times 6000\times 2\times 0.067}}=0.672''$$

This corresponds to a 4 diametral pitch; we will make the pinion 3-inch pitch diameter with 12 teeth, the gear will be 15-inch pitch diameter with 60 teeth.

# PART VII.—UNDER-BRACED JIB CRANE.

## Five-Ton Under-Braced Jib Crane.

This crane is to carry a load of 5 tons, 10,000 pounds, at a radius of 20 feet. The general clearance dimensions are given in the drawing. The frame is the one designed under "Frames and Girders," see p. 128.

The load is to be lifted 20 feet per minute. This will require a motor of the following horse-power (see p. 217):

$$\text{Horse-power} = \frac{5 \times 20}{10} = 10.$$

A No. 4 type K motor developing 10-horse-power with a rise in temperature of 40 degrees Centigrade in 15 minutes at 750 revolutions per minute will be used.

Chain.—Assuming the load carried by 2 chains,

$$\text{Load on each chain} = \frac{10,000}{2 \times .97 \times .98^3} = 5480 \text{ pounds.}$$

$$d = \sqrt{\frac{5480}{14,000}} = 0.625 \text{ inch} - \tfrac{5}{8} \text{ inch chain.}$$

Drum.—The drum diameter is $D \geqq 25d \geqq 25 \times 0.625 \geqq 16$ inches.

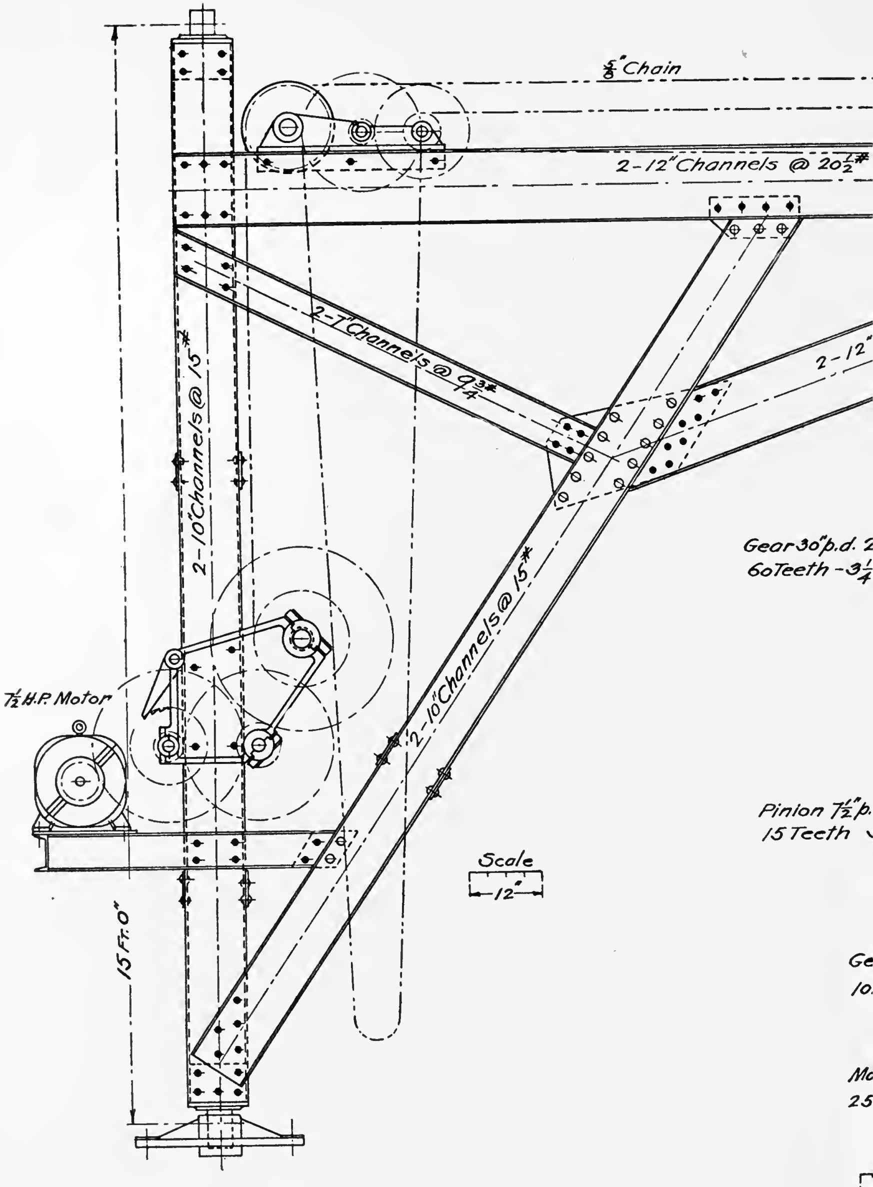
To raise the load 20 feet per minute when carried on 2 chains, the chain must be coiled on the drum 40 feet per minute, when

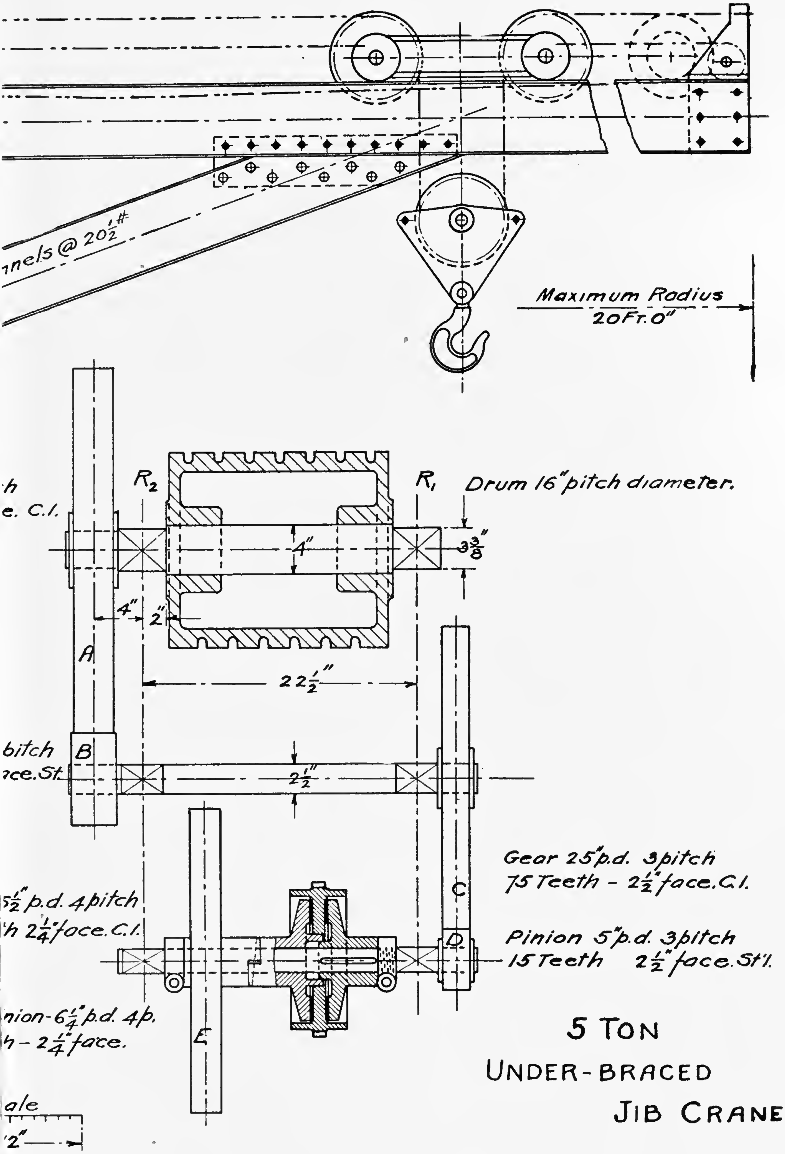
the drum speed is $\dfrac{40}{\frac{16}{12} \times 3.14} = 9.55$ revolutions per minute.

The reduction from the motor to the drum is $\dfrac{750}{9.55} = 78.5:1$.

This can be obtained by 2 reductions of 4 : 1 and 1 reduction of 5 : 1. This gives a total reduction of 80 to 1. The twisting moment on the drum shaft is, $M_t = 5480 \times 8 = 43,840$ inch-pounds.

Assuming a steel pinion the gear will have the weaker teeth,


5/8" Chain
2-12" Channels @ 20½#
2-7" Channels @ 9¾#
2-10" Channels @ 15#
2-10" Channels @ 15#
7½ H.P. Motor
Scale
12"
15 Ft. 0"




5 TON
UNDER-BRACED
JIB CRANE

being cast iron. The smallest gear having 60 teeth that will carry this twisting moment is found by the formula

$$p_c=\sqrt[3]{\frac{6.28 \quad M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times 43{,}840}{60\times 7000\times 2\left(0.124-\frac{0.684}{60}\right)}}$$

$p_c=1.43$ inches ⌒ No. 2 diametral pitch.

Pitch diameter of gear, 30 inches; No. teeth, 60; diametral pitch, 2; face, 3¼ inches. Cast-iron.

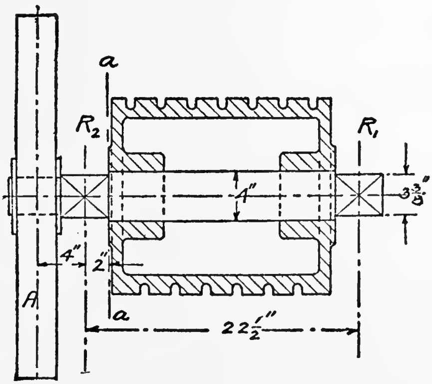
Pitch diameter of pinion, 6 inches; No. teeth, 12; diametral pitch, 2; face, 3¼ inches. Steel.

Gears.—C and D. Reduction 5 : 1.

Twisting moment on shaft carrying gear C.

$$M_t=\frac{43{,}840}{4\times .92}=11{,}900''\text{-lbs.}$$

Fig. 273.




The assumed velocity of gears at the pitch circle is 250 feet per minute, the allowable fiber stress for cast iron then is

$$s=8000\left(\frac{600}{600+v}\right)=5650 \text{ pounds.}$$

The smallest gear having 75 teeth is

$$p_c=\sqrt[3]{\frac{6.28\times M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times 11{,}900}{75\times 5650\times 2\times 0.115}}=0.92''$$

Gear C—25-inch pitch diameter—75 teeth, 3 pitch, 2½-inch face. Cast-iron.

Pinion D—5-inch pitch diameter—15 teeth, 3 pitch, 2½-inch face. Rolled steel.

Motor Pinion.—The velocity at the pitch circle will be assumed as 1000 feet per minute. The allowable working fiber stress is

$$s=8000\left(\frac{600}{600+v}\right)=8000\times\frac{600}{1600}=3000 \text{ pounds.}$$

The twisting moment on the gear E shaft is

$$M_t = \frac{43{,}840}{4 \times 5 \times .92^2} = 2590 \text{ inch-pounds.}$$

The pitch of the smallest 102-tooth gear is

$$p_c = \sqrt[3]{\frac{6.28 \times 2590}{102 \times 3000 \times 2 \times .117}} = .61 \text{ inch, say 4 pitch.}$$

Gear E—25½-inch pitch diameter—102 teeth—2¼-inch face—4 pitch. Cast-iron.

Pinion F—6¼-inch pitch diameter—25 teeth—2¼-inch face—4 pitch. Rolled steel.

The bending moment at the section *a–a* due to the force acting on the gear can be determined as follows:

The force on the gear face equals the twisting moment on the shaft divided by the pitch radius of the gear, or

$$F = \frac{M_t}{R} = \frac{43{,}840}{15} = 2920 \text{ pounds.}$$

The reaction at $R_1$ due to this force is

$$R_1 = \frac{2920 \times 4}{22.5} = 520 \text{ pounds.}$$

The bending at *a–a* is $R_1\,(22.5-2) = 10{,}660$ inch-pounds.

The bending due to the rope pull when the rope is at the end $R_2$ is found as follows, $R_2 = \frac{5480 \times (22.5-4)}{22.5} = 4500$ pounds.

The bending due to this force at the section *a–a* is

$$M = 4500 \times 2 = 9000 \text{ inch-pounds.}$$

These bending moments can now be combined as previously explained on p. 40. The combined moment is 12,800 inch-pounds.

The twisting moment previously found on this section is 43,840 inch-pounds. Combining these into an equivalent bending moment gives

$$M_{E.B.} = 0.35\,M_B + 0.65\sqrt{M_B^2 + M_T^2}$$

$$= (0.35 \times 12{,}800) + 0.65\sqrt{12{,}800^2 + 43{,}840^2} = 33{,}880 \text{ inch-pounds.}$$

This gives $d = \sqrt[3]{\frac{10\,M_{E.B.}}{f_B}} = \sqrt[3]{\frac{10 \times 33{,}880}{9000}} = 3.35 \text{ inches} - 3\tfrac{3}{8} \text{ inches.}$

Second Shaft.—Bending moment $=2{,}920\times4=11{,}680$ inch-pounds.

Twisting moment $=2{,}920\times\frac{7.5}{2}=10{,}950$ inch-pounds.

The equivalent bending moment is

$$M_{E.B.}=0.35\ M_B+0.65\sqrt{M_B^2+M_T^2}$$

$$=(0.35\times11{,}680)+0.65\sqrt{11{,}680^2+10{,}950^2}=14{,}500 \text{ inch-pounds.}$$

From this the shaft diameter is found to be

$$d=\sqrt[3]{\frac{10\ M_{E.B.}}{f}}=\sqrt[3]{\frac{10\times14{,}500}{9000}}=2.53 \text{ inches.}$$

Third Shaft.—This shaft carries the brake.

The twisting moment previously found is 2590 inch-pounds.

The force on the pinion face $=\frac{M_t}{R}=\frac{2590}{3.13}=830$ pounds.

The bending moment is $830\times3.25=2700$ inch-pounds.

The equivalent bending is

$$M_{E.B.}=(0.35\times2700)+0.65\sqrt{2700^2+2590^2}=3330 \text{ inch-pounds.}$$

This gives the diameter

$$d=\sqrt[3]{\frac{10\ M_{E.B.}}{f_B}}=\sqrt[3]{\frac{10\times3330}{9000}}=1.55 \text{ inches.}$$

Since the brake is placed on this shaft it will be made 1¾ inches in the journals and 2 inches in the body of the shaft.

# PART VIII.—INVERTED POST CRANE.

## Overhead Travelling Inverted Pillar Crane.

Load 4000 pounds to 5000 pounds. Estimated crane weight 5000 pounds. Gauge 6 feet 6 inches.

Wheel base 7 feet. Lifting speed 25 feet per minute.

Travel 150 feet per minute. Rotation by hand.

Inverted post. Bending on main section.

$$M = 5500 \times 84 = 462{,}000 \text{ inch-pounds.}$$

It is necessary to design the post hollow to allow the electrical conductors to pass through it. Assuming a 2-inch hole for this purpose, the easiest way to calculate the post sections is to assume several outside diameters and calculate the bending moments these sections will withstand. Working fiber stress 10,000 pounds per square inch.

Inertia of hollow cylinder $I = \dfrac{\pi(D^4 - d^4)}{64}$. $\dfrac{I}{e} \backsim \dfrac{D^4 - d^4}{10D}$

$$M = \frac{pI}{e} = \frac{p(D^4 - d^4)}{10D}$$

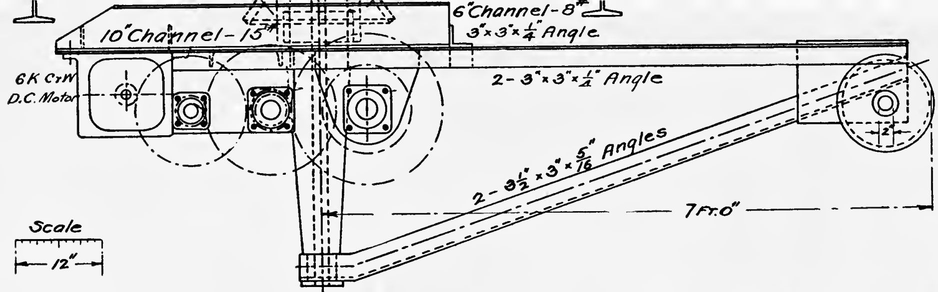
| Outside diameter *Inches* | Inside diameter *Inches* | $M = pI/e$ *Inch-pounds* |
|---|---|---|
| 8 | 2 | 510,000 |
| 7 | 2 | 340,710 |
| 6 | 2 | 213,330 |
| 5 | 2 | 121,800 |
| 4 | 2 | 60,000 |

As the bending moment varies uniformly along the post, the sections are readily fitted in their proper places.

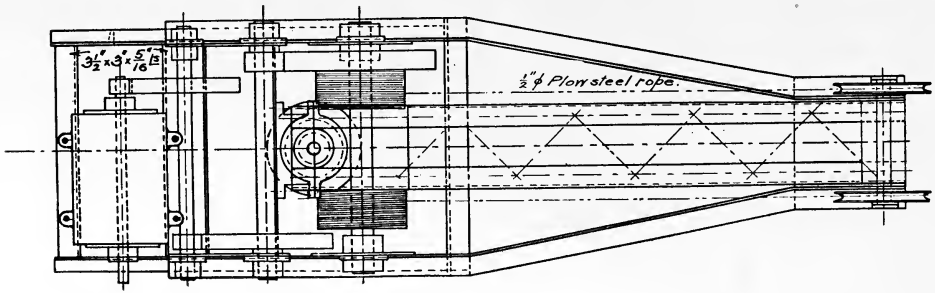
Hoisting Rope.—Trying extra flexible plow steel rope on 12-inch sheave, we have,

$$f_t = \left( \frac{2500}{152 \times \frac{3.14}{4} \times 0.027^2)} \right) + \left( \frac{3}{8} \times \frac{30{,}000{,}000 \times 0.027)}{12} \right)$$

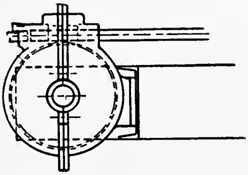
$$= 28{,}800 + 25{,}300 = 54{,}100 \text{ pounds, factor} = \frac{220{,}000}{54{,}100} = 4.08$$

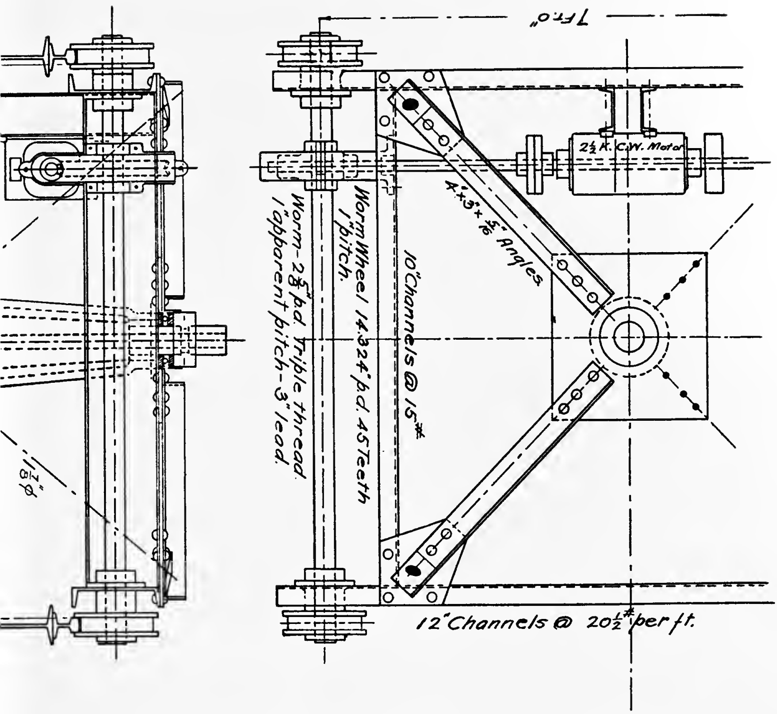



2¼ TON INVERTED POST CRANE.
7 FT. RADIUS.



VII.




7 Ft. 0"
2½ K. C.W. Motor
4"×3"×5/16" Angles
10" Channels @ 15#
Worm Wheel 14.324" p.d. 45 Teeth
1" pitch.
Worm-2⅝" p.d. Triple thread.
1" apparent pitch-3" lead.
1 7/8" φ
12" Channels @ 20½# per ft.

STRUT.—Under jib. Stress from triangle of forces acting at extremity of jib, 20,500 pounds. Trying 2 angles $3\frac{1}{2}\times 3\times\frac{5}{16}$ inches.

Radius of gyration, axis parallel to 3-inch leg, 1.10.

$$\frac{l}{r}=\frac{84}{1.1}=76.5$$

$$p^1=\frac{p}{1+\frac{1}{10,000}\left(\frac{l}{r}\right)^2}=\frac{10,500}{1.59}=6600 \text{ pounds.}$$

Net area required 1 angle $\frac{20,500}{2\times 6600}=1.55$ square inches.

Area of the rivet hole $\frac{3}{4}\times\frac{5}{16}\approx 0.25$ square inch.

Gross section $1.55+0.25=1.80$ square inches.

The area of the $3\frac{1}{2}\times 3\times\frac{5}{16}$ inch angle is 1.94 square inches which is ample. For the tension members $3\times 3\times\frac{1}{4}$ inch angles will be used.

PIN THROUGH SHEAVE.—The resulting diagonal pull due to combining the horizontal and vertical pulls on the rope running over the sheave is $1.41\times 2500=3500$ pounds.

$$M=p\frac{I}{e},\quad M=Pl=3500\times d$$

$$\frac{I}{e}\approx\frac{d^3}{10}\therefore M\approx\frac{pd^3}{10}=3500d\therefore d=\sqrt{\frac{10\times 3500}{9000}}\approx 2 \text{ inches.}$$

ROLLER BEARINGS.—These bearings take the horizontal reactions.

$$P_o=cl\delta \text{ taking } \delta=\tfrac{3}{4} \text{ inch, and } c=2000$$

Assuming 15 rollers in the bearing $P_o$, the pressure per roller is found to be $\frac{P_o}{P}=\frac{5}{\text{No. rollers}}\therefore P_o=\frac{5P}{\text{No. rollers.}}$

$P$ = total pressure on the bearing, pounds.

$c$ = constant depending upon the character of the rollers and their bushings.

$l$ = length of rollers in inches.

$\delta$ = diameter of roller in inches.

$$P_o=\frac{5\times 21,000}{15}=7000\qquad l=\frac{P_o}{c\delta}=\frac{7000}{2000\times .75}=5 \text{ inches.}$$

BALL-BEARING CARRYING POST:

| | Pounds. |
|---|---|
| Dead load, estimated | 3000 |
| Live load | 5000 |
| Total | 8000 |

Load per ball, 8000/15 = 530 pounds.

$$P = 4000\delta^2, \text{ from which } \delta = \sqrt{\frac{P}{4000}} = \sqrt{\frac{530}{4000}} = 0.36', \text{ say } \tfrac{1}{2}''$$

HOISTING MOTOR:

$$\text{H. P.} = \frac{\text{Load in Tons} \times \text{Hoisting Speed}}{10} = \frac{2.5 \times 25}{10} = 6.25 \text{ H. P.}$$

A No. 6K C. W. motor (series, D. C.) will carry 6-horse-power for about 25 minutes, rising 40 degrees Centigrade. Speed at 6-horse-power, and 220 volts, 650 revolutions per minute, torque 47 pounds at 1 foot radius. The calculation for the gearing, shafting, etc., does not differ materially from that of the motor-driven hoisting machinery previously given, to which reference can be made. No mechanical or retaining brake will be used on this crane's hoisting machinery. A solenoid brake will be placed on the motor shaft, and the brake arranged so that by operating a lever the pressure on the shoes can be relieved to permit the load to run down; the brake will be again applied automatically the moment the pull on the lever ceases.

TRAVEL.—The power required to move the crane measured at the wheels is

$$\text{H. P.} = \left[\frac{(W_T + L)\ (f + \mu r) \times S}{R \times 33{,}000}\right]$$

$$= \left[\frac{(5000 + 5200)(0.003 + (0.08 \times 1.5)}{6 \times 33{,}000}\right] \times 150 = \frac{10{,}200 \times 0.123}{1320} = 0.95$$

The tendency to twist upon the track, combined with the efficiency of the driving worm and wheel, can be assumed as giving a combined efficiency of about 33 per cent., hence the horse-power at the motor is,

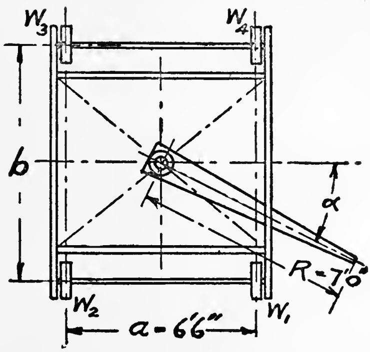
$$\text{H. P.} = \frac{0.95}{0.33} = 2.86 \text{ H. P.}$$

$$\text{R. P. M. of bridge wheels } \frac{150}{1 \times 3.14} \backsim 48.$$

A C. W. No. 2½K D. C. series motor will be chosen. Speed 700 revolutions per minute at 220 volts. Reduction motor to wheels $\frac{700}{48}=14.6$ to 1.

Force on front drivers $=\frac{(W_T+L)\ (f+\mu r)}{R\times e}=\frac{10,200\times 0.123}{6\times .75}=280$ lbs.

Reduction approximately 15 to 1. Trying 45 teeth on gear and triple thread on worm.




The wheel loads upon $W_1$, $W_2$ and $W_4$ will be found when the arm is parallel to the side $a$ of the frame, parallel to the side $b$ and when it is diagonally over the wheel $W_1$.

Let $W_T$ = weight of trolley, and L = live load.

Taking moments about rail under wheels $W_2$ and $W_3$ we have

$$W_1=\frac{W_1+W_4}{2}=\left[\left(W_T\times\frac{a}{2}\right)+L\left(\frac{a}{2}+R\right)\right]\div 2a$$

$$W_1=[\,(5000\times 3.25)+5200(3.25+7.0)\,]\div(2\times 6.5)$$

$$W_1=5350 \text{ pounds.}$$

Taking moments about wheels $W_3$ and $W_4$ when arm is parallel to side $b$,

$$W_1=\frac{W_1+W_2}{2}=\left[\left(W_T\times\frac{b}{2}\right)+L\left(\frac{b}{2}+R\right)\right]\div 2b$$

$$W_1=[\,(5000\times 3.5)+5200\times(3.5+7)\,]\div(2\times 7)=5150 \text{ lbs.}$$

Arm over wheel $W_1$, taking moments about center of post,

$$W_1=\frac{L\times R}{\sqrt{\left(\frac{a}{2}\right)^2+\left(\frac{b}{2}\right)^2}}=\frac{7000\times 7}{4.78}=10,200 \text{ pounds.}$$

From this the maximum wheel load is seen to be $W_1=10,200$ pounds, and from this equalling the combined weight of the crane and live load it is seen that the force for travelling may be exerted upon any single wheel.

For design of worm and wheel see p. 30. $K=c.p.f.$

Try 1-inch apparent pitch, assuming the velocity at the pitch line at 700 feet per minute.

$$a=\frac{13.12}{V}+0.42=\frac{13.12}{700}+0.42=0.44.$$

$$b=\frac{21{,}500}{540+700}-25=17.4-25=-7.6.$$

$$c=7.9\times a(t_1-t_2)+14.25b.$$

$$c=[7.9\times 0.44\times 100]+[14.25\times 7.6]=348-108=240.$$

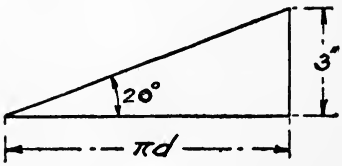
$$K=240\times 1.5p\times p \qquad K=240\times 1.5\times 1=360 \text{ pounds.}$$

This exceeds the pressure that will come upon it.

Lead $=3\times 1=3$ inches.

The angle of the thread should be about 20°. Tang. $20=0.364$.

tangent $20=3\pi d$




$$d=\frac{3}{0.364\times 3.14}=2.625'' \frown 2\tfrac{5}{8}''$$

Strength of the wheel, determined by Lewis's formula,

$$W=spf\left(0.124-\frac{0.684}{n}\right)$$

Assuming the maximum force 2 times the above,

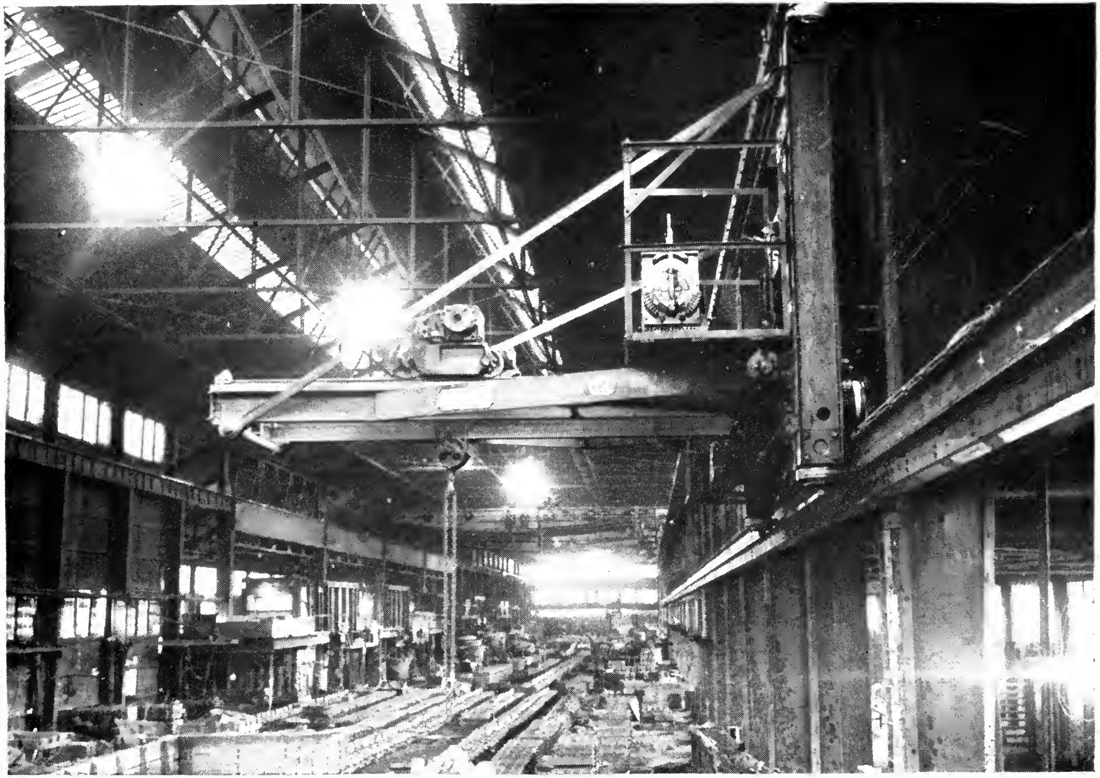
$$W=280\times 2=560 \text{ pounds.}$$

The factor for 45 teeth is $\left(0.124-\frac{0.684}{45}\right)=0.109.$

Solving for the fiber stress, we have

$$S=\frac{W}{p\times f\times 0.109}=\frac{560}{1\times 1.5\times 0.109}=3420 \text{ pounds.}$$

This fiber stress is sufficiently low for either cast-iron or bronze.



Three-Ton Wall Crane.

# PART IX.—WALL CRANE.

## Wall Crane.

Three-ton Wall Crane designed in accordance with the general description of crane used by the Pennsylvania Steel Company for carrying riveters (see Engineering Record October 3, 1903).

Estimated weight of frame and machinery, 5300 pounds.

The position of the center of gravity was not calculated, but was assumed as about ¼ the reach of the crane from the wall, or 6 feet 9 inches.

Live load 6,000 pounds. Trolley, block, etc., 400 pounds.

Design of Jib.—The length of the extended or cantilever portion of the crane is 10 feet. The weight of this portion is, $10\times80=800$ pounds.

The bending moment due to the live load at the maximum radius is, $M=P.l$ $M=6400\times9\times12=691{,}200$ inch-pounds.

The bending moment due to the extended part of the frame is

$$M=\frac{Wl}{2}=(800\times10\times12)\div2=48{,}000 \text{ inch-pounds.}$$

The total bending moment $M=691{,}200+48{,}000=739{,}200$ inch-pounds.

Allowing a fiber stress of 10,000 pounds per square inch, calls for a section modulus of $739{,}200\div10{,}000=73.9$ or 36.96 per channel, since 2 channels are used. This requires 15-inch channels weighing 33 pounds per foot. The supported span being 13 feet, it is evident the cantilever portion determines the beam section.

The ability of the web to resist the local bending due to the manner of carrying the trolley should be investigated. The wheel load can be considered as distributed over 18 inches of the length of the channel. The distance from the center of the track to the center of the web is 1.75 inches. The load on each trolley wheel is $\frac{6400}{4}=1600$.

Bending moment is, $M=1600\times1.75=2800$ inch-pounds.

Direct tension, $1600 \div 12 \times 0.4) = 334$ pounds, say 340 pounds; the web is 0.40 inches thick.

The extreme fiber stress due to bending is $p = \frac{Me}{I}$.

$$\frac{I}{e} = \frac{bd^2}{6} = \frac{18 \times 0.4^2}{6} = 0.48 \quad p = \frac{Me}{I} = \frac{2800}{0.48} = 5840 \text{ pounds.}$$

Combined tensile and bending stresses $= 5840 + 340 = 6180$ pounds.

The maximum reactions on the horizontal guide wheels will occur when the jib is at right angles to the wall and the live load is at its maximum radius.

$$R = \frac{(L \times b) + (G \times g)}{a}.$$

$$R = \frac{(6400 \times 314) + (5300 \times 89)}{96} = 25{,}800 \text{ pounds.}$$

The load equivalent to the live load and the crane weight concentrated at the apex where the jib joins the strut is

$$L^1 = \frac{6400 \times 23.5}{14.6} + \frac{5300 \times 4.75}{14.6} = 11{,}980 \text{ pounds, approximately}$$

12,000 pounds.

With this vertical load the triangle drawn at this apex gives 31,200 pounds as the stress in the strut and 29,000 pounds as the stress in the jib.

STRUT.—Trying two 12-inch I-beams, length 13 feet. The load per beam is $\frac{31{,}200}{2} = 15{,}600$ pounds. The radius of gyration of 12-inch I-beams at 31½ pounds for the minor axis is 1.01.

$$\frac{l}{r} = \frac{156}{1.01} = 154.$$

According to the column formula

$$p^1 = \frac{p}{1 + \frac{\left(\frac{l}{r}\right)^2}{10{,}000}} = \frac{10{,}000}{1 + 2.54} = 2820 \text{ pounds.}$$

The allowable load on 1 I-beam is $\text{Area} \times p^1 = 9.26 \times 2820 =$ 26,100 pounds.

Fig. I.

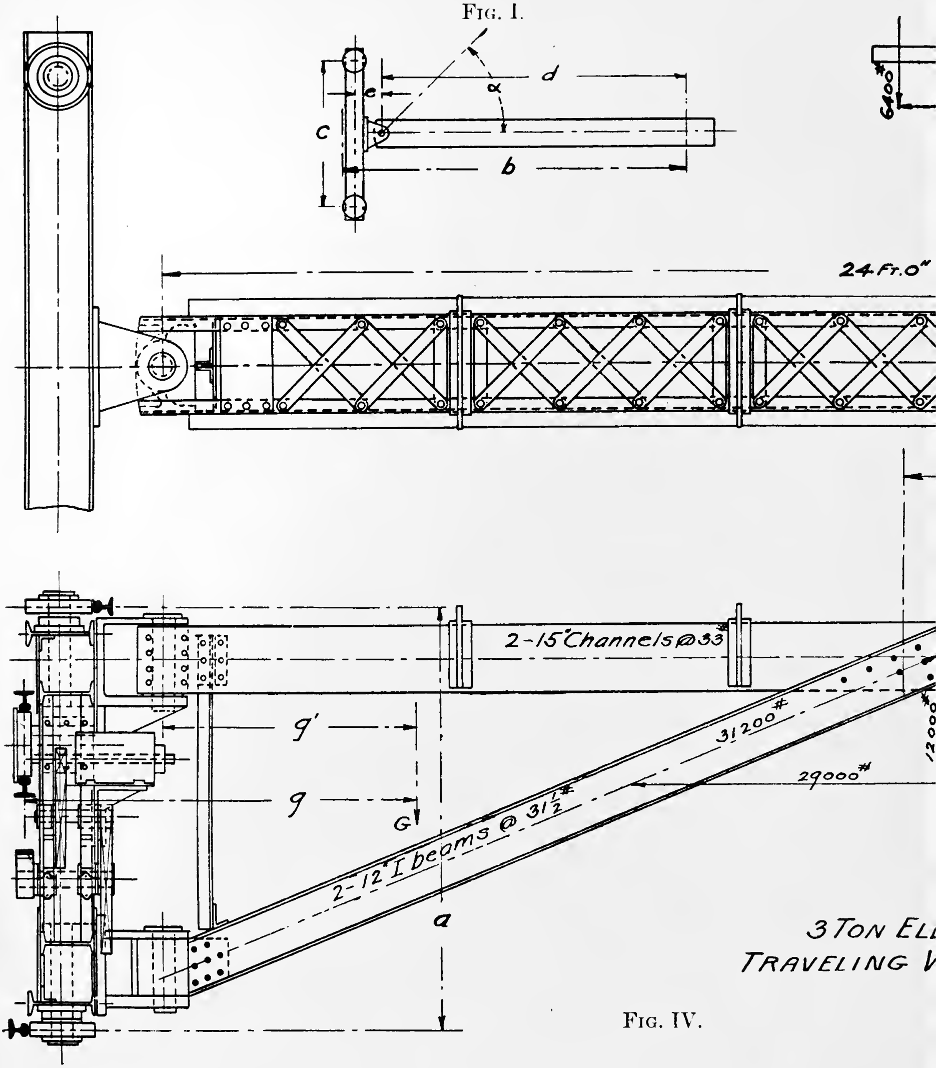

d
α
e
c
b
24 Ft.0"
6400#
2-15" Channels @ 33#
31200#
29000#
g'
g
G
2-12" I beams @ 31½#
a
3 Ton
Traveling

Fig. IV.

Fig. II.

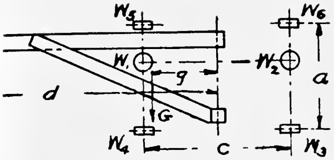
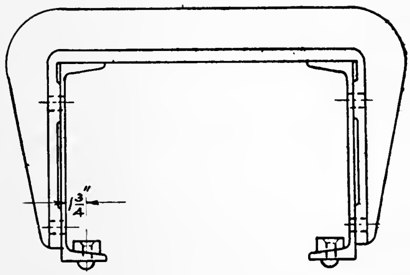
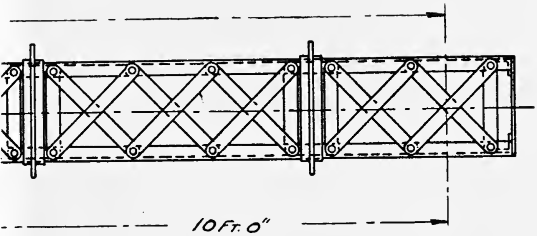
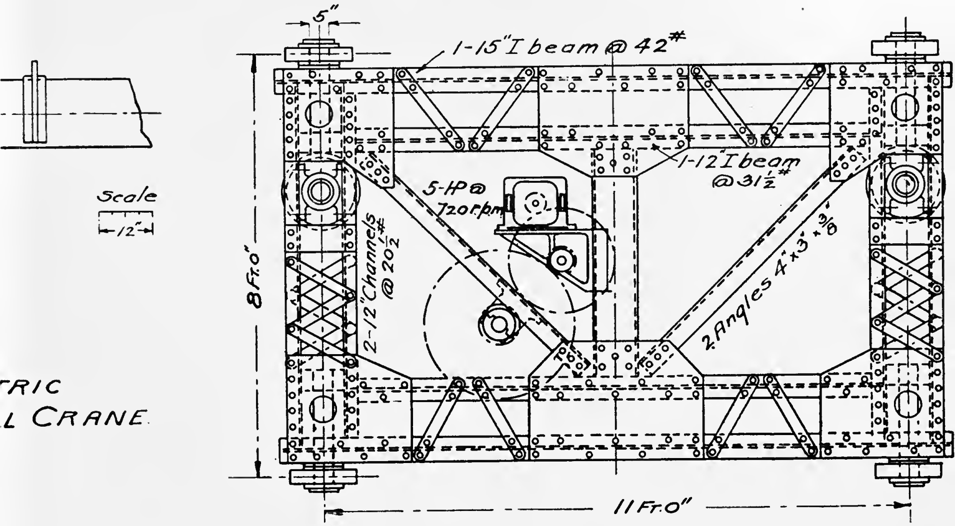



Fig. III.



This is almost double the load this beam will be subjected to, so it will be used although the $\frac{l}{r}$ value is high.

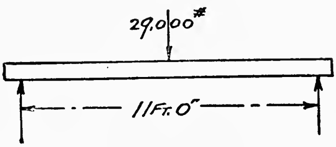
Frame Pins.—Load on pin, 29,000 pounds. The reaction at the end of each pin then is 14,500 pounds. Bending moment,

$$M = Wl = 14{,}500 \times 1.5d, \text{ taking } l \text{ at } 1.5d.$$

$$d^3 = \frac{10\,M}{p} = \frac{10 \times 14{,}500 \times 1.5d}{9000}.$$

$$d = \sqrt{\frac{10 \times 14{,}500 \times 1.5}{9000}} = 4.92 \text{ inches, say 5 inches.}$$

Longitudinal Frame:




$$M = \frac{Wl}{4} = \frac{29{,}000 \times 132}{4} = 957{,}000$$

inch-pounds.

$$\frac{I}{e} = \frac{M}{p} = \frac{957{,}000}{11{,}000} = 87.$$

| | |
|---|---|
| One 12-inch I-beam at 31½ pounds......... $\frac{I}{e}$ | $-36.0$ |
| One 15-inch I-beam at 42 pounds.......... $\frac{I}{e}$ | $-58.9$ |
| Total ................................ | 94.9 |

Bearing Wheels.—The maximum load occurs upon these when the jib with full load at maximum radius is parallel to the wall. (See Plate VIII, Fig. II.)

Finding the pressure on wheel No. 1 by taking moments about wheel No. 2.

$$L\left(d + \frac{c}{2}\right) + G\left(g^1 + \frac{c}{2}\right) = W_1 \times c.$$

$$W_1 = L\left(\frac{d}{c} + \frac{1}{2}\right) + G\left(\frac{g^1}{c} + \frac{1}{2}\right).$$

$$W_1 = \frac{Ld}{c} + \frac{Gg^1}{c} + \frac{L+G}{2}.$$

and in the same way,

$$W_2 = \frac{Ld}{c} + \frac{Gg^1}{c} - \frac{L+G}{2}.$$

$$W_1 = \frac{6400 \times 23.5}{11} + \frac{5300 \times 5}{11} + \frac{6400 + 5300}{2} = 21{,}930 \text{ pounds.}$$

$W_2 = 10{,}230$ pounds.

JOURNAL FOR BEARING WHEELS:

$M = 21{,}930 \times 5 = 109{,}650$ inch-pounds.

$$d = \sqrt[3]{\frac{10 \times M}{p}} = \sqrt[3]{\frac{10 \times 109{,}650}{10{,}000}} = 4.8, \text{ say } 4\tfrac{7}{8} \text{ inches.}$$

DIAGONAL BRACING.—Angle approximate y 45°.

Stress in the diagonal, $21{,}930 \times 1.4 = 30{,}700$ pounds.

Length 7 feet. Trying two $4 \times 3 \times \tfrac{3}{8}$-inch angles, braced across the 3-inch legs. $\frac{l}{r} = \frac{72}{1.26} = 57$.

$$p^1 = \frac{p}{1 + \frac{\left(\frac{l}{r}\right)^2}{10{,}000}}$$

$$p^1 = \frac{10{,}000}{1 + 0.325} = 7550 \text{ pounds.}$$

The allowable load $= p^1 \times$ area of angles $= 7550 \times 2 \times 2.49 = 37{,}600$ pounds.

This is well above the actual load.

POWER FOR CRANE TRAVEL.—The pressure upon the bearing and guide wheels will vary not only with the load, but also with the position of the trolley on the jib and of the jib relative to the frame. The angle $\alpha$, at which the jib carrying the full load at maximum radius must be placed to give the greatest total pressure upon the 6 wheels of the crane, can be estimated by assuming the tangent of the angle equal to $\frac{a}{c}$, tang. $\alpha = \frac{a}{c}$.

$a = 8$. $c = 11$. tang. $\alpha = \frac{8}{11}$. $\alpha = 36°$ 10 minutes.

The total pressure against the axles then is

$$F = \frac{2}{a}\Big[L\,(e + d\cos\alpha) + G(e + g^1\cos\alpha)\Big] + \frac{2}{c}\Big[Ld\sin\alpha + Gg^1\sin\alpha\Big]$$

$$F=\frac{2}{8}\left\{6400\,[2+(23.5\times .81)]+5300\,[2+(5.0\times 0.81)]\right\}$$
$$+\frac{2}{11}\left\{6400\times 23.5\times 0.59)+(5300\times 5\times 0.59)\right\}$$

$F=41{,}610+19{,}000=60{,}610$ pounds.

Assuming the wheel diameters 15 inches, the axles all 5 inches, the crane travel 50 feet per minute, then the horse-power to drive the crane is,

$$\text{Horse-power}=\frac{W(f+\mu r)v}{R\times 33{,}000\times \Sigma}.$$

$\Sigma$ = efficiency of machinery between wheels and motor.
$W$ = total load on wheels, pounds.
$f$ = coefficient of rolling friction, 0.003.
$\mu$ = coefficient of sliding friction, 0.10.
$r$ = radius of axle, inches.
$R$ = radius of wheel, inches.
$v$ = velocity of crane travel, feet per minute.

$$\text{Horse-power}=\frac{60{,}600\,[0.003+(0.10\times 2.5)]\times 50}{7.5\times 33{,}000\times 0.70}\backsim 4.5.$$

Use the Crocker-Wheeler motor No. 6K, 5-horse-power at 720 revolutions per minute.

A rack will be placed along the wall and the crane driven by a pinion meshing with this rack. The minimum pitch of an 18-tooth pinion meshing with the rack is found as follows:

$$W=s.p.f.\left(0.124-\frac{0.684}{n}\right).$$

Power delivered to the rack $=4.5\times 0.70=3.15$-horse-power.

$$\text{Force on the rack }\frac{3.15\times 33{,}000\times 3}{50}=6250\text{ pounds.}$$

The factor 3 is used to cover the sudden throwing on of the current in starting the crane in motion.

$$s^1=20{,}000\left(\frac{600}{600+50}\right)=18{,}500\text{ pounds.}$$

Substituting these values in the above formula,

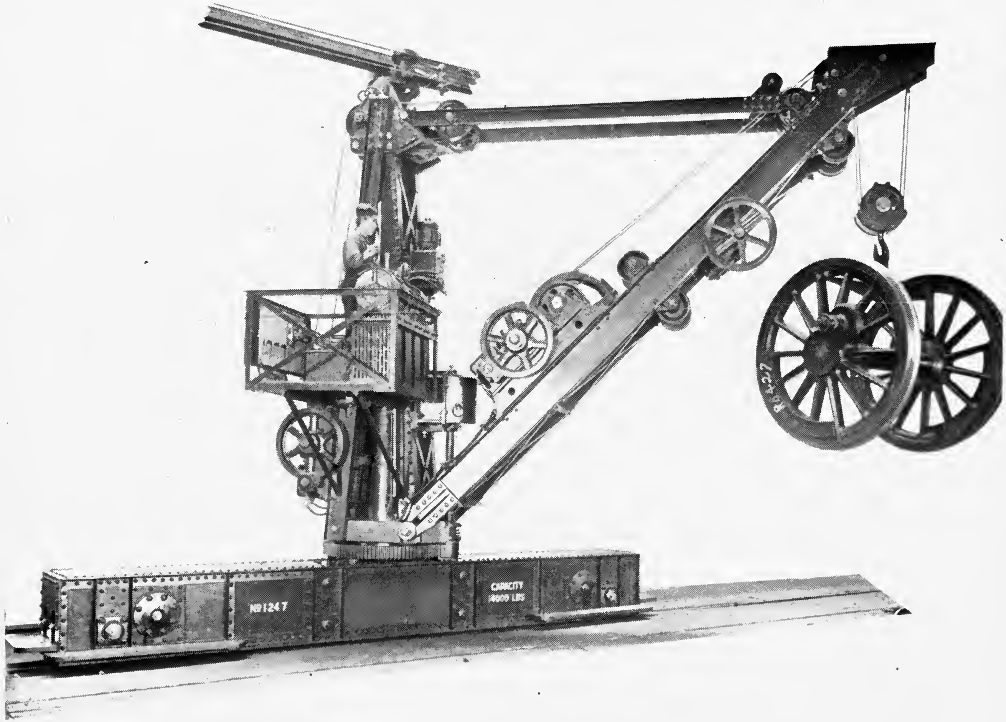
$$6250=18{,}500\times 2.25\,p^2\left(0.124-\frac{0.684}{18}\right).$$

$$p_c=\sqrt[2]{\frac{6250}{18{,}500\times 2.25\times 0.087}}=1.33 \text{ inches, use 2 diam. pitch.}$$

The calculations for the remaining gears and shafts are similar to those made for any motor-driven gearing for traversing.

## Velocipede Crane.

The accompanying cut illustrates a velocipede crane of 14,000 pounds capacity as built by Pawling & Harnischfeger. The general design would be carried out similarly to that of the Wall crane just described.




Velocipede Crane.

# PART X.—O. E. T. CRANE.

## Specifications for Electric Overhead Travelling Cranes.

The following is the specification for a 30-ton electric overhead travelling crane and is typical of specifications for this class of machinery.

SPECIFICATION NO. . . .
OF AN
ELECTRIC TRAVELLING CRANE
FOR

*Type.*—Standard, four-motor.

*Capacity.*—Main hoist, 30 net tons; test load, 36 net tons. Aux. hoist, 5 net tons; test load, 7½ net tons.

*Span.*—Center to center of runway rails, 60 feet.

*Hoist.*—Vertical movement of main block, 30 feet.

*Speeds.*—Approximate maximum speeds in feet per minute:

| | Hoist | | Trolley | Bridge |
|---|---|---|---|---|
| | Main | Aux. | | |
| Full load | 10 | 25 | 100 | 300 |
| No load | 25 | 62 | 125 | 350 |
| *Motors.*— | | | | |
| H. P. of Motors | 32 | 12 | 8 | 32 |
| Speed, r. p. m. | 485 | 575 | 630 | 485 |

*Voltage.*—Line voltage at crane, 220 direct current.

*Main Wires.*—Main line wires, anchors and carriers to be furnished by purchaser.

*Bridge Wheels.*—Number of wheels, 4; diameter, 30 inches; width of tread suitable for A. S. C. E. Standard 80-lb. rail.

*Rope and Sheaves.*—

| | Diam. Rope | No. Parts | Pitch Diam. Sheaves |
|---|---|---|---|
| | *Inches* | | *Inches* |
| Main hoist | 1 | 6 | 24 |
| Aux. hoist | $\frac{9}{16}$ | 4 | $13\frac{1}{3}$ |

*Drawings.*—Outside dimensions and extreme positions of hook will be as per accompanying drawing.

*General Specification.*—The attached General Specification, form 80, except as noted below, constitutes part of this proposal.

*Guarantee.*—It is guaranteed that the crane will be constructed of suitable materials in a substantial and workmanlike manner; also that it will handle the regular service load continuously at the highest speed given for such load,

Note:—This follows the specifications of The Shaw Electric Crane Co.

without dangerous heating of the motors and other electrical apparatus; and that the speeds of the various movements shall be readily controllable up to the limits specified. We will agree to furnish, without charge, new parts to replace any which shall prove defective in material or workmanship within one year from date of putting this crane in operation.

Respectfully submitted,

*Engineer.*

Date October 5, 1909.

*Bridge.*—The bridge usually consists of two box girders placed on top of steel truck beams. Chemical and physical properties of material are according to manufacturers' standard specifications for railway bridge steel. Workmanship is of the best quality.

For all ordinary spans and service, the bridge consists of two box girders, securely connected at the ends. Rails are attached centrally on the top flanges, and the space between the girders is left clear for hoisting ropes. Girder sections are made such that the stresses set up by the full load and the weight of the crane are not over 12,000 pounds per square inch of net section in tension, or 9,000 pounds per square inch in compression, but the compression strains are reduced below that figure when necessary to give sufficient lateral rigidity under all working conditions.

When span is not excessive, each flange member is usually of one piece. Web splices are made to give full strength to the section, and stiffeners are provided where needed. Diaphragms are provided at frequent intervals. Careful attention is given to rivet spacing.

When head room over runways will permit, the girders are placed on top of structural steel truck beams, and fastened with fitted bolts or rivets; but when head room is limited, we use a flush construction in which the girders are butted against the truck beams.

Long and heavy cranes, requiring four wheels under each end, have separate truck beams for each girder. The top flanges of the girders are connected at their ends by wide plates, reinforced by angles. This connection is rigid against racking strains but flexible vertically, allowing the wheels to follow any vertical irregularity or deflection of runway, and securing uniform distribution of the load on the four wheels.

*Bridge Driving Mechanism.*—Heavy chilled wheels of special iron are used. Cross shaft is driven near the center. A powerful foot brake is provided. Bridge motor gear and pinion are enclosed.

Treads of bridge wheels are ground true and to uniform circumference. The wheels are keyed to long and heavy quills, which are fitted with loose bronze bushings and run on fixed pins, giving most perfect lubrication and exclusion of dirt. Driving gears are keyed to the quills. The cross shaft is carried in capped bearings. The caps are planed in, and held by through bolts.

*Cage and Platform.*—The cage is large and convenient. Controllers are placed in rear. A platform extends full length of crane, and is provided with hand rail.

The operator's cage is constructed of heavy angles securely riveted to gusset plates, and is supported at one end of the crane, below and at one side of the bridge. The controllers are placed behind the operator and do not obstruct his view. Only the operating levers are in front of him. A ladder reaching the platform gives easy access to bridge driving mechanism and trolley, facilitating proper care of motors and machinery.

*Trolley.*—Trolleys are heavy and substantial. Construction is simple. All shafts are in capped bearings and all parts are accessible.

The trolley frame consists of two side frames connected by a cast steel girt. The side frames of trolleys of 30 tons capacity and smaller are of cast-iron; those for 40 tons capacity and larger are of cast steel. The members are finished in jigs, making them interchangeable, and are fastened together with through bolts.

A single hoisting drum is grooved right and left hand to give a vertical hoist. The grooves, which are wide and deep, are turned from solid metal.

All shafts are carried in capped bearings, and any shaft can be lifted out without disturbing any other. All caps are planed in and held by loosely fitting through bolts.

The upper sheaves are carried on a structural steel girt which forms no part of the trolley frame.

*Auxiliary Hoist.*—Auxiliary hoists are provided when required on cranes of ten tons capacity and larger.

*Load Brake.*—Load brake is of the multiple disc type and slow speed. Action is certain and without noise or shock.

All wearing parts are enclosed in an oil-tight casing, giving perfect lubrication and freedom from dirt. The brake, being located on the second shaft, runs at comparatively low speed. A gripping band has been substituted for ratchet and pawls. It is prompt in action, noiseless, and produces no shock. Load brakes are used on all sizes of hoists.

*Motor Brake.*—Motor brake is of the single solenoid type and is mounted on the motor.

Each hoist is provided with a motor brake which is released by the current which operates the motor and is applied by a spring when the current is interrupted.

The brake drum is carried on the commutator end of the armature shaft and the bracket is bolted to the motor frame. The brake frictions are adjustable for wear without throwing solenoids or brake spring out of adjustment. The brake applies no end pressure on the armature shaft.

*Limit Switch.*—Each hoist has an automatic limit switch which prevents over-travel of the block in hoisting. The limit contact device is enclosed in a neat box and mounted on the trolley. By means of an electric circuit a limit switch mounted on the switchboard is opened when the danger point is reached in hoisting. The operator can lower the block with the switch open, but cannot hoist it till the switch is closed.

*Lower Blocks and Ropes.*—All hooks are forged from tough refined iron. Sheaves are of large diameter. Rope is extra flexible plow steel.

All hooks of 15 tons capacity and over swivel on ball bearings. Sheaves have deep grooves and are well guarded to protect them from accidental injury and to keep the rope in place. Diameters are larger than specified by rope makers for the rope used, insuring durability of the rope.

A special plow steel rope is used, having 37 wires to the strand, six strands, and a hemp center, making it extra strong and very flexible. A factor of safety of about eight is provided in the rope.

*Gears.*—All gears are cut from solid blanks. Gears in hoisting train are steel. Pinions are steel forgings. All gears have sufficient strength to stand the overloads to which they are subjected by suddenly starting, stopping and reversing the motors.

*Bearings.*—All sheaves and wheels running on fixed pins are fitted with loose bronze bushings. All axle bearings are bronze. Bridge shaft bearings are lined with a good grade of babbitt metal, but all bearings in trolleys are bored in jigs and fitted with bronze bushings.

*Factor of Safety.*—A factor of safety of five will be provided in all parts, the fiber stresses of which are not specified.

*Motors.*—Our Type Z Motors are designed for hard service. Commutation is excellent under the worst conditions of load and speed variation. Armatures are built on quills so shafts are removable. Motors are completely enclosed yet very accessible. Bearings are ring oiled. Speeds are low, torque high and rating conservative. Mechanical construction is simple and strong and insulation is thorough.

The frame is of cast steel. Although the motor is completely enclosed, large hand holes with light swinging covers give access to brush holders and commutator. The upper half of frame can be quickly removed, exposing the armature.

All motors of three horse-power and over have four salient poles.

Bearings on all sizes of motors are ring oiled and are fitted with removable babbitt-lined shells.

Armature coils of four-pole motors are machine formed and removable. They are held in place by keys, and are thoroughly protected from oil and from mechanical injury. The armatures are so designed that the coils can be covered with canvas if necessary.

The motors are rated on a basis of 40° C. rise in temperature, measured by thermometer, after a thirty minutes' run with full load, but they are capable of carrying more than double the rated load for several minutes without injurious sparking or heating.

*Controllers.*—All electrical parts are removable from front and top, without moving the controller from its position in the cage. A strong magnetic blowout works over the whole range of contacts. All brushes and the blowout coil are carried on a single piece of insulating material. There are no external electrical parts. There are no gears or cams through which power is transmitted.

Our type S controller has been designed to meet the severest requirements of crane service. We use a special resistance material, which has a practically constant coefficient of resistance, regardless of variations of temperature. It is so arranged as to provide good ventilation, and is doubly insulated from the box. Any resistance card can be taken out through the door in front of the box by unbolting the electrical connections.

The blowout coil is carried with the brushes, making a short and powerful magnetic circuit, and concentrating the magnetic lines at each point where the electric circuit is broken. The block carrying the blowout coil and all brushes can be removed by loosening two bolts, and can be as quickly replaced. Contact strips are renewable.

As no electrical parts are exposed, the danger of accidental contact with "live" points is entirely removed.

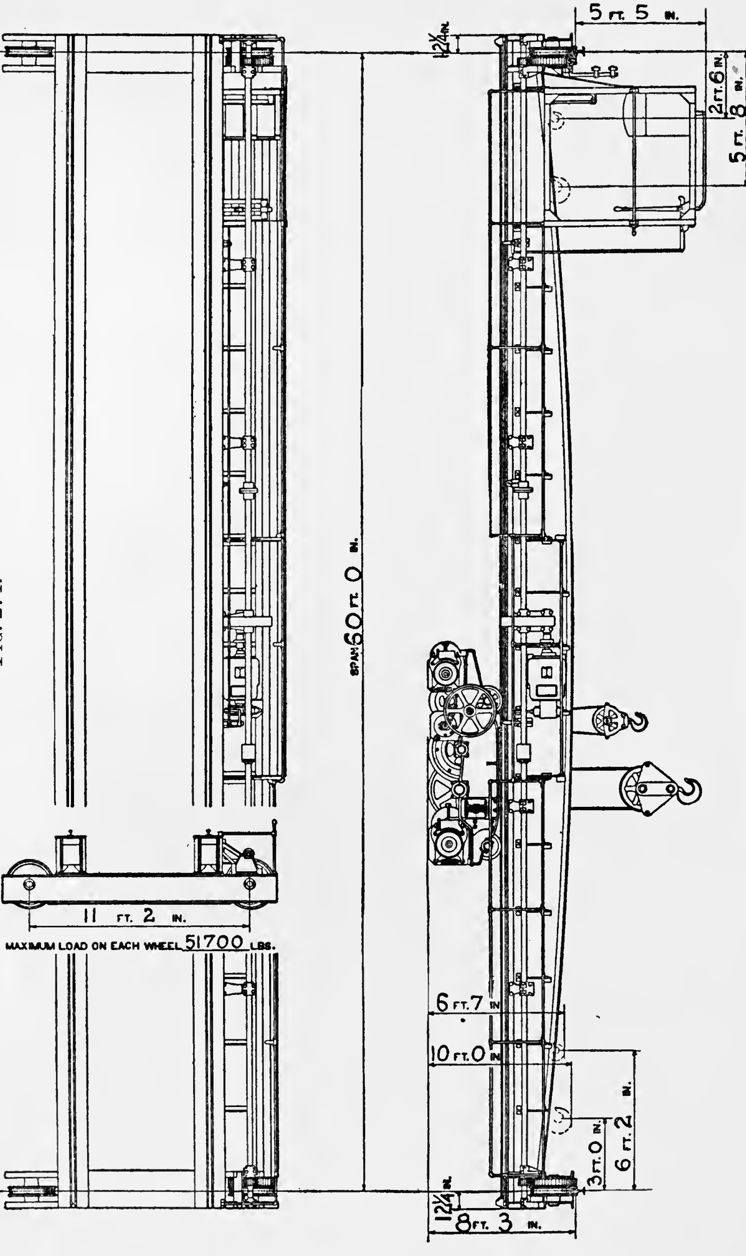



Fig. 274.

Clearance Diagram. 30-Ton Crane.

The controller is operated by a forward and backward movement of a long lever, standing in a vertical position and swinging through a small angle, a movement much to be preferred over a short arm, swinging around either a vertical or horizontal axis. As the motion is not transmitted through gears or cams which cause friction and lost motion, the movement is positive and very easy.

*Other Electrical Apparatus.*—Wires for moving contacts are of hard drawn bare copper; other wires are rubber covered and thoroughly protected.

Rolling contacts are provided for taking current from main conducting wires to the crane, and from bridge cross wires to the trolley.

A slate switchboard in cage is fitted with main switch, limit witch and fuses.

*Electrical Test.*—All electrical work, including motors and controllers, is tested with an alternating current of 1500 volts.

As has been previously stated under crane girders, crane loads run up to 10 tons (20,000 pounds) for light cranes and to 50, 75 or even 150 tons for heavy cranes. The spans range from 30 to 80 or 100 feet; by far the majority, however, are 60 feet or under.

Speeds.—Some idea of the usual speeds offered by manufacturers is given by the following tables taken from manufacturers' catalogues:

| Capacity (In Tons of 2,000 lbs.) | Speeds (in Feet Per Minute for Maximum Rated Capacity, and Light) | | |
|---|---|---|---|
| | Hoist | Bridge Travel | Trolley Rack |
| 2 | 15 to 30 | 200 to 250 | 100 to 150 |
| 3 | 15 to 30 | 200 to 250 | 100 to 150 |
| 5 | 15 to 30 | 200 to 250 | 100 to 150 |
| 7½ | 10 to 20 | 200 to 250 | 100 to 150 |
| 10 | 10 to 20 | 200 to 250 | 100 to 150 |
| 12½ | 10 to 20 | 200 to 250 | 100 to 150 |
| 15 | 10 to 20 | 200 to 250 | 100 to 150 |
| 20 | 10 to 20 | 200 to 250 | 100 to 150 |
| 25 | 10 to 20 | 200 to 250 | 100 to 150 |
| 30 | 10 to 20 | 200 to 250 | 100 to 150 |
| 35 | 10 to 20 | 200 to 250 | 100 to 150 |
| 40 | 8 to 15 | 150 to 200 | 75 to 100 |
| 50 | 8 to 15 | 150 to 200 | 75 to 100 |
| 60 | 8 to 15 | 150 to 200 | 75 to 100 |
| 75 | 6 to 10 | 150 to 200 | 75 to 100 |

SPEEDS FOR AUXILIARY HOIST ON MAIN TROLLEY

| CAPACITY (Tons of 2000 lbs.) | SPEEDS (Feet Per Minute) |
|---|---|
| 2 | 25 to 50 |
| 3 | 25 to 50 |
| 5 | 20 to 40 |
| 7½ | 20 to 40 |
| 10 | 20 to 40 |

| Size in Tons of 2000 lbs. | Standard Hoisting Speeds in ft. per Minute | Bridge Travel Speeds in ft. per Minute | Trolley Travel Speeds in ft. per Minute | Size of Aux. Hoist in Tons | Aux. Hoist Speeds in ft. per min. |
|---|---|---|---|---|---|
| 5 | 25 to 60<br>30 to 75<br>40 to 100 | 300 to 350<br>400 to 450 | 100 to 150 | | |
| 10 | 20 to 50<br>25 to 60<br>30 to 75 | 300 to 350<br>400 to 450 | 100 to 150 | 3 | 30 to 75 |
| 15 | 17 to 42<br>20 to 50<br>24 to 60 | 300 to 350<br>350 to 400 | 100 to 150 | 3 or 5 | 50 to 125<br>40 to 100 |
| 20 | 12 to 30<br>15 to 40<br>20 to 50 | 250 to 300<br>300 to 350 | 100 to 150 | 3 or 5 | 50 to 125<br>40 to 100 |
| 25 | 10 to 25<br>12 to 30<br>16 to 40 | 250 to 300<br>300 to 350 | 100 to 150 | 3<br>5<br>10 | 50 to 125<br>40 to 100<br>25 to 60 |
| 30 | 10 to 25<br>12 to 30<br>14 to 35 | 250 to 300<br>300 to 350 | 100 to 150 | 5 or 10 | 40 to 100<br>25 to 60 |
| 40 | 9 to 22<br>10 to 25<br>12 to 30 | 250 to 300<br>300 to 350 | 100 to 150 | 5 or 10 | 40 to 100<br>25 to 60 |
| 50 | 8 to 20<br>10 to 25<br>12 to 30 | 200 to 250<br>250 to 300 | 100 to 150 | 5 or 10 | 40 to 100<br>25 to 60 |
| 60 | 8 to 20<br>10 to 25<br>12 to 30 | 200 to 250<br>250 to 300 | 100 to 150 | 10 or 15 | 25 to 60<br>20 to 50 |
| 75 | 6 to 15<br>8 to 20<br>10 to 25 | 200 to 250 | 100 to 150 | 15 | 20 to 50 |
| 100 | 5 to 12<br>6 to 15<br>7½ to 18 | 200 to 250 | 100 to 150 | 20 | 20 to 50 |
| 125 | 5 to 12<br>6 to 15 | 200 to 250 | 100 to 150 | 25 | 20 to 50 |
| 150 | 5 to 12<br>6 to 15 | 200 to 250 | 100 to 150 | 25 | 20 to 50 |

Since the average crane load is probably under 25 per cent. of its rated load, the handling of light loads by heavy cranes is facilitated by equipping the trolleys with auxiliary hoists. These are preferably and generally driven by separate motors. The light loads are in this way more satisfactorily handled, less power is required, and a great amount of wear and tear is taken from the main hoist.

Some idea of the behavior of a crane under various loads is shown by the accompanying curves, which illustrate the action of a 25-ton crane under test.

FIG. 275.

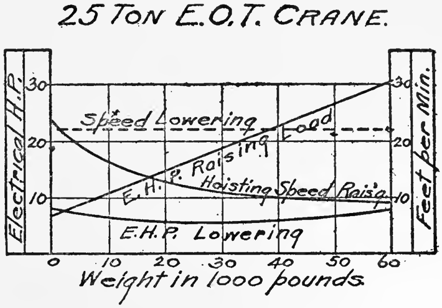



FIG. 276.

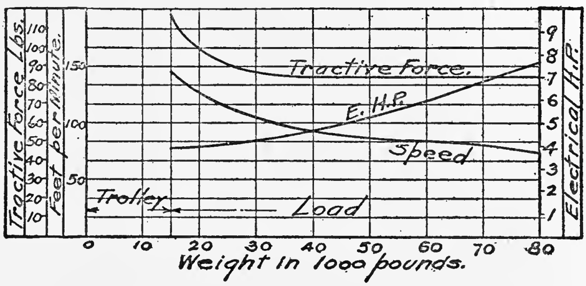
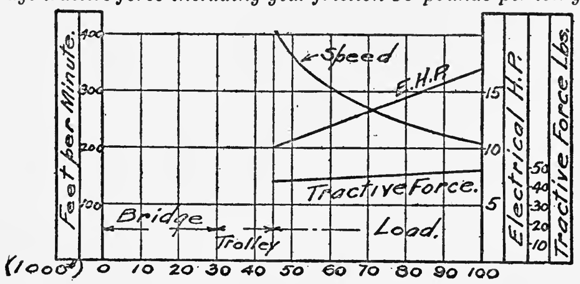



FIG. 277.

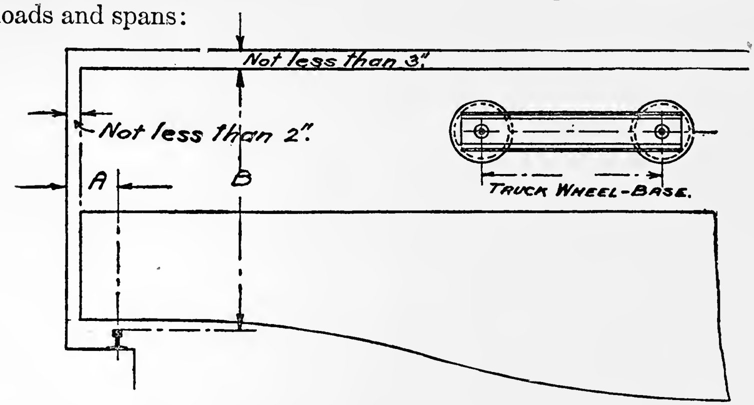
*Average tractive force including gear friction 80 pounds per ton gross.*




*Types of bridges suitable for cranes of various capacities and spans.*

| | Load Tons (2000 lbs.) | Span Feet. |
|---|---|---|
| Rolled Sections | 5 to 20 | up to 30 |
| Single Web Girder, stiffened laterally | 5 to 20 | 30 to 60 |
| Box Girders | 5 to 100 | 30 to 60 |
| Latticed Girders | 5 to 100 | over 60 |

The following table gives the weights of box girders for various loads and spans:




| Capacity Tons (2000 lb.) | Span | A | B | Wheel Base | Weight of Bridge 2 Girders | Weight of Trolley | Rail |
|---|---|---|---|---|---|---|---|
| | *Ft.* | *Inch.* | *Ft. & In.* | *Ft. & In.* | *Pounds.* | *Pounds.* | *Lbs. per Yard.* |
| | 40 | 8 | 4 9 | 8 0 | 8,600 to 13,000 | 4,500 | |
| 5 | 60 | 8 | 4 9 | 8 0 | 16,500 to 21,000 | to | 40 |
| | 80 | 8 | 4 9 | 8 0 | 28,000 to 33,000 | 5,000 | |
| | 40 | 8 | 5 0 | 8 6 | 12,300 to 16,000 | 6,000 | |
| 10 | 60 | 8 | 5 0 | 8 6 | 20,000 to 24,000 | to | 40 |
| | 80 | 8 | 5 0 | 9 0 | 32,000 to 37,000 | 8,000 | |
| | 40 | 8 | 5 6 | 9 0 | 14,000 to 23,000 | 9,000 | |
| 15 | 60 | 8 | 5 6 | 9 6 | 23,000 to 33,000 | to | 50 |
| | 80 | 9 | 5 6 | 10 0 | 37,000 to 48,000 | 10,000 | |
| | 40 | 9 | 6 0 | | 23,000 to 28,000 | 10,000 | |
| 20 | 60 | 9 | 6 0 | | 30,000 to 37,000 | to | 50 |
| | 80 | 9 | 6 0 | | 40,000 to 52,000 | 12,000 | |
| | 40 | 9 | 6 6 | 10 0 | 19,000 to 34,000 | 12,000 | |
| 25 | 60 | 9 | 6 6 | 10 6 | 29,000 to 45,000 | to | 60 |
| | 80 | 9 | 6 6 | 11 0 | 45,000 to 61,000 | 15,000 | |
| | 40 | 9 | 7 0 | 11 0 | 34,000 to 37,000 | 14,000 | |
| 30 | 60 | 9 | 7 0 | 11 6 | 44,000 to 49,000 | to | 60 |
| | 80 | 9 | 7 0 | 12 0 | 58,000 to 66,000 | 17,000 | |
| | 40 | 9 | 8 0 | 12 6 | 43,000 to 49,000 | 16,000 | |
| 40 | 60 | 9 | 8 0 | 13 0 | 60,000 to 63,000 | to | 80 |
| | 80 | 9 | 8 0 | 13 3 | 70,000 to 82,000 | 20,000 | |
| | 40 | 9 | 8 6 | 12 6 | 48,000 to 57,000 | 24,000 | |
| 50 | 60 | 9 | 8 6 | 13 0 | 66,000 to 73,000 | to | 100 |
| | 80 | 9 | 8 6 | 13 6 | 85,000 to 95,000 | 30,000 | |
| | 40 | 10 | 9 0 | 15 0 | 78,000 | | |
| 60 | 60 | 10 | 9 0 | 15 3 | 70,000 to 95,000 | 32,000 | |
| | 80 | 10 | 9 0 | 15 6 | 100,000 to 126,000 | | 100 |
| | 40 | 10 | 9 6 | 15 6 | 101,000 | | |
| 75 | 60 | 10 | 9 6 | 15 6 | 80,000 to 120,000 | 40,000 | to |
| | 80 | 10 | 9 6 | 15 6 | 120,000 to 144,000 | | |
| | 40 | 12 | 10 0 | 15 6 | 134,000 | | 150 |
| 100 | 60 | 12 | 10 0 | 15 6 | 94,000 to 161,000 | 56,000 | |
| | 80 | 12 | 10 0 | 15 6 | 125,000 to 187,000 | | |

The girders are of the box type. Sizes over 60 tons usually have 4 wheels at each end of the bridge instead of 2. For 2 wheels at each end maximum wheel load =

$$\frac{\text{Weight of Bridge}}{4}+\frac{\text{Weight of Trolley}+\text{Live load}}{2}.$$

## Crane Motors.

Both direct- and alternating-current motors are available for crane service, although comparatively few installations are equipped with alternating-current motors.

The direct-current motors are series wound, the usual voltage being either 220 or 500 volts. The manufacturers place a nominal rating based upon a rise in temperature of the motor of 40 degrees Centigrade above the surrounding air, the temperature of the air not exceeding 25 degrees Centigrade; the duration of the test is 30 minutes.

The usual crane motor is of the enclosed type, and the test is made with the motor fully enclosed.

Crane service demands a heavy starting torque; this the series motor amply supplies. In the series motor there is a definite current and speed corresponding to any load and voltage. The current used depends upon the applied and the back electromotive force and the motor resistance.

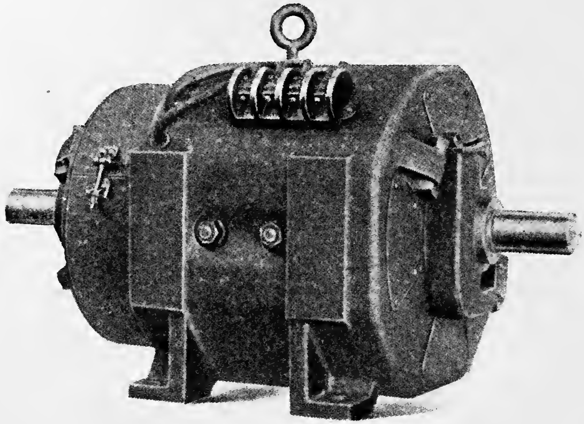
$$I=\frac{E-e}{R}$$

$I$ = current in amperes.
$E$ = voltage across motor terminals.
$e$ = voltage due to rotation of the motor armature in its own field, which produces a counter electromotive force to that driving the motor.
$R$ = resistance of motor in ohms.

Since $e$ depends upon the speed of the armature, it is evident that with the armature at rest or in slow motion $e$ becomes zero or very low, and in this event I would be very great, resulting in practically a short circuit. It is therefore necessary in starting a series motor to cut down the electromotive force applied across the terminals by the introduction of resistance in the circuit. The motor is started by means of a controller and suitable resistance. The first point on the controller closes the circuit through the entire resistance. Gradually throwing the controller around to succeeding points, as the motor speeds up, cuts out the resistance until finally the motor is operating at the speed corresponding to the load with the full voltage across the terminals. To prevent excessive current being sent through the motor by throwing the controller around too rapidly, fuses or preferably circuit-breakers should be placed in each motor circuit. Circuit-breakers, although more expensive than fuses, permit the circuit to be quickly closed, while trouble with fuses not infrequently results in copper wire being used instead of the fuse wire.

Crane service is severe on motors, due to the frequent starting, stopping and reversing. They usually run in the hottest part of a room or building, under the roof. Special cranes may even oper-

Fig. 278.

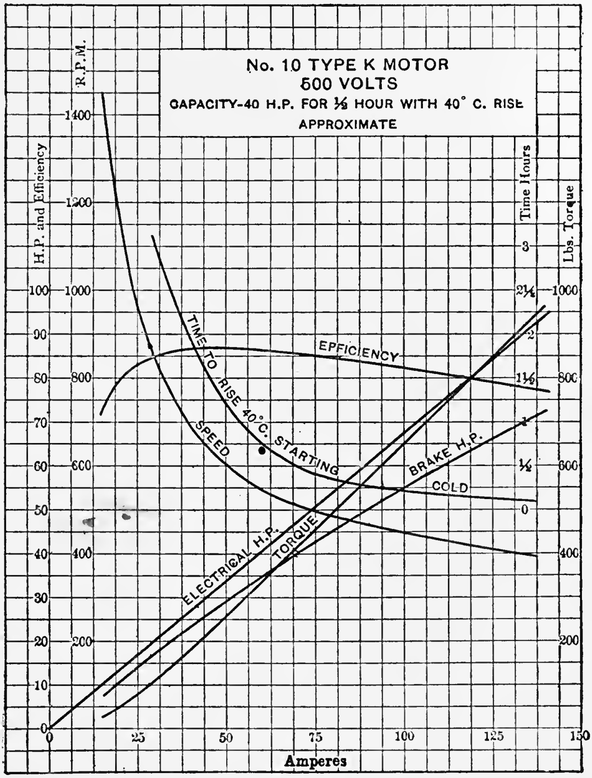


Westinghouse No. 10 Type K Crane Motor.

ate above or near furnaces, in which case special consideration must be given to the heating of the motor. In selecting the hoisting motor, it must be borne in mind that the average load will possibly not exceed 20 per cent. of the full load; the motor characteristic should be such that as much work as possible be done by the motor running with full voltage across its terminals, *i.e.*, no resistance in the circuit. In the bridge and trolley travels the power required when the hook is empty is usually a considerable portion of that required when fully loaded, so that the selection of these motors is not so difficult.

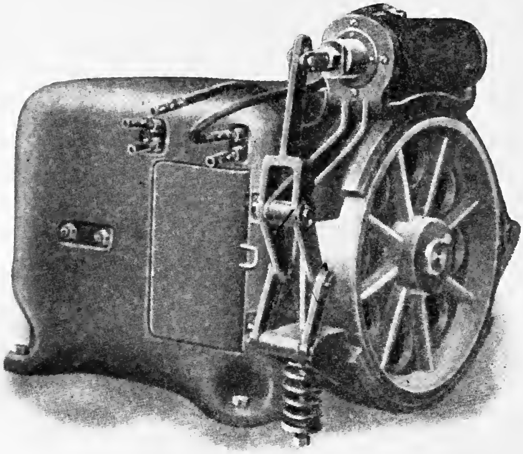
The accompanying curves show the motor characteristics of the 10-K motor built by the Westinghouse Electric and Manufacturing Company.

It will be noted that this motor when developing 40-brake horse-power uses 70 amperes, develops a torque of 410 foot-pounds and runs at 515 revolutions per minute, and that when developing a torque of 100 foot-pounds it would run at 875 revolutions per minute and use 30 amperes.



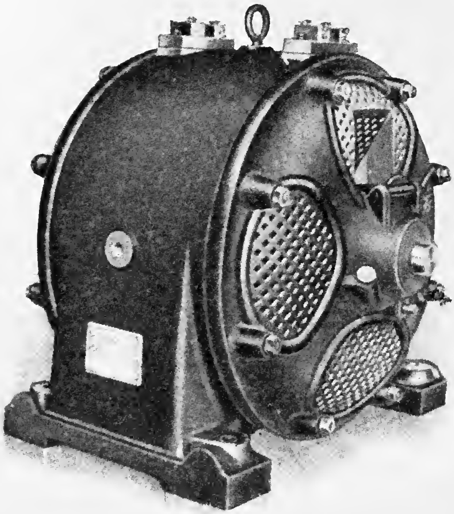

In ordinary cases of crane driving the standard motors may be chosen in accordance with manufacturers' characteristics. When, however, the service is unusually severe, and when the crane or hoist working is sufficiently known to permit of plotting velocity and power curves, it is better to plot such curves and have the motor either designed or selected with special reference to the work it is to perform.

Fig. 279.



The above illustrates a Crocker-Wheeler crane motor with solenoid brake.

Fig. 280.



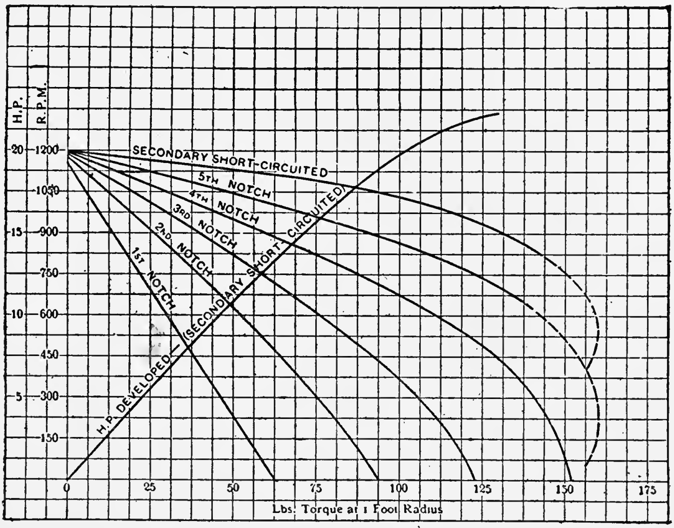
The cut shows a variable-speed induction motor as built by the Westinghouse Electric & Manufacturing Company.

## Alternating-Current Motors.

Alternating-current motors are available of the induction type both single and polyphase and of 200 and 400 volts.

The greatest recommendation of the alternating-current motors lies in their simplicity. Their maintenance costs practically nothing. The motor has no commutator or brushes. The speed is controlled by auto-transformers, which serve the same usefulness as direct current controller resistance. They act by cutting down the voltage across the motor. They differ from the resistance, however, in causing only a slight loss of power. There is therefore no objection to operating the motors at low speeds due to the controller through the auto-transformer.

The diagram shows the torque developed by an induction motor working under different notches of the controller.




CHARACTERISTIC CURVES OF AN INDUCTION MOTOR.

## Electrical Wiring of Cranes.

The requirements of the "National Electrical Code" demand that, wherever possible, all wires shall be insulated and run in wrought-iron conduits or in metal conduit fittings.

The following diagrams show the wiring of a crane for both direct and alternating current:

Fig. 281.

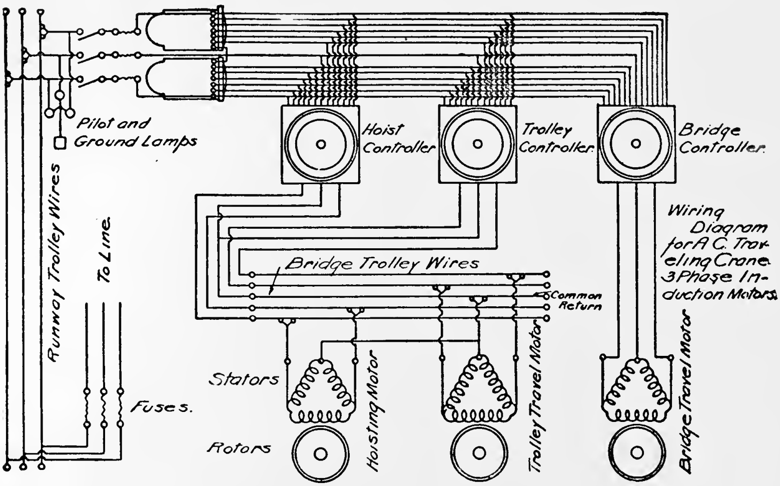
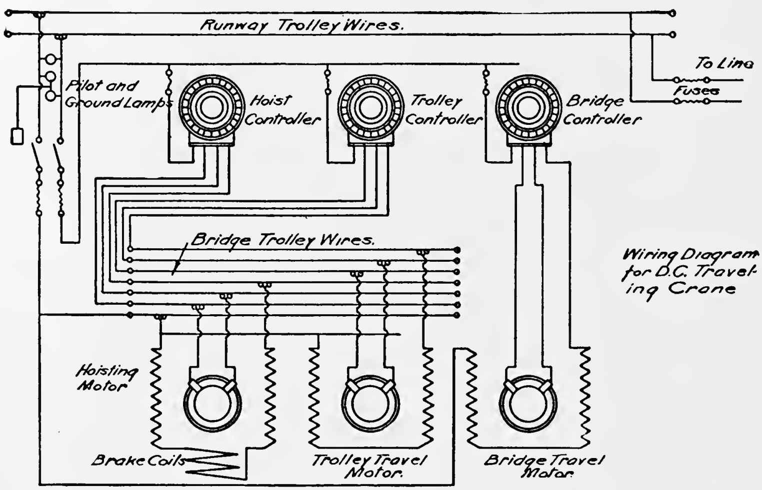



Fig. 282.



The following table gives the capacities of copper wires, double braid, rubber covered, voltage 0 to 600 when run in National Electrical Code S —— conduit:

| No. Wire B & S Gauge | Area of Wire Circ. Mills | Safe Current Amperes | Wt. of 1000 feet. Bare Wire. Lbs. | Resistance at 68° F. Ohms per 1000 Ft. | Size of Conduit. Inches. | | |
|---|---|---|---|---|---|---|---|
| | | | | | 1 Wire in Conduit. | 2 Wires in Conduit. | 3 Wires in Conduit. |
| 18 | 1,624 | 3 | 4.92 | 6.37 | ½ | ½ | ½ |
| 16 | 2,583 | 6 | 7.82 | 4.01 | ½ | ½ | ½ |
| 14 | 4,107 | 12 | 14.43 | 2.52 | ½ | ½ | ¾ |
| 12 | 6,530 | 17 | 19.77 | 1.59 | ½ | ¾ | ¾ |
| 10 | 10,380 | 24 | 31.43 | 1.00 | ½ | ¾ | 1 |
| 8 | 16,510 | 33 | 49.98 | 0.63 | ½ | 1 | 1 |
| 6 | 26,250 | 46 | 79.56 | 0.39 | ¾ | 1 | 1¼ |
| 5 | 33,100 | 54 | 100.2 | 0.31 | ¾ | 1¼ | 1¼ |
| 4 | 41,740 | 65 | 126.4 | 0.25 | ¾ | 1¼ | 1½ |
| 3 | 52,630 | 76 | 159.3 | 0.20 | ¾ | 1¼ | 1½ |
| 2 | 66,370 | 90 | 200.9 | .156 | ¾ | 1½ | 2 |
| 1 | 83,690 | 107 | 253.3 | .124 | 1 | 1½ | 2 |
| 0 | 105,500 | 127 | 319.5 | .098 | 1 | 2 | 2 |
| 00 | 133,100 | 150 | 402.8 | .078 | 1 | 2 | 2 |
| 000 | 167,800 | 177 | 508.0 | .062 | 1¼ | 2 | 2½ |
| 0000 | 211,600 | 210 | 640.5 | .049 | 1¼ | 2 | 2½ |

The accompanying table gives the approximate full load current in amperes, used by motors of various horse-power.

| H. P. of Motors. | Direct Current. | | Alternating Current. | | | | | |
|---|---|---|---|---|---|---|---|---|
| | | | Single-phase | | Two-phase (4 wires) | | Three-phase (3 wires) | |
| | 220 volts. | 500 volts. | 220 volts. | 500 volts. | 220 volts. | 500 volts. | 220 volts. | 500 volts. |
| 1 | 5 | 2 | 8 | 3 | 3 | 2 | 4 | 2 |
| 2 | 9 | 4 | 13 | 5 | 6 | 3 | 7 | 3 |
| 3 | 13 | 6 | 17 | 8 | 8 | 4 | 9 | 4 |
| 5 | 21 | 9 | 26 | 12 | 13 | 6 | 15 | 6 |
| 7½ | 31 | 13 | 38 | 16 | 20 | 8 | 22 | 9 |
| 10 | 37 | 18 | 49 | 22 | 23 | 11 | 25 | 13 |
| 15 | 57 | 25 | .. | .. | 34 | 15 | 39 | 17 |
| 20 | 76 | 34 | .. | .. | 44 | 20 | 52 | 23 |
| 30 | 114 | 49 | .. | .. | 68 | 30 | 78 | 33 |
| 40 | 152 | 67 | .. | .. | 90 | 39 | 107 | 46 |
| 50 | 183 | 83 | .. | .. | 102 | 44 | 119 | 52 |
| 75 | 277 | 123 | .. | .. | 155 | 69 | 179 | 78 |
| 100 | 369 | 162 | .. | .. | 206 | 91 | 236 | 105 |
| 150 | 556 | 245 | .. | .. | 308 | 137 | 356 | 157 |
| 200 | 736 | 326 | .. | .. | 410 | 183 | 472 | 210 |

In wiring motors the National Electric Code requires when the motor develops a high starting torque that the conductor capacities shall be designed for 125 per cent. of the rated current for the motor. Trolley wires are taken about as follows:

Cranes up to 20 tons capacity require a No. 4 trolley wire, while larger sizes call for a No. 2 or a No. 0 wire.

## Crane Cage.

The accompanying cut shows a crane cage as built by the Shaw Electric Crane Company. The controllers with their resistances back of them are shown on the floor of the cage. On a panel in the rear are shown the switch and circuit-breaker.

The operator's cage is made up of angles braced to make the cage rigid. The controllers, switchboard, and brake lever for operating bridge brake are located in the most convenient manner. The cage is of sufficient size to allow the operator ample room.

For outdoor service, the lower portion of the cage is enclosed with sheet-steel covering and the upper portion with glass windows.

## Trolley Frames and Bridge Trucks.

Trolley frames are made either of structural material, beams, channels, angles and plates, or of cast-iron or steel castings; the structural frame can be made lighter than the cast-iron frames, and the bearings and machinery located on the former are generally more readily accessible. The design is best illustrated by an example.

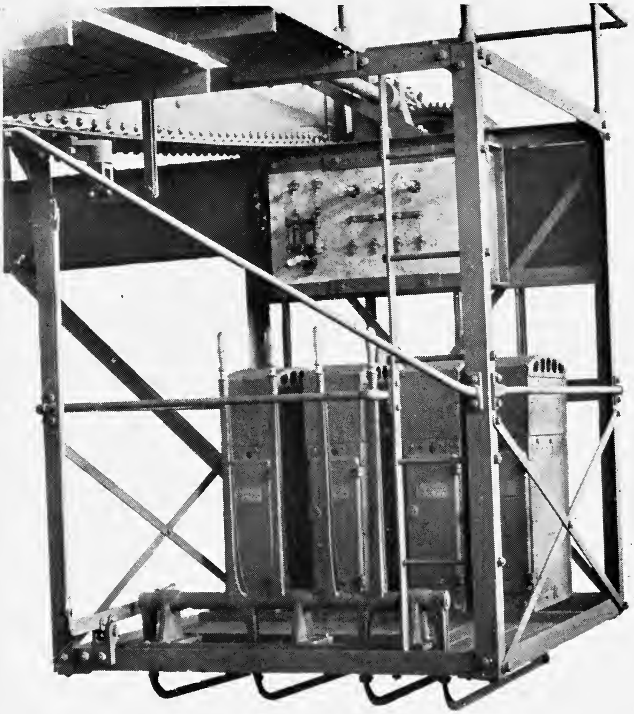
**Example.**—The trolley is for a 10-ton (20,000 pounds) crane. Design a frame of cast-iron in which the maximum fiber stress shall not exceed 2000 pounds per square inch. Then design a frame of structural material using a fiber stress of 10,000 pounds per square inch.

The method of determining the bending moment on the frame will vary somewhat with the type of the trolley. With the one in question it will be most readily done by finding the reaction at the wheel B and then determining the bending under the shaft $S_2$. As the load will hang symmetrically with respect to the 2 sides of the trolley, ½ of it can be taken as acting upon each side. Taking moments about A,

$$\text{Reaction at B} = \frac{20{,}000}{2} \times \frac{22.5}{47} = 4788 \text{ pounds.}$$

The bending under $S_2 = M_1 = 4788 \times 17.5 = 83{,}790$ inch-pounds.

Fig. 283.



Cage.

Fig. 284.

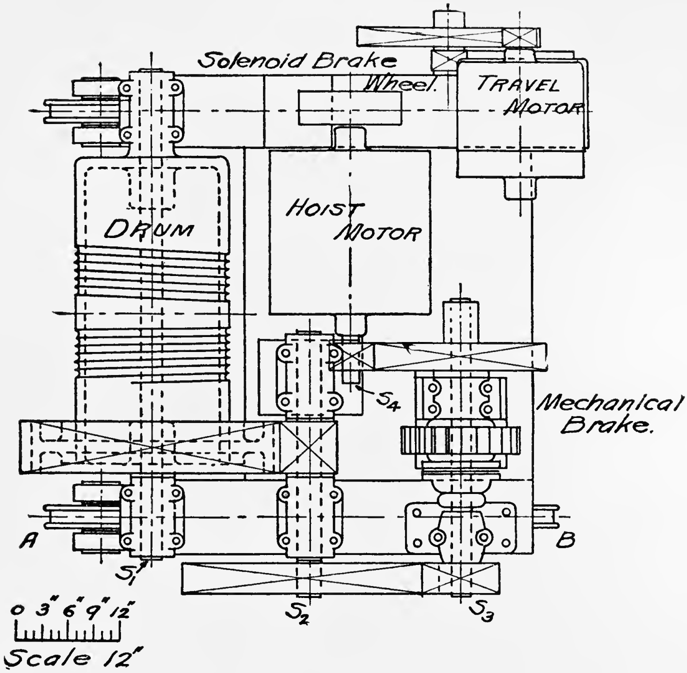
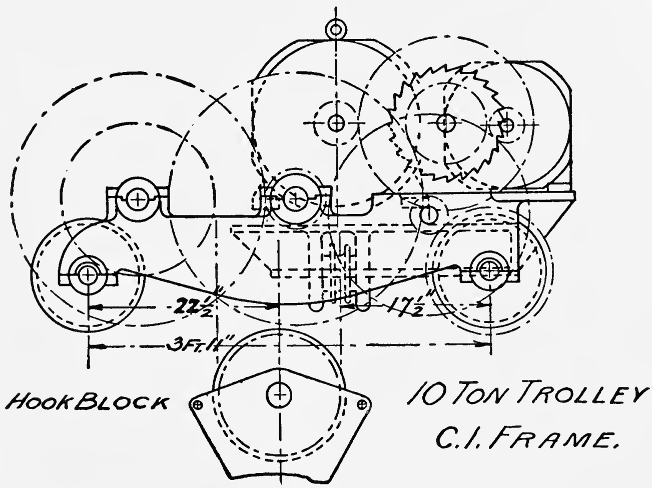



Fig. 285.



The bending produced by the weight of the trolley will be assumed as approximately that due to a uniform load. The trolley weight is 6,000 pounds. The load on each side then is 3,000.

$$M = \frac{WL}{8} = \frac{3000 \times 50}{8} = 18{,}750 \text{ inch-pounds.}$$

The total bending $= 83{,}790 + 18{,}750 = 102{,}540$ inch-pounds.

If the fiber stress is to be limited to 2000 pounds per square inch the resistance must be

$$\frac{I}{e} = \frac{M}{p} = \frac{102{,}540}{2000} = 51.27.$$

The section must now be determined by trial. The frame will be assumed as a hollow rectangular section, and the section will afterwards be slightly rearranged to facilitate casting.

Fig. 286.

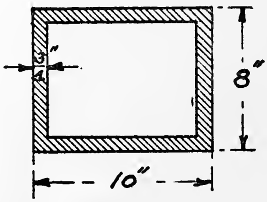
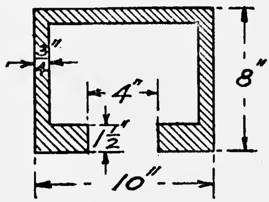



Fig. 287.




The inertia of Fig. 286

$$I = \frac{B.D^3}{12} - \frac{bd^3}{12} = \left(\frac{10 \times 8^3}{12}\right) - \left(\frac{8.5 \times 6.5^3}{12}\right) = 235.$$

$$\frac{I}{e} = \frac{235}{4} = 58.75.$$

This is sufficiently near the 51.27 required.

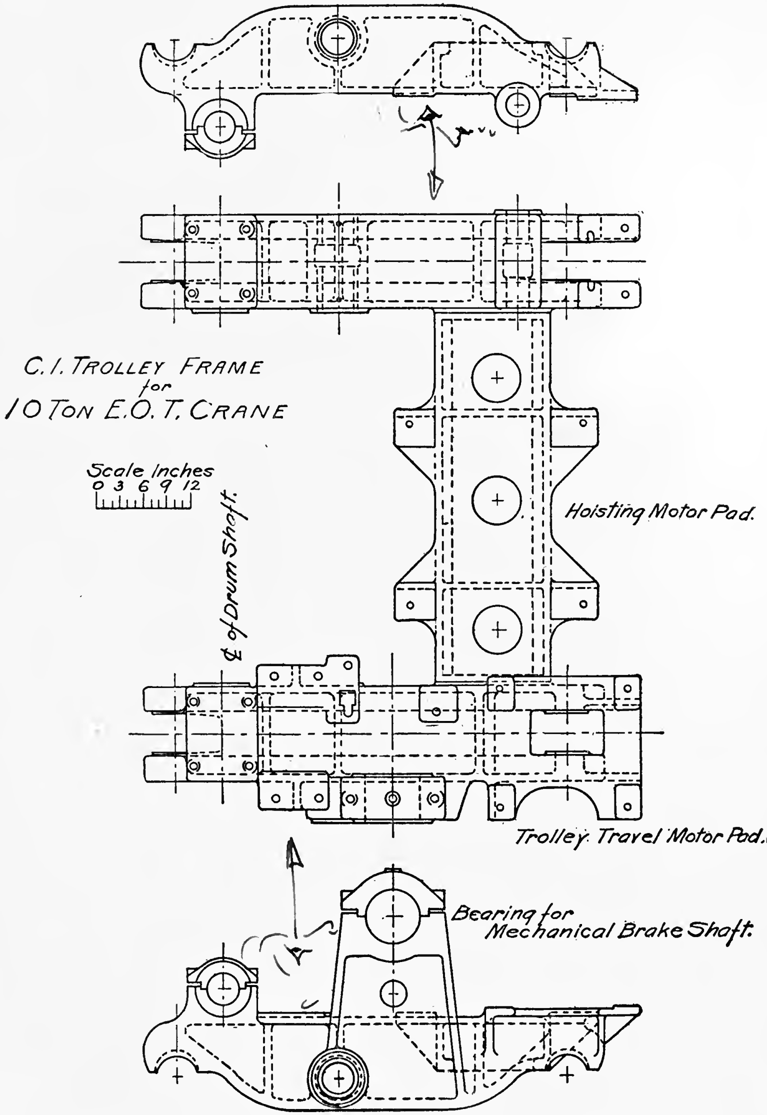
Had it been decided to use 2 channels instead of the above casting, the size channels required would have been determined as follows:

$$\frac{I}{e} = \frac{102{,}540}{2 \times 10{,}000} = 5.13.$$

Referring to manufacturer's hand-book we find that two 7-inch channels weighing 9¾ pounds per foot and having a section modulus about their principal axis of 6 will be needed.

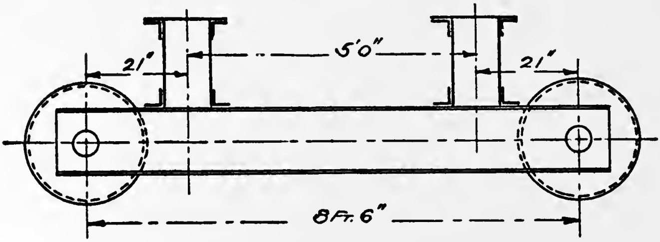
It must be understood that these are the minimum sections, and they may be enlarged to more conveniently carry the shaft bearings.

FIG. 288.


C. I. TROLLEY FRAME
for
10 TON E.O.T. CRANE
Scale Inches
0 3 6 9 12
℄ of Drum Shaft.
Hoisting Motor Pad.
Trolley Travel Motor Pad.
Bearing for
Mechanical Brake Shaft.

Bridge Trucks.—Bridge trucks are designed in much the same way. Assume that the crane for which the trolley frame was designed has a bridge-wheel base of 8 feet 6 inches, and that the maximum wheel load is 21,000 pounds.

Fig. 289.



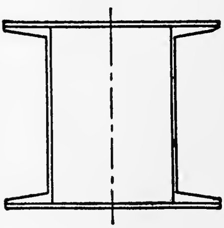

Bending moment $M_B = 21{,}000 \times 21 = 441{,}000$ inch-pounds.

$$\frac{I}{e} = \frac{441{,}000}{2 \times 10{,}000} = 22.05.$$

This requires two 12-inch channels 25 pounds per foot.

Sometimes a section, shown in Fig. 290, is used, in which case the inertia must first be found and then the section modulus.

Fig. 290.



| | Inertia. |
|---|---|
| Two 12-inch [ s ] 20½ pounds at 128.. | 256 |
| $\frac{bd^3}{12}$ for flats neglected............... | —— |
| Area flats $\times h^2 = (2 \times 12 \times \frac{1}{4}) \times 6.125^2$... | 224 |
| Inertia | 480 |

$$\text{Section modulus} = \frac{I}{e} = \frac{480}{6.25} = 76.7$$

This section could have carried 36,000 pounds instead of the 21,000 pounds carried by the other section.

## Design of a Twenty-Ton Crane.

We will now proceed to design a 20-ton crane, using the principles previously laid down.

Span 60 feet. Girder to be 4 feet deep.

Fiber stress due to direct loading not to exceed 12,000 pounds per square inch. This must be properly reduced for compression pieces.

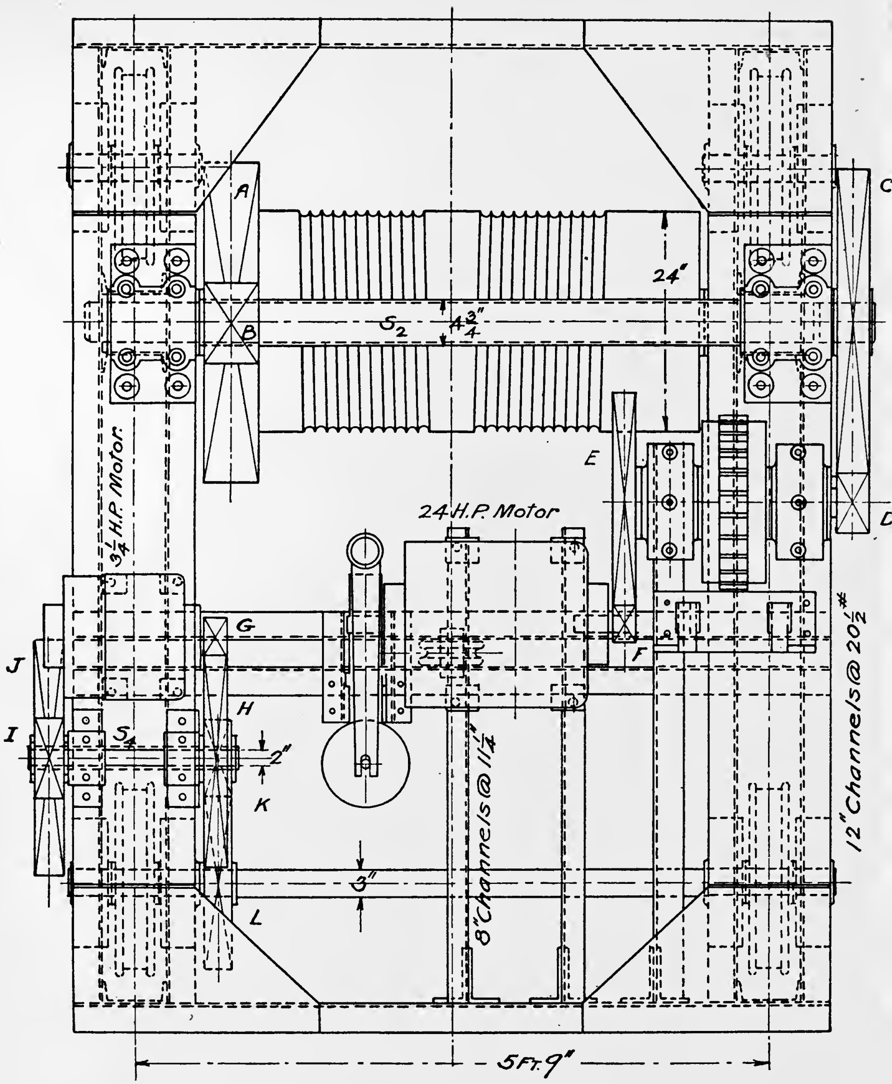
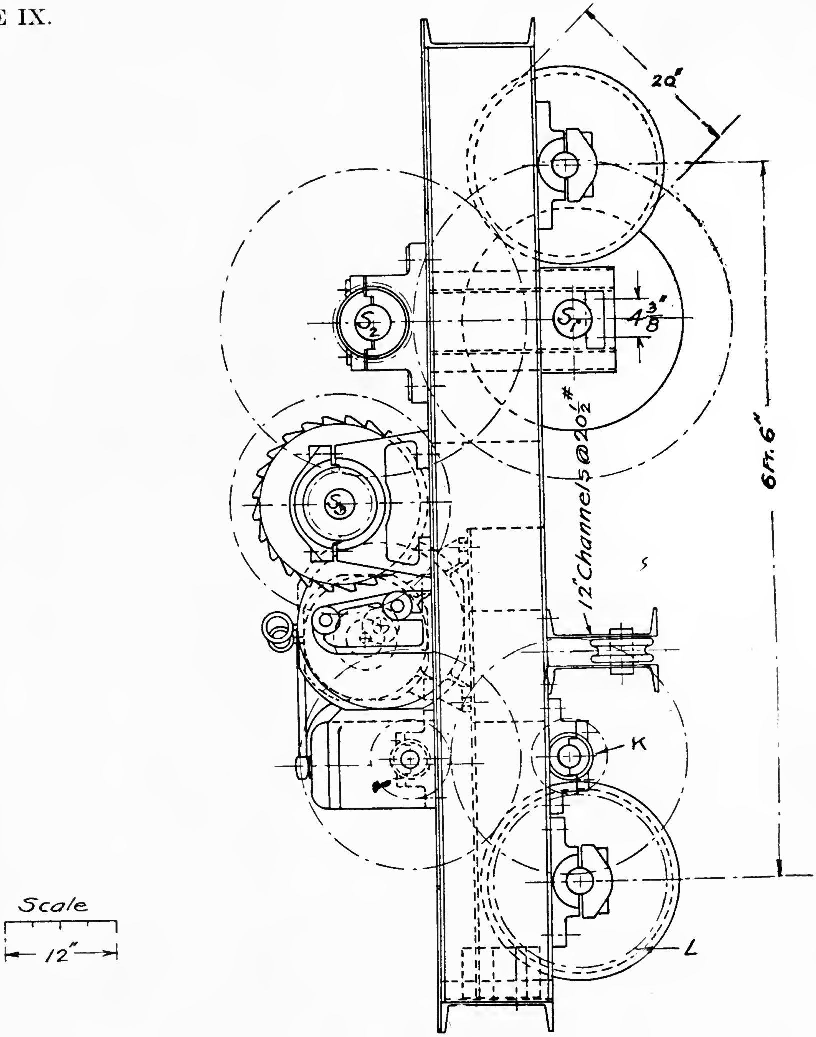

A
B
C
D
E
F
G
H
I
J
K
L
$S_2$
$4\frac{3}{4}''$
24"
24 H.P. Motor
$3\frac{1}{4}$ H.P. Motor
$S_4$
2"
3"
8" Channels @ $11\frac{1}{4}$
12" Channels @ $20\frac{1}{2}$ #
5 Ft. 9"

PLATE IX.




TROLLEY FOR
20 TON O.E.T CRANE.

| | Feet per minute. |
|---|---|
| Hoisting speed, full load, | 12 |
| Bridge travel, full load | 250 |
| Trolley travel, full load | 100 |

The bridge girders to be of the fish-belly type, of box girders. Their flange width must not be under 1/40 of the span. The truck for the bridge, and the trolley frame are to be made of structural material.

Series motors designed for 220 volts, direct current, will be used.

The following weights will be assumed (see p. 294):

Weight of trolley, 12,500 pounds.

Weight of bridge, 37,000.

Maximum wheel load—

$$\frac{W_B}{4}+\frac{(W_T+L)}{2}=\text{maximum wheel load.}$$

$$\frac{37,000}{4}+\frac{(12,500+40,000)}{2}=35,500 \text{ pounds.}$$

### Selection of Motors.

Hoisting.—Horse-power of hoisting motor =

$$\frac{\text{Load in tons}\times\text{hoisting speed feet per minute}}{10}.$$

$$\text{Horse-power}=\frac{20\times 12}{10}=24\text{-horse-power.}$$

Selecting a No. 6 type K 220-volt motor, developing 24 horse-power at 480 revolutions per minute and using 110 amperes at this loading. The motor characteristics are shown by the accompanying figure.

This motor when carrying the maximum load will rise 40° C. in temperature after running continuously for 13 minutes. Considering how intermittent the service is, this should prove satisfactory.

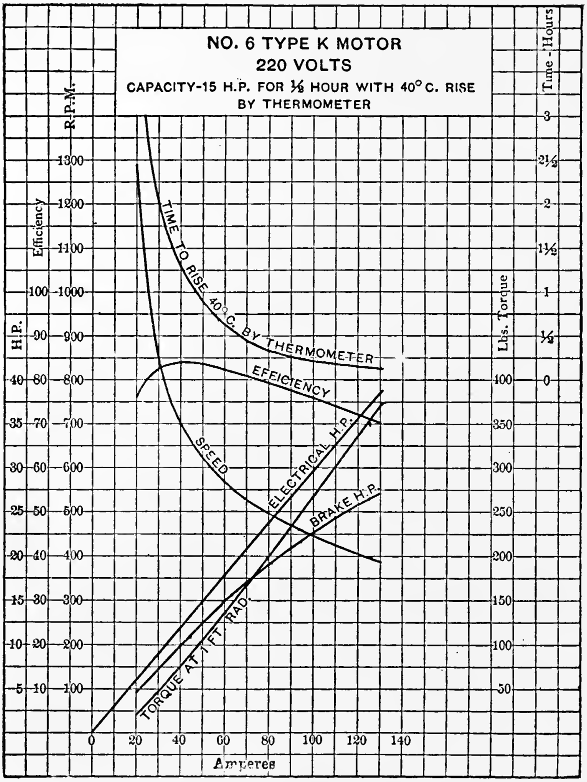
Trolley Travel.—Minimum diameter of chilled wheels for trolley. Rails are steel.

$$W=500.b.D$$

Using 40-pound rails, whose width of head is 1⅞ inches. W will be assumed as ¼ the combined weight of load and trolley.

Hence $W = \frac{12{,}500 + 40{,}000}{4} = 13{,}125$ pounds.

$$D = \frac{13{,}125}{500 \times 1.5} = 17\tfrac{1}{2} \text{ inches.}$$




We will use 20-inch diameter wheels.

Assuming the diameter of the axle journal as 3 inches and substituting in formula on p. 83, we have,

$$H.\ P. = 1\tfrac{1}{4}\left[\frac{W_T + L)\ (f + \mu r) \times S}{R \times 33{,}000}\right]$$

$$H.\ P. = 1\tfrac{1}{4}\left[\frac{(12{,}500 + 40{,}000)\ [.002 + (0.08 \times 1.5)] \times 100}{10 \times 33{,}000}\right]$$

$$H.\ P. = \tfrac{5}{4} \times \frac{52{,}500 \times 0.122 \times 100}{330{,}000} = 2.42 \backsim 3\tfrac{1}{4}\ H.\ P.$$

A motor of the same type as that chosen for the hoisting will be used of 3¼ horse-power at 820 revolutions per minute.

BRIDGE TRAVEL.—In a similar way the maximum wheel load having been estimated at 35,500 pounds and assuming a 60 pound rail whose head width is 2⅜ inches, the minimum diameter of the wheels should be,

$$D=\frac{W}{900\times b}=\frac{35,500}{900\times 2}=19.7 \text{ inches.}$$

Hence the wheel diameter should exceed 20 inches. We will make it 24 inches. Assuming the diameter of the axle journals as 4 inches, the horse-power to move the bridge will be,

$$\text{H. P.}=1.5\,\frac{(W_B+W_T+L)\,(f+\mu r)\,S}{R\times 33,000}$$

$$=1.5\,\frac{(39,000+12,500+40,000)\,[.002+(0.08\times 2)]\,250}{12\times 33,000}=13.8.$$

Use the same type motor as for hoisting, but No. 5K, developing 13.8 bridge horse-power at 600 revolutions per minute.

HOISTING DRUMS, GEARS, SHAFTING, ETC.—Limiting the force acting at the circumference of the drum to about 20,000 pounds, it will be necessary to carry the load upon 4 ropes.

$$\text{Load per rope}=\frac{20\times 2000}{4}=10,000 \text{ pounds.}$$

Now assume some drum and rope diameter and determine the fiber stress in the rope. Trying a 24-inch pitch diameter drum and a ⅞ inch plow steel rope, 6 strands of 37 wires each. From the formula p. 57 we have

$$f_T=\left(\frac{S}{i\times\frac{\pi\delta^2}{4}}\right)+\left(\tfrac{3}{8}\,\frac{E\times\delta}{D}\right).$$

The approximate diameter of the individual wires in this rope is $\frac{0.875}{21}=0.0416$ inch.

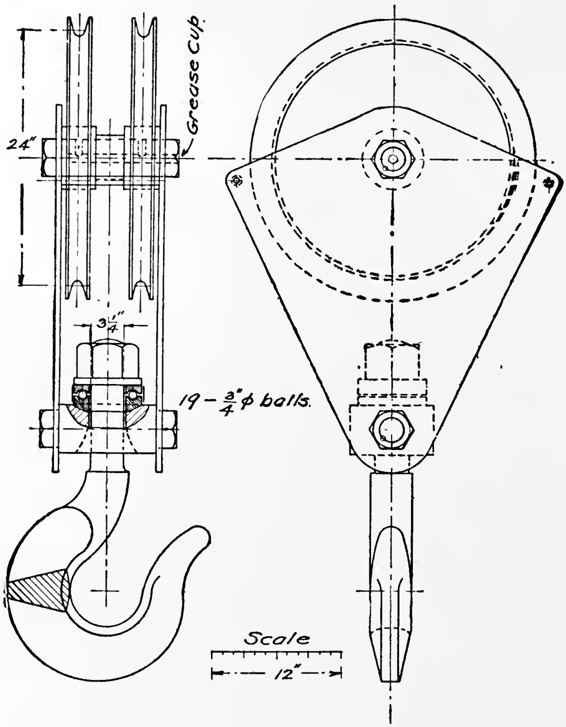
$$f_T=\left(\frac{10,000}{222\times\frac{3.14\times 0.0416^2}{4}}\right)+\left(\tfrac{3}{8}\,\frac{30,000,000\times 0.416}{24}\right)$$

$$=33,000+19,500=52,500 \text{ pounds.}$$

The ultimate strength of plow steel being 220,000 pounds per square inch, the factor of safety is $\frac{220,000}{52,500}=4.20$; this is satisfactory.

The minimum width of drum will depend upon the length of lift. Taking this at 30 feet. The length of rope wound upon each side of the drum will be $2\times 30=60$ feet.

FIG. 291.




LOWER BLOCK and HOOK.

The turns of rope on the drum $\frac{60}{2\times 3.14}=9.55 \backsim 10$.

Allowing 12 turns at 1⅛ inches would take 13½ inches, so that if the two ropes wound on the drum within 9 inches of each other, the total width would be $(2\times 13\frac{1}{2})+9=36$ inches.

The reduction motor to drum. Finding first the revolutions per minute of the drum.

Revolutions per minute of drum =

$$\frac{\text{Velocity of load} \times \text{Reduction in tackle}}{3.14 \times \text{Pitch diameter of drum (feet)}}.$$

Velocity of load in feet per minute.

Revolution per minute of drum $\frac{12 \times 2}{3.14 \times 2} = 3.82.$

The speed of the hoisting motor is 480 revolutions per minute at 24 horse-power.

The reduction motor to drum 480/3.82 = 126.

Three reductions will be necessary, so we will try 1–4 to 1, and 1–5.25 : 1 and 1–6 : 1, making the total 126 to 1.

Drum Gear.—As the drum gear is a slow-running gear we will use 13 teeth in the pinion, making 52 teeth in the gear. The gears will be steel castings. The velocity being very low, about 30 feet per minute, the allowable fiber stress taken for 100 feet per minute is,

$$S = 14{,}000\left(\frac{600}{600+V}\right) = 14{,}000\left(\frac{600}{600+100}\right) = 12{,}000 \text{ pounds.}$$

The pull on the drum neglecting friction is 20,000 pounds, there will be 2 efficiencies of about 97 per cent. each, so that pull on the drum $= \frac{20{,}000}{0.97^2} = 21{,}300$ pounds.

The twisting moment on the drum,

$$Mt = 21{,}300 \times 12 = 255{,}600 \text{ inch-pounds.}$$

Using the formula for the pitch of the smallest gear having 52 teeth, we have

$$p = \sqrt[3]{\frac{6.28 \times Mt}{n.s.c.\left(0.124 - \frac{0.684}{n}\right)}} = \sqrt[3]{\frac{6.28 \times 255{,}600}{52 \times 12{,}000 \times 3.0 \times 0.111}}$$

$$= 1.98 \text{ inches center pitch.}$$

This corresponds to $\frac{\pi}{1.98} = \frac{3.14}{1.98} = 1.59$ diameter pitch.

The pitch diameter of the pinion $\frac{13}{1.50} = 8.67$ inches.

The pitch diameter of the gear $\frac{52}{1.50} = 34.67$ inches.

Width of face of gear $2.0 \times 3 = 6$ inches.

The pinion will be rolled steel and if necessary it can be strengthened by using a face slightly wider than that of the gear. The required width can be found from Lewis's formula,

$$W = s.p.f.\left(0.124 - \frac{0.684}{n}\right)$$

$$W_1 \text{ force on teeth } = \frac{255{,}600 \times 2}{34.67} = 14{,}700 \text{ pounds.}$$

$$\text{Allowable stress on steel } S = 20{,}000\left(\frac{600}{600+V}\right).$$

$$S = 20{,}000 \times \frac{600}{700} = 17{,}100 \text{ pounds per square inch.}$$

From the above formula

$$f = \frac{W}{s.p.\left(0.124 - \frac{0.684}{n}\right)} = \frac{14{,}700}{17{,}100 \times 2.1\left(0.124 - \frac{0.684}{13}\right)}.$$

$f = 5.8$ inches.

This being less than 6 inches, the width of the gear face, both will be made the same, 6 inches.

The maximum twisting moment on the second shaft is

$$M_t = \frac{255{,}600}{4 \times 0.92} = 69{,}500 \text{ inch-pounds.}$$

The second pinion can have 16 teeth, so that this gear will have 84 teeth. The pitch velocity of the pinion, assuming a 6-inch pinion, would be

$$\frac{480}{6} \times \frac{6 \times 3.14}{12} = 126 \text{ feet} \backsim 150.$$

The allowable stress for steel casting is

$$S = 14{,}000\left(\frac{600}{600+V}\right) = 14{,}000\left(\frac{600}{600+150}\right) = 11{,}200 \text{ pounds.}$$

Again using the formula for the minimum gear, the pitch is

$$p = \sqrt[3]{\frac{6.28 \times M_t}{n.s.c.\left(0.124 - \frac{0.684}{n}\right)}} = \sqrt[3]{\frac{6.28 \times 69{,}500}{84 \times 11{,}200 \times 3 \times 0.116}}$$

$$= 1.10 \text{ inches.}$$

$$\text{Diametral pitch } \frac{3.14}{1.10} = 2.85, \text{ use } 2\tfrac{1}{2} \text{ diametral pitch.}$$

The pitch diameter of the gear is $\frac{84}{2.5}=33.6$ inches; face $1.10\times 3 \backsim 3\frac{1}{2}$ inches.

The pitch diameter of the pinion is $\frac{16}{2.5}=6.4$ inches; face $\backsim 3\frac{1}{2}$ inches.

Motor Pinion.—The velocity will be approximately 600 feet per minute. The pinion will be cut from a steel round, a rolled section.

Allowable stress $20{,}000\left(\frac{600}{600+V}\right)=10{,}000$ pounds.

The gear will be a steel casting.

Allowable stress $14{,}000\left(\frac{600}{600+V}\right)=7000$ pounds.

Reduction 6 to 1.

The twisting moment on the gear shaft $=\frac{69{,}500}{5.25\times .92}=14{,}400$ pounds.

The twisting moment on the motor shaft $=$

$$\frac{14{,}400}{6}\times 0.93=2560 \text{ inch-pounds.}$$

We must now determine the pitch required by the gear, then by the pinion, and use whichever is the larger.

Using the formula for the smallest pinion or gear, the pinion being assumed as having 16 teeth, we find,

$$\text{Gear } p=\sqrt[3]{\frac{6.28\, M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}=\sqrt[3]{\frac{6.28\times 14{,}400}{96\times 7000\times 2.5\times 0.117}}=0.773 \text{ inch.}$$

$$\text{Pinion } p=\sqrt[3]{\frac{6.28\, M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}$$

$$=\sqrt[3]{\frac{6.28\times 2560}{16\times 10{,}000\times 2.5\times 0.081}}=0.792 \text{ inch.}$$

Hence the steel pinion is the weaker tooth and will require 4 diametral pitch.

Pinion, pitch diameter 4-inch face 2.5 inches.

Gear, pitch diameter 24-inch face 2.5 inches.

The method used makes no provision for the acceleration forces due to the motor excepting so far as they may be covered by the safety factors. Some designers estimate the efficiency of the hoisting train, increase the live load by this efficiency, and then determine the stresses upon the other gears with this force as a basis, but neglect efficiencies between drum and motor. In this problem the efficiency being calculated at about 70 per cent., the load upon the ropes would be,

$$\frac{40,000}{4\times .70}=14,300 \text{ inch-pounds.}$$

The twisting moment on the drum,

$$M_t=2\times 14,300\times 12=343,200 \text{ inch-pounds.}$$

While the twisting moment on the first pinion shaft is,

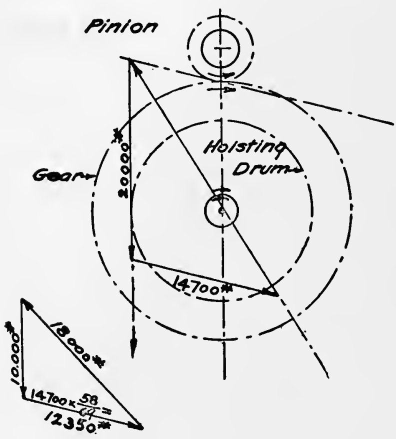
$$M_t=\frac{343,000}{4}=85,750 \text{ inch-pounds.}$$

### Hoisting Machinery (Shafts).

See drawing of Trolley. (Plate IX.)

SHAFT $S_1$.—As the main gear is attached to the drum, the shaft $S_1$ will not be subjected to torsion; it will, however, have to resist the bending due to the load and gear and will have to afford ample surface for bearing.

FIG. 292.




The reaction at the bearing near the drum gear will be due to the 20,000-pound rope pull on the drum and the 14,700-pound force acting on the gear. Taking moments about the other bearing, the vertical reaction is 10,000 pounds and the reaction due to both forces is $14,700\times\frac{58}{69}=12,350$.

The resultant of these 2 forces is 18,00C pounds, found by the triangle in Fig. 292.

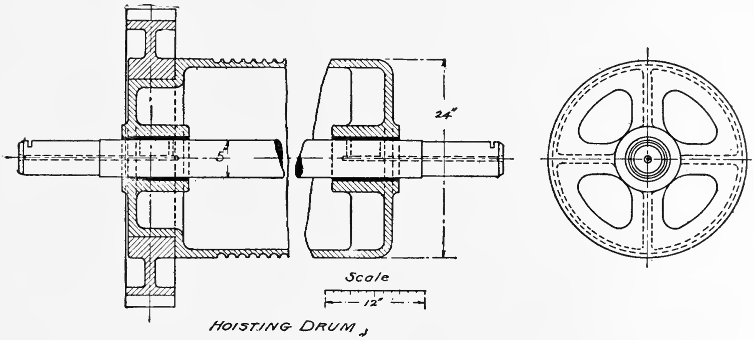
The diameter of the shaft is given by $d=\sqrt[3]{\frac{10.2\times M_B}{f_B}}$

$$M=18,000\times 8=144,000 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10\times 2\times 144{,}000}{12{,}000}}=4.96 \text{ inches} \backsim 5 \text{ inches.}$$

The speed of this shaft being very low, the pressure on the bearing will be the maximum permitted on the materials, as for steel on bronze (see p. 46), say 800 pounds per square inch.

FIG. 293.




$$\text{Projected area of bearing}=\frac{18{,}000}{800}=22.5 \text{ square inches.}$$

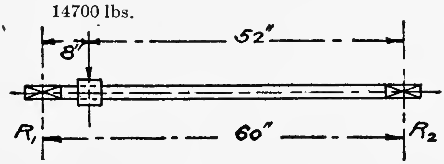
$$\text{Length of bearing}=\frac{\text{area}}{\text{diameter}} \backsim \frac{22.5}{5}=4.5.$$

Hence the length of the bearing should equal or exceed $4\frac{1}{2}$ inches.

SHAFT $S_2$.—This shaft is subjected to both bending and twisting, so we must first find the equivalent bending moment. The maximum forces will be those near the pinion.

The force at the pitch line is 14,700 pounds.

The shaft is shown in the sketch. The reaction $R_1$ is,




$$R_1=\frac{14{,}700\times 52}{60}=12{,}700$$

pounds.

$$M=12{,}700\times 8=101{,}600 \text{ inch-pounds.}$$

$$M_{E.B.}=0.35M_B+0.65\sqrt{M_B^2+M_T^2}.$$

$$=(0.35\times 101{,}600)+0.65\sqrt{101{,}600^2+69{,}500^2}.$$

$$=35{,}500+80{,}000=115{,}500 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10.2\times M_{E.B.}}{f_B}}=\sqrt[3]{\frac{10.2\times 115{,}500}{12{,}000}}=4.62 \text{ inches} \backsim 4\tfrac{3}{4} \text{ inches,}$$

Limiting the bearing pressure to 400 pounds per square inch, the length of the bearing should be, if the shaft diameter is made 4 inches at the bearing, $\frac{13,750}{400\times4}=8.6$ inches for babbitted bearing.

SHAFT $S_3$.—The twisting moment previously found for this shaft is 14,400 inch-pounds.

$$\text{Force on pinion face}=\frac{\text{Twisting moment}}{\text{Pinion radius}}=\frac{14,400}{3.2}$$
$$=4500 \text{ pounds.}$$

Estimated bending moment $=4500\times6=27,000$ inch-pounds.

Equivalent bending moment $=$

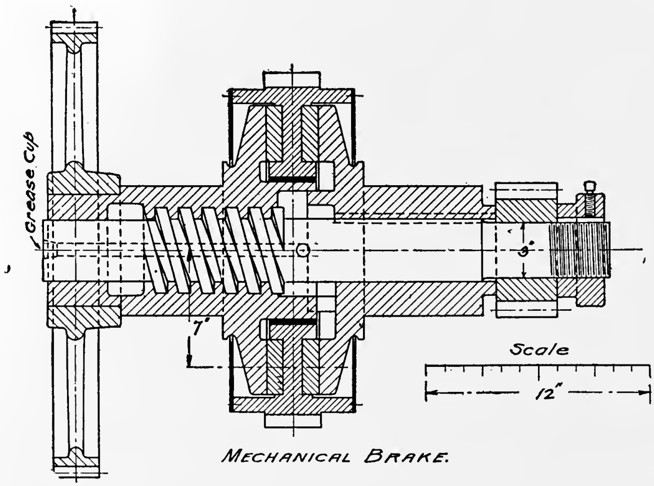
$$(0.35\times27,000)+0.65\sqrt{27,000^2+14,400^2}.$$

$M_{E.B.}=29,400$ inch-pounds.

$$d=\sqrt[3]{\frac{10.2\times M_{E.B.}}{f_B}}=\sqrt[3]{\frac{10.2\times29,400}{10,000}}=3.11 \text{ inches.}$$

The mechanical brake goes on this shaft or on an extension to it. Use a simple type of plate brake (see p. 176).

FIG. 294.




MECHANICAL BRAKE.

The twisting moment on this shaft is 14,400 inch-pounds.
The force acting at the pinion teeth is $P=4500$ pounds.

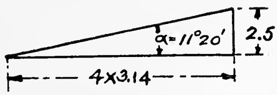
We will try the following dimensions:
$r = 2$ inches; $r_1 = 1\frac{1}{2}$ inches; $\mu_1 = 0.05$; $\mu_2 = 0.10$; $Pa - \mu r_1 P \leqq 2\mu_1 KR$

$$14{,}400 - (0.05 \times 1.5 \times 4500) \leqq 2 \times 0.10 \times K \times 7.$$

$$K \geqq \frac{14{,}063}{1.4} = 10{,}050 \text{ pounds.}$$

Also since $r$ tang $(\alpha + \rho) \leqq \mu_1 R$ we have by trying a double threaded screw, pitch 1.25 inches or $2\frac{1}{2}$ inch lead.

$$\text{tang}\,\alpha = \frac{2.5}{4 \times 3.14} = 0.20 : \alpha = 11° \, 20'.$$




Assuming $\mu = 3°$, we have

$$r \text{ tang } (\alpha + \rho) \leqq \mu_1 R \;\therefore\; 2 \times \text{tang } (11° \, 20' + 3° \, 0') \leqq \tfrac{1}{10} \times R.$$

$$2 \times 0.256 \leqq \frac{R}{10} \;\therefore\; R \geqq 5.12 \text{ inches.}$$

Hence 7 inches is more than ample.

Allowing 3000 foot-pounds of work absorbed per minute per square inch of friction surface would call for the following surface:

Work to be absorbed $40{,}000 \times 12 \times .90 \times .92 = 397{,}000$ foot-pounds.

Surface required $\dfrac{397{,}000}{2 \times 3000} = 66.3$ square inches.

Assuming the outer radius $8\frac{1}{2}$ inches and the inner radius $5\frac{1}{2}$ inches, the area of the circular ring is $226 - 95 = 131$ square inches.

The unit pressure is $\dfrac{10{,}050}{131} = 76.3$ pounds per square inch.

## Machinery for Driving Trolley.

The driving motor selected is $3\frac{1}{4}$ horse-power, the torque at 1 foot radius when developing $2\frac{1}{2}$ horse-power at 820 revolutions per minute is 16 pounds. Assuming the maximum torque at 3 times this gives $16 \times 3 = 48$ pounds.

Diameter of driving wheels 20 inches. Revolutions per minute of driving wheels corresponding to a speed of 100 feet per minute is

$$\text{Revolutions per minute} = \frac{100 \times 12}{20 \times 3.14} = 19.1.$$

$$\text{Reduction between wheels and motor } \frac{820}{19.1} = 43$$

Taking the reduction at the motor at 6 to 1, the maximum twisting moment at the motor shaft is $48\times12=576$ inch-pounds. Assuming 16 teeth in the pinion, the pitch of the smallest gear will be given by

$$p=\sqrt[3]{\frac{6.28\ M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}$$

For a cast-iron gear, $s=8000\left(\frac{600}{600+v}\right)$

$$s=8000\left(\frac{600}{600+800}\right)\backsim 3400.$$

Taking $c=3$, we have

$$p=\sqrt[3]{\frac{6.28\times572\times6}{96\times3400\times3\times0.116}}=0.58 \text{ inch} \backsim 4 \text{ diametral pitch, } 2\tfrac{1}{2}\text{-inch face.}$$

| | Pitch Diam. | Diam. Pitch | No. Teeth | Width Face |
|---|---|---|---|---|
| Pinion G...... | 4 inches | 4 | 16 | 2½ inches |
| Gear H ...... | 24 inches | 4 | 96 | 2½ inches |

SECOND SET OF GEARS.—It will be necessary to use large diameter gears here, so that the bearings will fall outside the frame beams. Assuming both pinion and gear of cast-iron, the pinion will be the weaker tooth. Considering a pinion of 25 teeth, the smallest diameter pinion will be given by

$$p=\sqrt[3]{\frac{6.28\times M_T}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}$$

The twisting moment on the pinion shaft is

$$M_T=572\times6\times.92=3160 \text{ inch-pounds.}$$

$$p=\sqrt[3]{\frac{6.28\times3160}{25\times5000\times3\times0.097}}=0.82 \text{ inch} \backsim 3\tfrac{1}{2} \text{ diametral pitch.}$$

Use 3½ diametral pitch with 3-inch face.

| | Pitch Diam. | Diam. Pitch | No. Teeth | Width of Face | |
|---|---|---|---|---|---|
| Pinion I........ | 8.57 | 3½ | 30 | 3 | C.I. |
| Gear J ........ | 25.71 | 3½ | 90 | 3 | C.I. |

The gear on the driving axle will be taken about the same diameter as the driving wheel, 20 inches. Gear ratio $\frac{43}{6\times3}=2.39$.

The force acting on the gear teeth is

$$\frac{\text{Turning moment}\times\text{Gear reduction}\times\text{Efficiency}}{\text{Radius of driving wheel}}=\frac{572\times 43\times .92^2}{10}$$
$$=2080 \text{ pounds.}$$

The gears being cast-iron and having a velocity of 100 feet per minute, the allowable fiber stress is

$$s=8000\left(\frac{600}{600+V}\right)=8000\left(\frac{600}{700}\right)=6850 \text{ pounds.}$$

Using Lewis's formula,

$$W=s.p.f.\left(0.124-\frac{0.684}{n}\right)$$

Taking $n=16$; $\left(0.124-\frac{0.684}{n}\right)=0.081.$

$$W=2080=6850\times p\times 3\times 0.081.$$

$$p=\frac{2080}{6850\times 3\times 0.081}=1.25 \text{ inches, use 2 diametral pitch,}$$

face 3 inches.

The gear ratio desired is 2.39 to 1. Using 38 teeth in gear and 16 teeth in pinion gives $\frac{38}{16}=2.375$, which is close enough.

| | Pitch Diam. *Inches* | | Diam. Pitch | Teeth No. | Width of Face *Inches* | Material |
|---|---|---|---|---|---|---|
| Pinion ........ | 8 | K | 2 | 16 | 3 | C.I. |
| Gear ........ | 19 | L | 2 | 38 | 3 | C.I. |

The shaft nearest the motor. The gears will be placed outside of the bearings and the lever arm assumed as 4 inches.

Twisting moment on shaft $572\times 6\times .92=3160$ inch-pounds.

Force acting on pinion teeth $\frac{3160}{4.29}=740$ pounds.

Bending moment $=740\times 4=2960$ inch-pounds.

Equivalent bending moment is,

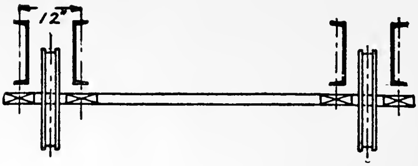
$$M_{E.B.}=0.35\,M_B+0.65\sqrt{M_B^2+M_T^2}$$
$$=(0.35\times 2960)+(0.65\sqrt{2960^2+3160^2}$$
$$=1040+2820=3860 \text{ inch-pounds.}$$

The shaft diameter is given by

$$d=\sqrt[3]{\frac{10.2\times M_B}{f_B}}=\sqrt[3]{\frac{10.2\times 3860}{9000}}=1.64 \text{ inches.}$$

Use a 2-inch diameter shaft, $S_4$.

### Shaft on Driving Wheels.




Twisting moment on shaft,
$M_T = 572 \times 43 \times .92^3$
$= 19{,}150$ inch-pounds.

One-half of this goes to each driving wheel, or

$$M_T = \frac{19{,}150}{2} = 9575 \text{ inch-pounds.}$$

$$\text{Bending moment } M_B = \frac{WL}{8} = \frac{13{,}125 \times 12}{8} = 19{,}690 \text{ inch-pounds.}$$

Equivalent bending moment

$$M_{E.B.} = (0.35 \times M_B) + 0.65\sqrt{M_B^2 + M_T^2}$$
$$= (0.35 \times 19{,}690) + (0.65\sqrt{19{,}690^2 + 9575^2})$$
$$= 6{,}900 + 14{,}200 = 21{,}100 \text{ inch-pounds.}$$

From which the diameter is found to be,

$$d = \sqrt[3]{\frac{10.2 \times M_B}{f_B}} = \sqrt[3]{\frac{10.2 \times 21{,}100}{9000}} = 2.90 \text{ inches} \backsim 3 \text{ inches.}$$

The brake will be the usual solenoid brake, purchased with the motor, and will not be designed here.

INTERMEDIATE SHAFT.—Twisting moment on axle 19,150 inch-pounds. Reduction from axle to intermediate shaft 38 to 16.

Twisting moment on intermediate shaft $\frac{19{,}150}{.92} \times \frac{16}{38} = 8780$ inch-pounds.

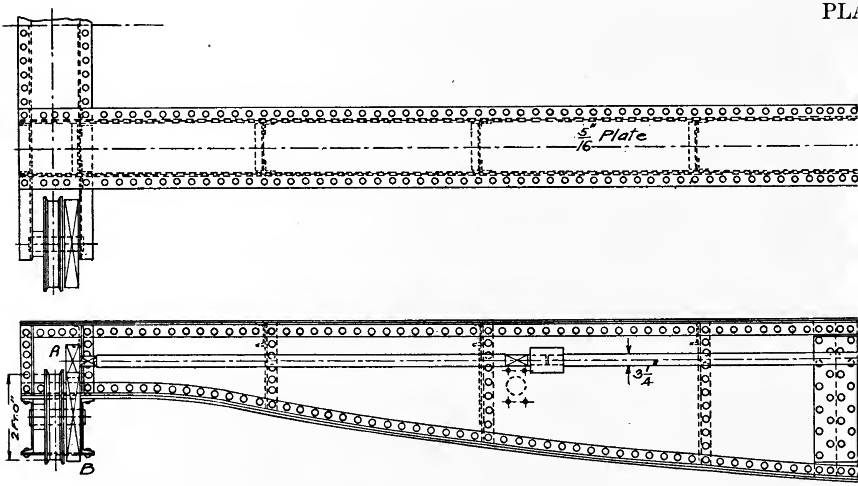
Force on pinion teeth 2080 pounds, $M = 2080 \times 4 = 8320$ inch-pounds.

Selecting shaft diameter from diagram p. 39, corresponding to a bending moment of 8320 inch-pounds and a twisting moment of 8780 inch-pounds, gives a shaft 2⁹⁄₁₆ inches in diameter, with 9000 pounds fiber stress.

### Bridge Girders.

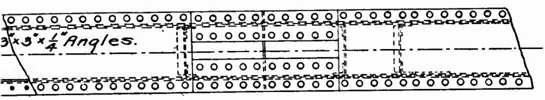
The maximum end shear is practically equal to the maximum wheel load, *i.e.*, 35,500 pounds. The girder will be box type with web plates ¼ inch thick, according to specification, 4 feet deep at the middle and 18 inches at the end.

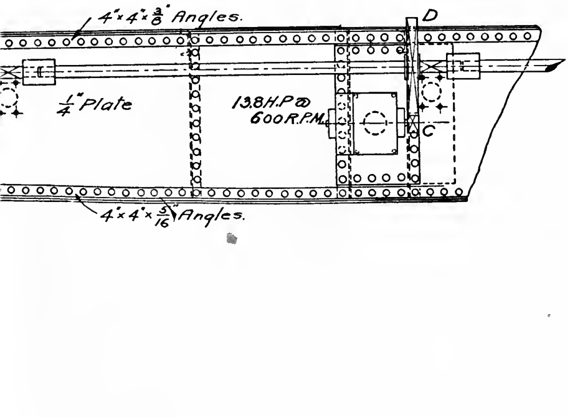
PLA

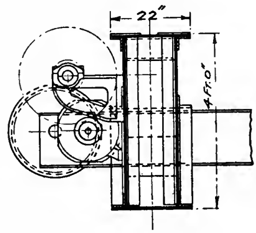


BRIDGE for
20 TON ELECTRIC
TRAVELING CRANE.
BOX GIRDERS SPAN 60 ft.

Scale
12"



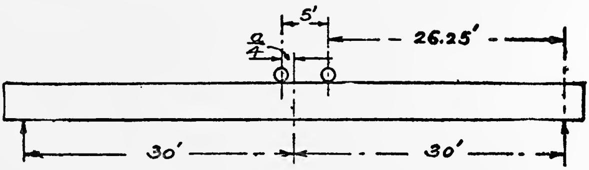
The unit shear at the end is

$$\frac{35,500}{2\times\frac{1}{4}\times18}=3945 \text{ pounds per square inch,}$$

which is sufficiently low.

We will assume the trolley-wheel base as 5 feet, although when we lay it out we may not be able to do quite so well.

The maximum bending will occur when a wheel is the distance $\frac{a}{4}$ from the center of the span. (See p. 11.)




$$M=\frac{W}{8l}\left(l-\frac{a}{2}\right)^2+\frac{Gl}{8}+\frac{\text{Weight of motor}\times l}{4}.$$

$$M=\frac{52,500}{8\times60\times12}\left(720-\frac{60}{2}\right)^2+\frac{37,000\times720}{2\times8}+\frac{1200\times720}{4}.$$

$$M=4,350,000+1,665,000+216,000=6,231,000'' \text{ lbs.}$$

The girders are to be of the fish-belly type, making them approach girders of uniform strength, *i.e.*, the flange force will not decrease towards the supports, but will remain more nearly equal to that acting at the middle of the girder.

Consulting the curves for compression flanges (see p. 111), and remembering that the conditions of this flange are somewhere between curves 1–1 and 4–4, and assuming that the flange width is 22 nches, or about $\frac{1}{33}$ of the span, we will take a compressive stress of about $\frac{3}{4}$ the tensile stress, or

$$\frac{3}{4}\times12,000=9,000 \text{ pounds.}$$

To make an approximation to the flange areas, we will first calculate the areas upon the basis of 10,500 pounds fiber stress per square inch, and afterwards enlarge the upper flange.

$$M=\text{Flange area square inch}\times\text{fiber stress}\times d.$$

Where $d$ is distance $c$–$c$ of flanges. 48 inches − 1 inch = 47 inches.

$$6,231,000=\text{Area}\times10,500\times47$$

$$\text{Area}=\frac{6,231,000}{10,500}\times47=12.65 \text{ square inches.}$$

If the web is assumed as resisting bending, $\frac{1}{8}$ of its section can be considered as flange area.

21

$\frac{1}{8} \times$ Web area $= \frac{1}{8} \times 2 \times \frac{1}{4} \times 48 = 3$ square inches.

The flange area can then be taken as

| | |
|---|---|
| $\frac{1}{8}$ Web area .................. | 3.00 |
| 1 Cover plate $22 \times \frac{1}{4}$ .............. | 5.50 |
| 2 Angles $4 \times 4 \times \frac{5}{16}$ inches at 2.41 | 4.82 |
| Total .................... | 13.32 square inches. |

If a fiber stress of 10,500 pounds per square inch were permissible in both flanges this design would be satisfactory. About the lightest sections desirable have been used in the tension flange, so that it will not be reduced any. In the compression flange the allowable stress is 9000 pounds, and it has been designed for a mean stress of 10,500 pounds per square inch. The maximum stress will slightly exceed this average stress, so that we will assume the average compressive stress to be 8500 pounds per square inch.

$$\frac{\text{Flange area required}}{\text{Flange area determined}} = \frac{10,500}{8,500}$$

From which

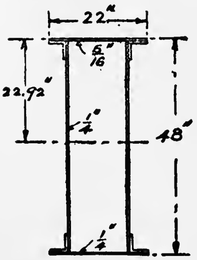
$$\text{The flange area required is} = \frac{10,500}{8,500} \times \text{flange area determined}$$

$$= \frac{10,500}{8,500} \times 12.65 = 15.6 \text{ sq. in.}$$

This will require

| | |
|---|---|
| $\frac{1}{8}$ Web area ................................ | 3.00 |
| 1 Cover plate $22 \times \frac{5}{16}$ ......................... | 6.88 |
| $2\text{-}4 \times 4 \times \frac{3}{8}$ Angles at 2.86 .................. | 5.72 |
| Total .................................. | 15.60 |

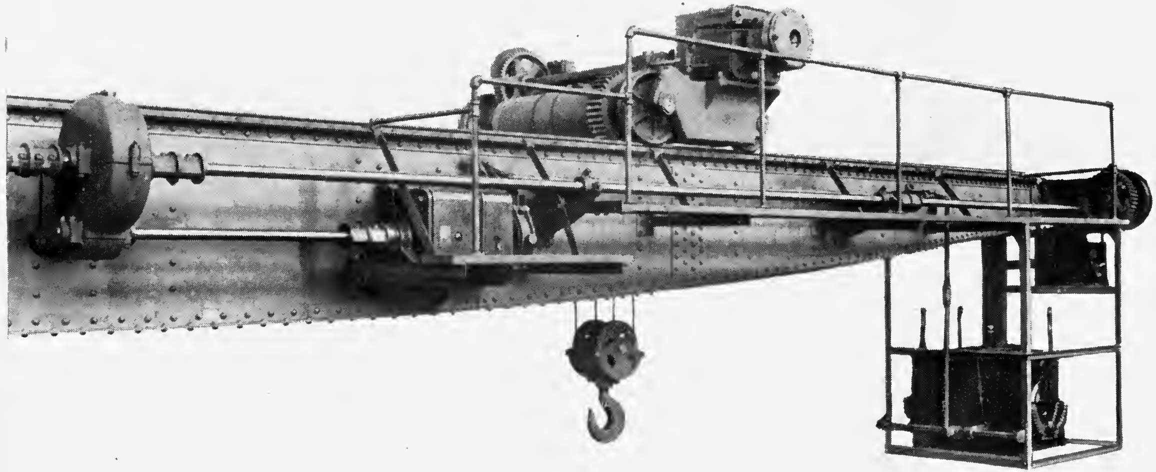
If desired this section can now be checked by means of the moment of inertia.




CENTER OF GRAVITY

| | Area sq. in. | | Mom. |
|---|---|---|---|
| $22 \times \frac{5}{16}''$ plate ......... | $6.87 \times$ | $0.156 =$ | 1.70 |
| Upper angles $4 \times 4 \times \frac{3}{8}''$ | $5.72 \times$ | $1.45 =$ | 8.28 |
| 48″ Web plates ........ | $23.50 \times$ | $24.00 =$ | 564.00 |
| Lower angles $4 \times 4 \times \frac{5}{16}''$ | $4.82 \times$ | $46.63 =$ | 224.76 |
| $22 \times \frac{1}{4}''$ plate .......... | $5.50 \times$ | $47.88 =$ | 263.33 |
| Total ..... | 46.41 | | 1061.44 |

FIG. 295



BRIDGE.

Deducting for rivet holes

$$0.68\times1.75=1.19\times\ 0.34=\ 0.40$$
$$0.56\times1.75=0.98\times47.72=46.80$$

| | | | | |
|---|---|---|---|---|
| Total. . | 2.17 | 47.20 | 2.17 | 47.20 |
| Net . . . . . . . . . . . . . . . . . . . . . | | | 44.24 | 1014.24 |

$$x=\frac{1014.24}{44.24}=22.92 \text{ inches.}$$

MOMENT OF INERTIA:

| | | |
|---|---|---|
| Upper flange plate | $\frac{bd^3}{12}$ (negligible) | |
| | $Ah^2\,6.87\times(22.92-0.16)$ . . . . . . . . . | 3559.00 |
| Flange angles | 2 angles at 4.36 . . . . . . . . . . . . . . . . . . . | 8.72 |
| | $Ah^2=5.72\ (22.92-1.45)^2$ . . . . . . . . . . . . | 2637.00 |
| Web plates | $ld^3\div12\ \frac{2\times.25\times47^3}{12}$ . . . . . . . . . . . . . . . . . . | 4326.00 |
| | $Ah^2=0.5\times47\times1.08^2$ . . . . . . . . . . . . . . . . . . | 27.40 |
| Lower flange angles | 2 angles . . . . . . . . . . . . . . . . . . . . | 7.42 |
| | $Ah^2=4.82\times23.71^2$ . . . . . . . . . . . . | 2710.00 |
| Lower flange plate | $bd^3\div12$ (negligible) | |
| | $Ah^2=0.25\times22\times24.96^2$ . . . . . . . . . | 3426.00 |
| | | I = 16,701.54 |

From this must be deducted the inertia representing rivet hole areas.

Rivet holes in (deducting the $Ah^2$ value only):

Upper flange $(2\times0.875\times0.68)\times22.58^2=607.00$

Lower flange $(2\times0.875\times0.56)\times24.80^2=602.00$

1,209.00

As the web-stiffener rivets will not come in line with these rivets, no allowance will be made for them.

$$I=16,701-1,209=15,492$$

The extreme fiber stress in compression then will be

$$M=p.\frac{I}{e}. \qquad p_c=\frac{Me}{I}.$$

$$p_c=\frac{6,231,000\times22.92}{15,496}=9200 \text{ pounds.}$$

This will be close enough to the desired amount.

In a similar way the extreme fiber stress in tension is

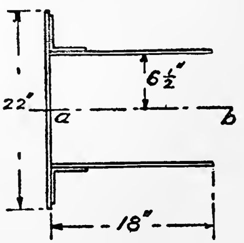
$$p_t = \frac{6,231,000 \times 25.08}{15,496} = 10,400 \text{ pounds.}$$

Before this section is finally adopted, it should be examined for the lateral bending that may come upon it due to suddenly stopping the fully loaded crane. If the brake operating upon the bridge driving shaft can exert a retarding force sufficient to stop the crane in 5 feet, the crane can be checked as rapidly as is advisable.

WEIGHTS IN POUNDS.

| | |
|---|---|
| Bridge | 37,000 |
| Trolley | 12,500 |
| Motor | 1,200 |
| Live load | 40,000 |
| Total | 90,700 |

The moment of inertia of the upper flange must now be found.




Inertia about axis $ab$

$$\frac{bd^3}{12} = \frac{0.31 \times 22^3}{12} = 275.00$$

$$\left\{ \begin{array}{ll} 2 \text{ angles } 4 \times 4 \times \tfrac{3}{8}'' & = 8.72 \\ Ah^2 \; 5.72 \times 7.64^2 & = 334.00 \end{array} \right.$$

$$2 \text{ plates } Ah^2 \; 18 \times \tfrac{1}{2} \times 6.38^2 = 366.00$$

$$\text{Total } I = 983.72$$

As the assumed width 18 inches is a mere assumption, no account will be made of rivet holes.

When the crane is fully loaded lateral bending stresses will be produced proportionally to their respective weights. The material of the girders will produce stresses in both flanges, while the trolley and load will be assumed as producing lateral stress in the upper flange only.

Kinetic energy of bridge $\frac{Wv^2}{2g} = \frac{37,000}{64.4} \times \left(\frac{250}{60}\right)^2 = 9980$ ft.-lbs.

Kinetic energy of trolley and live load $\frac{Wv^2}{2g} = \frac{53,700}{64.4} \times \left(\frac{250}{60}\right)^2 =$ 14,500 ft.-lbs.

Retarding force one girder bridge $\frac{9980}{2 \times 2 \times 5} = 499$ pounds.

Retarding force one girder, trolley and load,

$$\frac{14{,}500}{2\times2\times5}=725 \text{ pounds.}$$

$$M=\frac{W_1L}{8}+\frac{W_2L}{4}=\frac{2\times499\times60\times12}{2\times8}+\frac{2\times725\times60\times12}{4}$$

$$=306{,}000 \text{ inch-pounds.}$$

Since $M=\frac{pI}{e}$ $\therefore$ $p=\frac{Me}{I}$.

$$p=\frac{306{,}000\times11}{983.7}=3420 \text{ pounds.}$$

It was previously estimated that with a fiber stress of 9000 pounds in the upper flange this might be increased to 12,000 pounds by column action, and the extreme fibers will now have this further increased by this 3420 pounds.

This maximum fiber stress occurs only in the corners of the upper flange and is 12,000+3420=15,420 pounds.

This should not be too high, considering that it is the worst condition possible and that the stresses have been fully determined.

Some designers assume that each girder must carry, in addition to the vertical loading, a horizontal load of 1/10 the live load capacity of the crane. This is assumed as acting upon the upper flange.

## Bridge Girder Flange Riveting.

The girder (Fig. 296) is drawn approximately for uniform strength by determining $d$ in the formula on page 147 for the several sections. The moments are given by Figs. 152 and 153. Draw the diagram of maximum shears (Fig. 297), accounting for both dead and live loads (see p. 144).

From p. 152, we have $S=\sqrt{S_1^2+S_2^2}$.

$$S_1=\frac{13{,}125}{36}=365 \text{ pounds.}$$

$$S_2=\frac{V}{H}=\frac{30{,}300}{21.75-(2\times0.875)}=\frac{30{,}300}{20}=1515 \text{ pounds.}$$

$$S=\sqrt{365^2+1515^2}=1560 \text{ pounds.}$$

In a box girder the web plate being double, ½ this amount will come upon the rivets in each web plate. Using ¾-inch rivets in ¼-inch web plates, single shear, and taking a shearing value of

$\frac{3}{4} \times 11{,}000 = 8250$ pounds per square inch, and a bearing value of 16,500 pounds per square inch, we have the shearing value (single shear) of a ¾-inch rivet is 3650 pounds and the bearing value in a ¼-inch plate is 3100 pounds.

$$p = \frac{\text{Rivet value}}{\text{S}} = \frac{3100}{780} = 3.97 \text{ inches} \backsim 4 \text{ inches.}$$

Fig. 296.

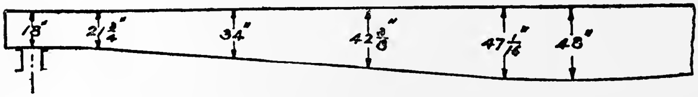
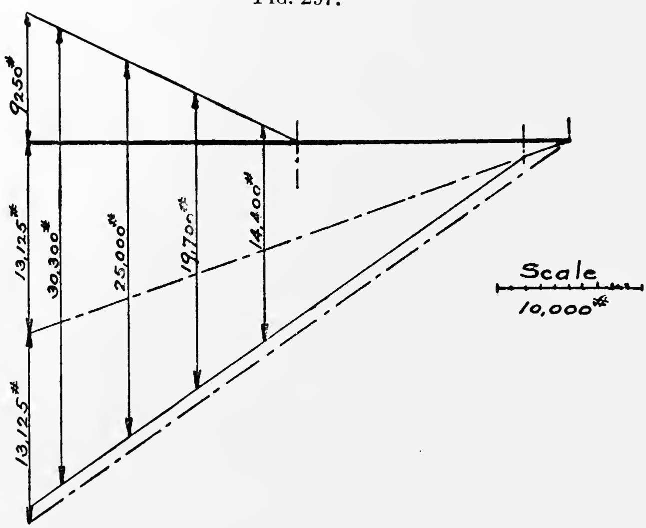



Fig. 297.




As the rivet spacing should not exceed 16 times the thickness of the outside plate, this rivet spacing of 4 inches should be used entirely across the girders, excepting at the extreme ends, where the rivets must be spaced closer to provide properly for the maximum end shear concentration at the supports. Note.—This method of determining the rivet spacing is used because the girder is only approximately one of uniform strength.

Stiffening Angles.—As previously explained, no very satisfactory method can be given for calculating stiffeners. 3 inches

$\times 3$ inches$\times\frac{1}{4}$-inch angles not exceeding 48 inches long, secured so they can only fail about an axis parallel to one leg, will carry

$$p_1 = \frac{p}{1+\frac{1}{1000}\times\left(\frac{l}{r}\right)^2} \text{ pounds per square inch.}$$

$$\text{This gives } p_1 = \frac{12{,}000}{1+\frac{1}{10{,}000}\times\left(\frac{48}{0.93}\right)^2} = 9400 \text{ pounds.}$$

Two such angles will carry $2\times 1.44\times 9400 = 27{,}100$ pounds.

This is almost equal to the maximum end shear and should be ample for the intermediate stiffeners. At the ends additional stiffeners are used in accordance with usual practice.

## Gearing and Shafting for Bridge Drive.

Driving wheels 24-inch diameter. Travel 250 feet per minute.

Revolution per minute of driving wheels =

$$\frac{\text{Travel in feet per minute}}{\text{Circumference of driving wheel in feet}}$$

$$\text{Revolutions per minute} = \frac{250}{2\times 3.14} = 39.8 \sim 40.$$

From the motor characteristic curves the speed of the motor when developing 13.8 horse-power is 600 revolutions per minute. The reduction motor to driving wheel is $\frac{600}{40} = 15:1$. Making the reduction at the motor $6:1$, the reduction required at the driving wheels is $\frac{15}{6} = 2.5:1$.

The torque from the motor delivering 13.8 horse-power at 600 revolutions per minute is 120 pounds at 1-foot radius; the current is 60 amperes.

The maximum stress upon the bridge driving mechanism may occur either from the driving motor or the brake. As a maximum torque from the motor assumed 3 times the torque corresponding to the normal running.

In this case $3\times 120 = 360$ foot-pounds at the motor shaft.

The torque measured at the main driving pinions will be found, assuming that the pitch diameter of the gear equals the diameter of the driving wheels, by

$$\frac{360\times 2\times 3.14\times 600\times .92\times .90}{250} = 4500\text{-pounds.}$$

The torque due to the brake measured at the same pinions will be found as follows, upon the assumption of stopping the crane under full load in 5 feet:

Kinetic energy in crane when fully loaded and travelling at full speed $=\frac{Wv^2}{2g}$.

$$\frac{Wv^2}{2g}=\frac{90{,}700\times\left(\frac{250}{60}\right)^2}{2\times 32.2}=24{,}400 \text{ foot-pounds.}$$

If the crane is stopped in 5 feet by being uniformly retarded, the force acting at the wheels to do this will be

$$\frac{24{,}400}{5}=4880 \text{ pounds.}$$

From this must be deducted the frictional work of the crane, which, in order to be on the safe side, will be taken low, say 30 pounds per ton or a total of $45\times30=1350$ pounds measured at the driving wheels.

The force that must be exerted at the wheels by the brake then is

$$4880-1350=3530 \text{ pounds.}$$

It is evident that the gear must be designed for the former. The twisting moment on this main shaft is

$$M_t=\frac{4500\times 12}{2.5}=23{,}500 \text{ inch-pounds.}$$

This twisting moment will be taken by each end proportionally to the loads at the ends. The maximum wheel load $=35{,}500$ pounds; the reaction at this end of the bridge, 71,000 pounds. Total weight of crane and live load $=90{,}700$ pounds. Reaction at light end, 19,700 pounds. Twisting moment on pinion at end with maximum load,

$$23{,}500\times\frac{71{,}000}{90{,}700}=18{,}400 \text{ inch-pounds.}$$

The allowable fiber stress, using a steel casting, will be, for a velocity of 250 feet per minute,

$$S=14{,}000\left(\frac{600}{600+v}\right)=14{,}000\times\frac{600}{850}=10{,}000 \text{ pounds.}$$

Now finding the pitch of the smallest pinion having 16 teeth, and a face width equal to twice the circular pitch, we have

$$p=\sqrt[3]{\frac{6.28\times 18{,}400}{16\times 10{,}000\times 2\tfrac{1}{4}\times 0.081}}=1.57 \text{ inches} \backsim 2 \text{ diam. pitch.}$$

| | No. Teeth | Diam. Pitch | Pitch Diam. | Width of Face |
|---|---|---|---|---|
| Pinion A.. | 16 | 2 | 8 inches | 3.5 inches |
| Gear B ... | 40 | 2 | 20 inches | 3.5 inches |

The brake being on the main shaft, the motor pinion and its gear can be designed for the maximum motor torque. Assume a 16-tooth pinion cut from rolled steel. Assume a velocity on the pitch line of 600 feet per minute, the allowable fiber stress is

$$S=20{,}000\left(\frac{600}{600+v}\right)=20{,}000\frac{600}{1200}=10{,}000 \text{ pounds.}$$

The twisting moment at the motor is $360\times12=4320$ inch-pounds. Again using the formula for the pitch of the smallest pinion, we have, if the tooth face $=3p$,

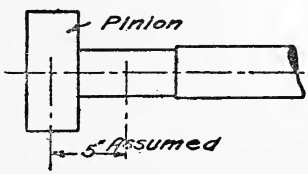
$$p=\sqrt[3]{\frac{6.28\times432}{15\times10{,}000\times3\times0.078}}=0.88 \text{ inch } c.p.$$

We will use 3 diametral pitch and make the face 3 inches.

| | No. Teeth | Diam. Pitch | Pitch Diam. | Width of Face |
|---|---|---|---|---|
| Pinion C.. | 15 | 3 | 5.00 inches | 3 inches |
| Gear D ... | 90 | 3 | 30.00 inches | 3 inches |

## Main Shaft.

FIG. 298.




The main shaft at the pinion is subjected to both torsion and bending.

Twisting moment $=18{,}400$ inch-pounds.

Force acting on pinion teeth $=$

$$\frac{18{,}400}{4}=4600 \text{ pounds.}$$

Bending moment $=4600\times5.5=25{,}300$ inch-pounds.

Finding the equivalent bending moment

$$M_{E\cdot B}=0.35\ M_B+0.65\sqrt{M_B^2+M_T^2}.$$
$$=(0.35\times25{,}300)+0.65\sqrt{25{,}300^2+18{,}400^2}$$
$$=26{,}600 \text{ inch-pounds.}$$

$$d=\sqrt[3]{\frac{10.2\ M_B}{f_B}}=\sqrt[3]{\frac{10.2\times26{,}600}{9000}}=3.12 \text{ inches.}$$

Before accepting this, it should be checked for stiffness (see

p. 37). Limiting the torsional deflection to 0.075 degree per foot of shaft length,

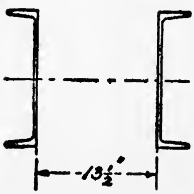
$$\Delta = 0.075 \times 30 = 2.25 \text{ degrees.}$$

$$d = \sqrt[4]{\frac{M_T \times L}{1660 \times \Delta}} = \sqrt[4]{\frac{18{,}400 \times 30}{1660 \times 2.25}} = 3.48 \text{ inches.}$$

This demands a 3½-inch diameter shaft, which can be reduced to 3¼ inches at the bearings.

### Axle for Driving Wheels.

Fig. 299.




In the bridge-wheel drive used the axle is subjected to bending only, the gear being either fastened directly to the driving wheel or both keyed to the same sleeve. The bending will be assumed as due to a uniformly distributed load, hence $M = \frac{WL}{8}$.

$$M = \frac{35{,}500 \times 13.5}{8} = 60{,}000 \text{ inch-pounds.}$$

The shaft diameter is given by $d = \sqrt[3]{\frac{10.2 \times M}{p}}$.

$$d = \sqrt[3]{\frac{10.2 \times 60{,}000}{9000}} = 4.09 \text{ inches.}$$

The diameter must now be checked for bearing pressure, which should not exceed 800 pounds per square inch. The length of the bearing is 12½ inches, hence if the shaft is taken 4¼ inches diameter the pressure per square inch of projected area is

$$\frac{35{,}500}{l \times d} = \frac{35{,}500}{12.5 \times 4.25} = 670 \text{ pounds, which is satisfactory.}$$

The force acting at the wheels to stop the crane in 5 feet has been found to be 3530 pounds. Measured at the brake placed upon the main driving shaft, the force is

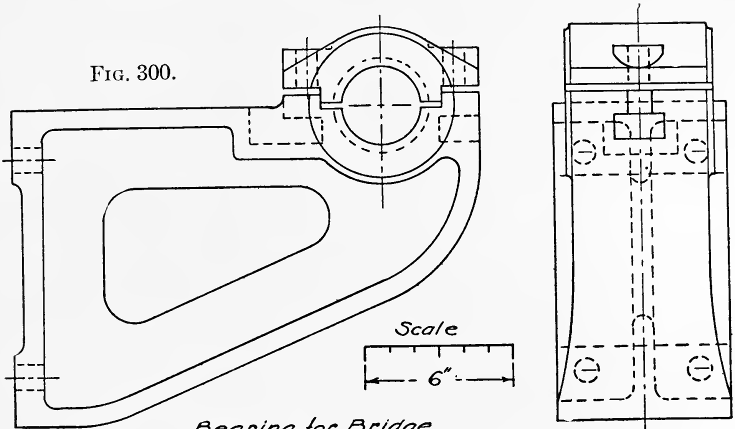
$$F = \frac{3530 \times 12 \times .92}{10 \times 2.5} = 1550 \text{ pounds.}$$

The brake used will be a form of wooden block brake, which will act with the brake wheel running in either direction. (See Fig. 301.) The energy to be absorbed by the brake is

$$E = 3530 \times 5 \times .92 = 16{,}350 \text{ foot-pounds.}$$

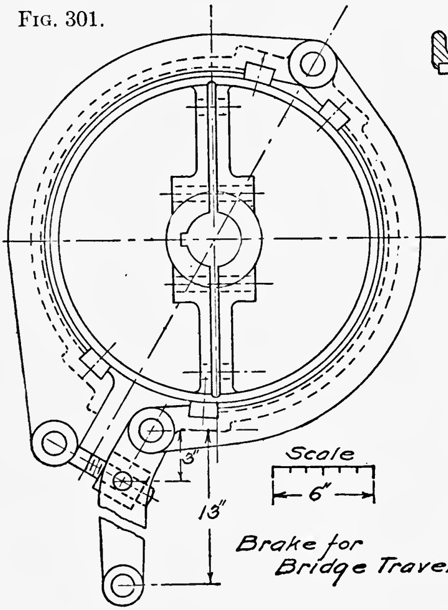
## DETAILS FOR 20 TON O.E.T. CRANE.

Fig. 300.



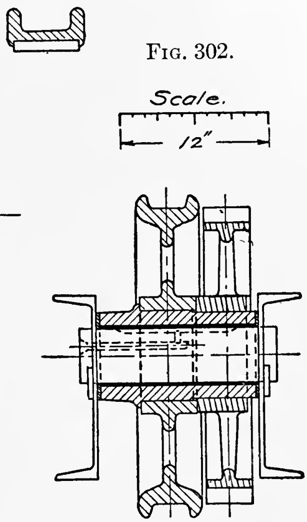

Bearing for Bridge Driving Shaft.

Fig. 301.




Brake for Bridge Travel.

Fig. 302.




Bridge Wheels.

Allowing 250 foot-pounds absorbed per square inch of brake shoe, we find the area of one shoe to be

$$A=\frac{16,350}{2\times 250}=32.7 \text{ square inches.}$$

Allowing a coefficient of friction between wood and cast-iron brake shoe equal to $\frac{3}{10}$, the pressure on one shoe must be

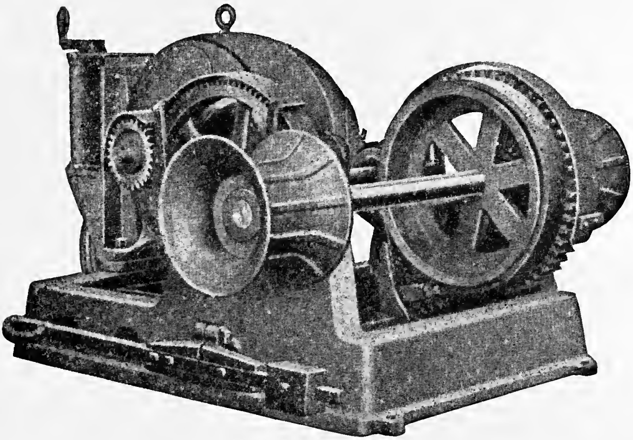
$$P=\frac{1550}{2\times .3}=2580 \text{ pounds.}$$

Taking moments about the pin A of the brake, the pull on the lever is $\frac{2580\times 3}{13}=595$ pounds. From this point to the cage a system of levers must be used to bring the force to operate the brake within the capacity of an operator, say under 50 pounds at the foot lever, including the pull due to the releasing spring or weight.

# PART XI.—HOISTING ENGINES.

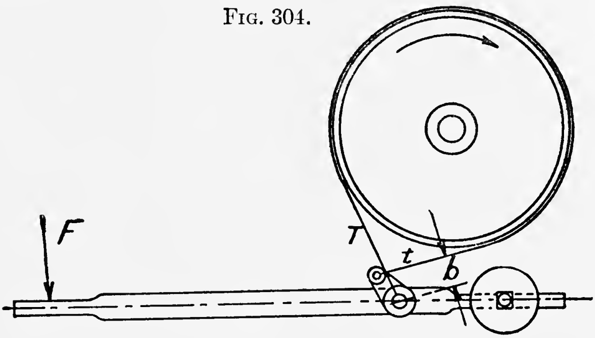
## Electrically Driven Hoists.

Fig. 303.



For selection of motor see Electric Overhead Travelling, p. 295.

Fig. 304.




The brake used is shown in Fig. 304. Here $a=o$ and the pressure F required upon the foot lever is given by

$$Wr \leqq PR, \quad t=\frac{P}{e^{\mu\alpha}-1} \text{ and } T=\frac{Pe^{\mu\alpha}}{e^{\mu\alpha}-1}$$

hence $$F=\frac{t \times b}{L}=\frac{P}{e^{\mu\alpha}-1} \times \frac{b}{l}.$$

## Steam Driving.

Hoisting engines may be either single cylinder or duplex engines. Single cylinder engines have the diasdvantage of having dead centers and of furnishing a less uniform turning moment than the duplex engines. The duplex engines, having their cranks set at 90,° have no dead centers and their starting is prompt at all crank positions. The engines for small hoisting drums are not usually reversing engines, the loads being lowered by a brake. The engine is coupled to the drum by a clutch, which is disengaged when the load is lowered. When reversing engines are used the the reversing is usually accomplished by Stephenson's links. A Marshall valve gear is also sometimes used, as is also a type of reversing gear in which the D slide valve has neither inside nor outside lap, and the reversing is accomplished by interchanging the live and exhaust steam parts.

With the common slide valve the cut-off will range from ½ to ⅘ of the stroke. The usual steam gauge pressure ranges from 75 to 100 pounds per square inch.

The ratio between the initial steam pressure on the piston and the average pressure throughout the stroke is given by expansion tables as follows.

TABLE OF RATIOS OF $p_m/p$ IN PER CENT.

| $\epsilon$ | Clearance | | | $\epsilon$ | Clearance | | |
|---|---|---|---|---|---|---|---|
| | 1/100 | 1/20 | 1/10 | | 1/100 | 1/20 | 1/10 |
| 10 | 35 | 41 | 49 | 50 | 85 | 86 | 87 |
| 15 | 45 | 50 | 57 | 60 | 91 | 92 | 93 |
| 20 | 54 | 58 | 62 | 62½ | 92 | 93 | 93 |
| 25 | 60 | 64 | 68 | 70 | 95 | 95 | 96 |
| 30 | 67 | 70 | 73 | 75 | 97 | 97 | 97 |
| 33⅓ | 71 | 74 | 76 | 80 | 98 | 98 | 98 |
| 40 | 77 | 79 | 81 | 100 | 100 | 100 | 100 |

$\varepsilon$ = percentage of stroke completed at cut-off.
$p$ = initial pressure pounds per square inch absolute.
$p_m$ = mean pressure pounds per square inch.

The initial pressure will be less than the boiler pressure owing to friction in the steam piping, valves, steam chest and slide valve; this drop can be assumed at from 5 to 10 pounds per square inch. The back pressure $p_b$ in non-condensing engines can be taken at from 16 to 18 pounds absolute.

The efficiency $\eta_2$ of the engine will vary with the size and character of the engine, but will range from 70 to 85 per cent. The efficiency of each pair of gears can be assumed at 90 per cent. The power indicated by the engine will fall short of the theoretical amount due to rounding of the corners of the card, etc.; this efficiency can be taken at 90 to 95 per cent.

**Nomenclature.**

$M_T$ = Twisting moment, (inch-pounds) on drum shaft.
$M_E$ = Twisting moment at engine shaft.
A = Area of piston in square inches.
S = Stroke of piston. Inches.
$n$ = Number of strokes of 1 piston per minute.
N = Revolutions per minute of crank shaft.
$d$ = Diameter of piston. Inches.
$\eta_1$ = Efficiency of indicator card. (Card factor.)
$\eta_2$ = Efficiency of engine parts.
$\eta_3$ = Efficiency of gearing and hoisting machinery.
H.P. = Effective horse-power at drum.
$p_m$ = Mean effective pressure. Pounds per square inch.
$p_b$ = Back pressure. Pounds per square inch.
$p$ = Pressure in valve chest, pounds per square inch gauge.
T = Piston travel, feet per minute, 1 piston.
R = Gear ratio. Crank shaft to drum shaft.

$$\propto = \text{Ratio}, \quad \frac{\text{Minimum turning moment}}{\text{Mean turning moment}}.$$

The minimum twisting moment of a duplex hoisting engine measured at the drum shaft is given by

$$M_T \text{ (Minimum)} = \frac{2\,(p_m - p_b), \text{A.S.R.}\ \eta_1 . \eta_2 . \eta_3 . \propto}{3.14} \qquad (1)$$

$\propto$ will vary with the pressure, clearance, etc., but for ¾ cut-off and 90 pounds absolute initial pressure it can be taken at 75 to 80 per cent. Having found $M_T$ (minimum), the load the drum can pull at its circumference can be found by dividing $M_T$ by the drum radius, or if this pull is P pounds and the drum diameter is D inches, we have

$$P = \frac{2\,M_T \text{ (minimum)}}{D}.$$

The horse-power required to raise the load where V = velocity of the load, feet per minute,

$$\text{Horse-power} = \frac{P \times V}{33{,}000 \times \eta_2 \times \eta_3}.$$

The problem most frequently requires solving for the cylinder dimensions. In this case the twisting moment on the drum will be known and the gear reduction between the drum and the crank shaft can be assumed. In equation (1) let $S = \beta d$ and $A = \frac{\pi D^2}{4}$

then
$$D = \sqrt[3]{\frac{2\,M_T\ (\text{minimum})}{\beta\ (p_m - p_b)\,R.\eta_1.\eta_2.\eta_3.\propto}} \qquad (2)$$

$\beta$ ranges from 1 to 1.5.

**Problem.**—What size duplex engine operating under 75 pounds gauge pressure at ¾ cut-off will be required to raise 2400 pounds on a 30-inch diameter drum? The following assumptions will be made: $\eta_1 = 95$ per cent., 1 reduction 5 : 1, $\eta_3 = 90$ per cent., $\eta_2 = 80$ per cent. and $\propto = 80$ per cent.

The twisting moment required is

$$M_T = 2400 \times 15 = 36{,}000 \text{ inch-pounds.}$$

Taking $\beta = 1.45$, we have

$$(p_m - p_b) = [(90 \times 0.95) - 18] = 67.5 \text{ pounds.}$$

Hence

$$D = \sqrt[3]{\frac{2 \times 36{,}000}{1.45 \times 67.5 \times 5 \times 0.95 \times 0.80^2 \times 0.90}} = 6.46 \text{ ins.} \backsim 6\tfrac{1}{2} \text{ ins.}$$

Stroke $6.5 \times 1.45 = 9.43$ inches $\backsim$ 9½ inches.

The nearest usual size to this is 7 inches × 10 inches.

The indicated horse-power (I.H.P.) of this duplex engine at any particular speed is

$$\text{I. H. P.} = \frac{(p_m - p_b),\ A.S.N.\eta.}{99{,}000} \qquad (3)$$

The horse-power developed at the crank shaft of the engine is

$$\text{Horse-power} = \frac{(p_m - p_b),\ A.S.N.\eta_1 \times \eta_2.}{99{,}000}$$

The gearing, shafting, etc., should be designed by using the maximum twisting moment due to the engines; this can be assumed as approximately K times the mean twisting moment.

$$M_T\,(\text{maximum}) = \frac{2\,(p_m - p_b)\,A.S.R.\eta_1.\eta_2.\eta_3.K}{3.14}.$$

Here K = 1.4.

According to Böttcher, the twisting moment given at the crank shaft should be assumed as

$$M_E = \frac{D^2 \times p \times S}{5}.$$

Equating this with the twisting moment due to the load acting on the drum and solving for the diameter of the cylinder, we have

$$D = \sqrt[3]{\frac{5\,M_t}{R.\eta_3 \times p \times \beta}}$$

The selection of a boiler for a hoisting engine is affected by the following considerations:

1. Steam consumption per horse-power developed by engine.
2. Character of service.
3. Operation of boiler.

The following table illustrates hoisting engine practice as built by several manufacturers:

| Nominal H. P. | Engine | | | Drum | | Load | Nominal Speed of Load, | Gear Reductions | Estimated Boiler Heating Surface | $\frac{\text{Load}\times V}{33000}$ rated H.P. | Heating Surface per | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | Diam. | Stroke | R.P.M. Nom'l | Diam. | Nom'l R.P.M. | | | | | | Rated H.P. | Nom'l H.P. |
| | *Ins.* | *Ins.* | | *Ins.* | | *Lbs.* | *Feet.* | | *Sq. Feet.* | | | |
| 6 | 5 | 6 | ... | 12 | .. | 2000 | ... | ..... | 90 | .. | .... | 15 |
| 10 | 5½ | 8 | 150 | 10 | 32 | 5000 | ... | 75:16 | 135 | 15 | 9 | 13.5 |
| 12 | 6¼ | 8 | ... | 14 | .. | 3000 | ... | ..... | 130 | .. | .... | 11 |
| 16 | 6¼ | 10 | 150 | 12 | 37 | 6000 | ... | 4:1 | 190 | 21 | 9 | 12 |
| 20 | 7 | 10 | 150 | 12 | 37 | 7000 | ... | 4:1 | 235 | 25 | 9.4 | 12 |
| 20 | 7 | 10 | ... | 33 | .. | 1000 | 600 | 3:1 | ... | 18 | .... | .... |
| 20 | 7 | 10 | ... | 14 | .. | 5000 | ... | ..... | 215 | .. | .... | 11 |
| 30 | 8¼ | 10 | 150 | 14 | 30 | 8000 | ... | 5:1 | 365 | 27 | 13.5 | 12 |
| 30 | 8¼ | 10 | ... | 14 | .. | 8000 | ... | ..... | 245 | .. | .... | 8 |
| 35 | 8 | 12 | 175 | 48 | 30 | 3500 | 375 | 5.85:1 | ... | 40 | .... | .... |
| 35 | 9 | 10 | ... | 14 | .. | 9200 | ... | ..... | 325 | .. | .... | 9.3 |
| 40 | 8½ | 12 | ... | 16 | .. | 10000 | ... | ..... | 395 | .. | .... | 10 |
| 50 | 10 | 12 | ... | 16 | .. | 12000 | ... | ..... | 475 | .. | .... | 9.5 |
| 50 | 10 | 12 | ... | 16 | .. | 13000 | ... | ..... | 470 | .. | ... | 9.4 |
| 50 | 10 | 12 | 166 | 54 | 28 | 4500 | ... | 5.85:1 | ... | 55 | .... | .... |

1. The steam consumption per horse-power developed is high—for simple non-condensing engines with ordinary D slide valve 35 to 45 pounds steam per horse-power hour.

2. The service demanded of the average hoisting engine is very intermittent, thus cutting down the hourly steam consumption.

3. As it frequently happens in the use of small hoisting engines that the man who operates the engine fires the boiler, it is necessary to have the boiler of ample size, thus requiring a minimum of attention from the operator.

The upright tubular boiler is the type commonly used. The accompanying list gives the sizes manufactured by the Lidgerwood Manufacturing Company.

VERTICAL TUBULAR BOILERS.

| Number | Horse-power | Diameter of Boiler | Height of Boiler | Number Tubes (all 2-inch Diameter) | Length of tubes | Thickness of Iron in Shell and Furnace | Thickness of Tube Heads | Estimated Weight of Boiler without Fixtures | Estimated Weight of Boiler and Fixtures Complete |
|---|---|---|---|---|---|---|---|---|---|
| | | *Inches* | *Inches* | | *Inches* | *Inches* | *Inches* | *Pounds* | *Pounds* |
| 1 | 5½ | 28 | 63 | 40 | 40 | 1/4 | 3/8 | 1075 | 1550 |
| 2 | 6½ | 28 | 69 | 40 | 45 | 1/4 | 3/8 | 1150 | 1675 |
| 3 | 7½ | 30 | 72 | 44 | 48 | 9/32 | 3/8 | 1400 | 1950 |
| 4 | 8½ | 32 | 75 | 48 | 50 | 9/32 | 3/8 | 1550 | 2175 |
| 5 | 10 | 34 | 78 | 52 | 53 | 9/32 | 3/8 | 1725 | 2400 |
| 6 | 11 | 36 | 75 | 57 | 50 | 5/16 | 3/8 | 1885 | 2700 |
| 7 | 12 | 36 | 81 | 57 | 57 | 5/16 | 3/8 | 2025 | 2925 |
| 8 | 13 | 38 | 81 | 68 | 57 | 5/16 | 3/8 | 2250 | 3075 |
| 9 | 15 | 40 | 75 | 80 | 50 | 5/16 | 3/8 | 2160 | 3250 |
| 10 | 17 | 40 | 84 | 80 | 57 | 5/16 | 3/8 | 2500 | 3600 |
| 11 | 21 | 42 | 90 | 88 | 63 | 5/16 | 3/8 | 2775 | 3900 |
| 12 | 29 | 48 | 96 | 115 | 68 | 11/32 | 7/16 | 3600 | 5400 |
| 13 | 31 | 48 | 102 | 115 | 72 | 11/32 | 7/16 | 3750 | 5450 |
| 14 | 35 | 50 | 102 | 124 | 72 | 3/8 | 7/16 | 4275 | 5900 |
| 15 | 40 | 50 | 114 | 124 | 84 | 3/8 | 7/16 | 4700 | 6400 |
| 16 | 40 | 53 | 102 | 150 | 72 | 3/8 | 1/2 | 5100 | 6900 |
| 17 | 50 | 53 | 120 | 150 | 87 | 3/8 | 1/2 | 5775 | 7800 |
| 17½ | 55 | 60 | 108 | 180 | 75 | 3/8 | 1/2 | 6150 | 8400 |
| 18 | 60 | 60 | 120 | 180 | 87 | 3/8 | 1/2 | 6850 | 9200 |

In general design the following proportions will hold.

Revolutions per minute—100 to 200 (nominal). Piston speed 200–400.

Single reduction 1:3 to 1:6.

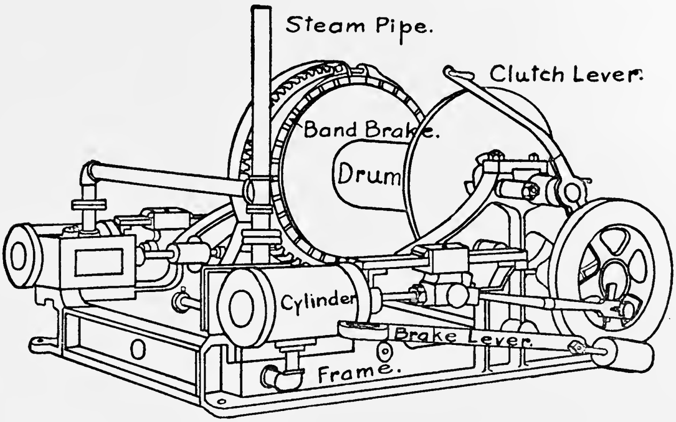
Boiler heating surface rated per horse-power, ordinary service 9 to 10 square feet.

Boiler heating surface per rated horse-power pile-driving service, 15 square feet and over.

Fig. 305 illustrates a simple hoisting engine. This type is frequently accompanied with a boiler as shown in Fig. 306. The latter is largely used by contractors in erection work. The power is furnished by 2 steam engines with cranks at right angles. These engines run in one direction only; the lowering is performed either

**Hoisting Engines.**

FIG. 305.




Double Cylinder Friction Drum Engine.

against a strap brake or against the friction drive, which is then released sufficiently to prevent the lowering of the load. Each drum carries a ratchet wheel which holds the drum and the load when the pawl engages the wheel. The pawl is commonly counterweighted to hold it clear of the ratchet wheel when the load is being raised.

The band brake used in lowering the load should be made to operate as readily as possible, as it is preferable for it to get the wear, thus saving the driving clutch. The brake is usually operated by a foot lever.

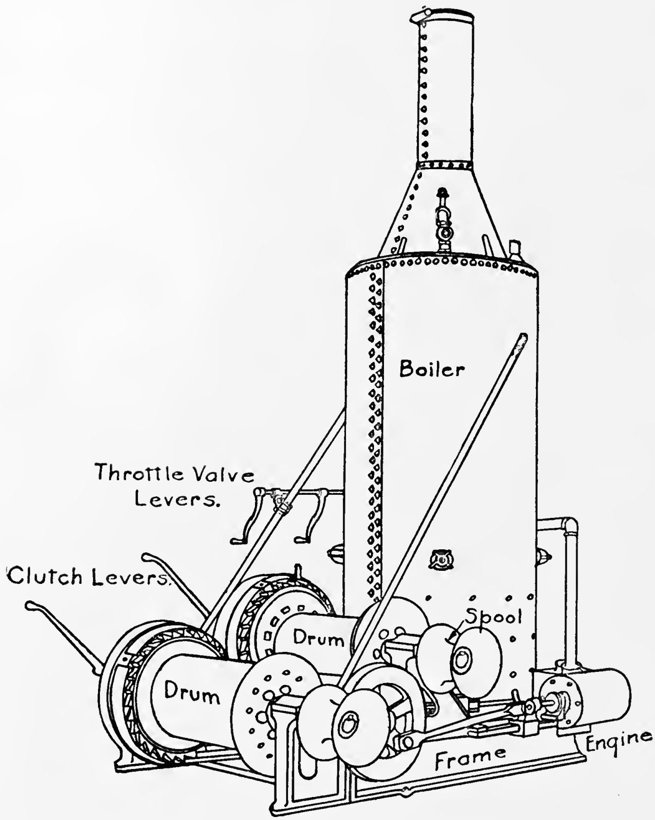
In a double drum hoist the full load can be raised by either drum, so that both drums and all gears and shafting should be designed for the maximum turning effort that the engines can exert. The usual proportions are about as follows:

$$\frac{\text{Piston diameter}}{\text{Length of stroke}} = \frac{1}{1.25 \text{ to } 1.50}.$$

$$\frac{\text{Length of stroke}}{\text{Length of connecting rod}} = \frac{1}{2.5 \text{ to } 3.5}.$$

HOISTING ENGINES.

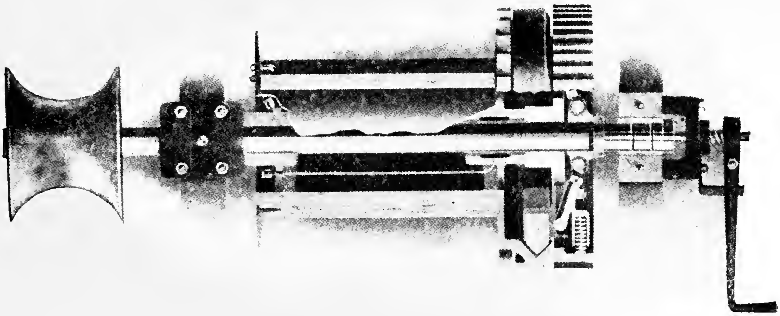
FIG. 306.




The vital part of such engines is the friction clutch; these are generally one of two types, either the radial wedge friction Fig. 221 or the axial cone friction Fig. 220. In both types the gear meshes with the pinion on the crank shaft, while the drum rotates freely on its shaft.

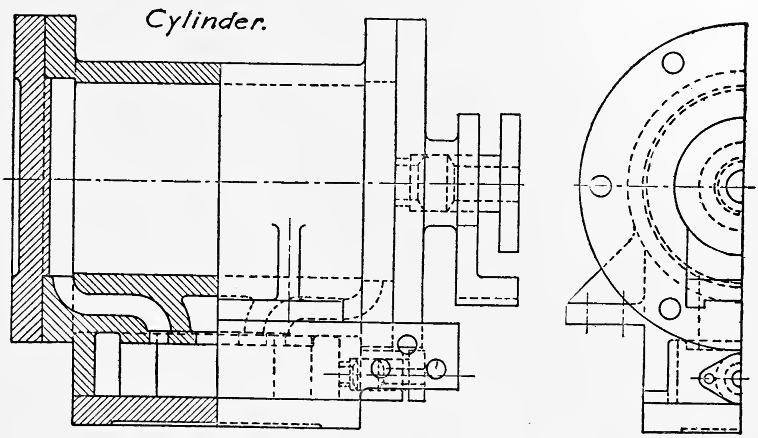
In Fig. 221 the screw E when turned in the fixed nut pushes the collar D axially along the shaft. This is accomplished by the stem of the screw extending into a central hole in the shaft and

Fig. 307.



pushing upon a flat located in a slot G in the shaft. The arrangement of links forms toggle movements which drive the friction wedges into the grooves, thus securing the drum to the gear.

Fig. 308.




The action of the second clutch is quite similar to the one just described, excepting that the axial motion of the shaft forces the wedge surfaces together without the intervention of the toggle movement.

See pp. 185 to 193 for mechanics of clutches.

## Engine Proportions.

For a more detailed account of steam engine design see any of the numerous books upon this subject.

The rotative speed of small hoisting engines depends upon the load and the operator. Engines of 6-inch or 8-inch stroke can be assumed as having a piston speed of 300 feet per minute, larger engines up to 18 inches stroke can have their piston speeds taken at 450 feet per minute.

The principal parts of the ordinary hoisting engine are shown in the illustrations.

Fig. 309.

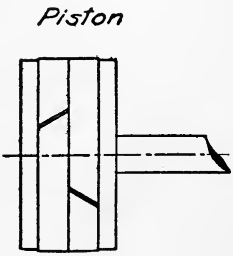



Fig. 310.

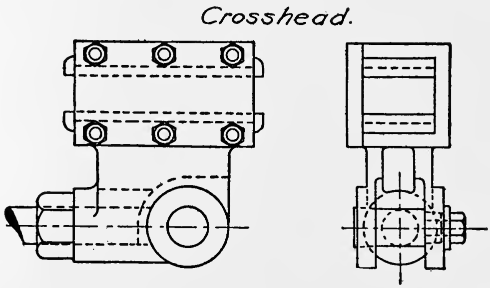
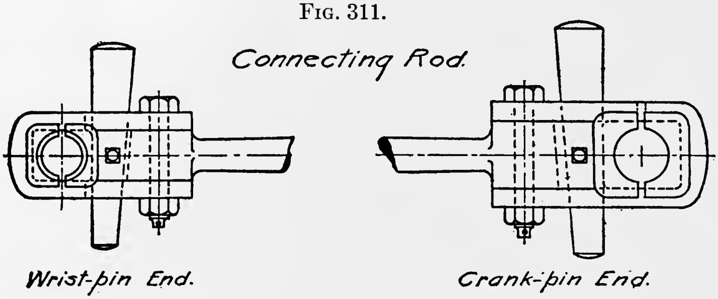



Fig. 311.




The following engine proportions by various authorities can be used as a guide:

Cylinder walls—

$$t = 0.04D + 0.3 \text{ inch to } 0.06\,D + 0.3 \text{ inch.}$$

$$t = \frac{p\,D}{4000} + 0.6 \text{ inch.}$$

$$\text{Area of ports} = \begin{cases} \text{Steam } 0.08a \text{ to } 0.09a. \\ \text{Exhaust } 0.15a \text{ to } 0.20a. \end{cases}$$

Diameter of steam pipe $= 0.25D + \frac{1}{2}$ inch.

Diameter of exhaust pipe $= \frac{1}{3}$ D.
Width of piston $= 0.46$ D.
Crank pins. Where $l = 0.25d$ to $0.30\,d$, $d = 0.18\,D$ to $0.2\,D$.
Piston Rods.

$$d = 0.145\sqrt{DL}.$$
$$d = 0.14\,D \text{ to } 0.17\,D.$$

Crank Shaft.

$$d = 7.25\sqrt[3]{\frac{I.H.P.}{N}}$$

Connecting Rods—Round Rods.

Center dimensions, $d = 0.032\sqrt{DL\,p^{\frac{1}{2}}}$
at wrist pin, $d = 0.011\,D\sqrt{p}.$

Rectangular Rods.

Center $h = 0.04\sqrt{DL\,p^{\frac{1}{2}}}$
Wrist pin end, $h^1 = 0.015\;D\sqrt{p}.$

The thickness of rectangular connecting rods $t^1 = \frac{4}{9}h$ (constant).
The notation used is as follows:

$a$ = area of piston in square inches.
$p$ = maximum pressure in the cylinder in pounds per square inch.
D = diameter of the cylinder.
$d$ = diameter of part considered.
$l$ = length of crank pin.
L = length of connecting rod.
N = revolutions per minute.
I.H.P. = indicated horse-power.
$t$ = thickness of cylinder walls.
$t^1$ = thickness of connecting rod.
$h$ = height of connecting rod at center.
$h^1$ = height of connecting rod at small end.

All dimensions are in inches.

# PART XII.—LOCOMOTIVE CRANES.

## Locomotive Cranes.

Locomotive cranes are commonly steam-driven, but a few are electrically driven. Steam locomotive cranes are driven by double cylinder engines with the cranks set at 90°. The several motions of rotation, transfer on the track, moving the load and boom are ordinarily accomplished by the use of friction clutches; the engine is then of the non-reversing type. The boiler is placed behind the engine, thus serving to counterweight the crane. The fuel-and-water tanks for the boiler are also placed to serve in this capacity. As previously explained for hoisting engines, the boiler should be large, so as to demand only occasional attention from the operator, and for the same reason ample fuel and water should be carried on the crane. The following are usual specifications for the locomotive cranes:

Gauge of track....either 4 feet 8½ inches or 8 feet.
Boiler pressure....100 pounds to 125 pounds gauge.
Cut-off.........................6/10 to 8/10 of stroke.
Revolutions per minute (engine).......80 to 200
Car wheels ....................24-inch diameter.
Track speed..........300 to 500 feet per minute.
Track power level track........3 to 4 loaded cars.
Slewing speed............4 revolutions per minute.

Owing to the limitations of the counterweight the crane will raise its greatest load when working at its shortest radius. The crane parts must therefore be designed for the stresses due to the maximum load.

The cranes are also designed to move several loaded cars along a level track, thus greatly facilitating loading and unloading cars.

The maximum direct stress will occur in the boom when it is in its highest position; the maximum lateral stress, however, may come upon it when in its lowest position and beginning to be rotated, as the accelerating force must then act upon it to produce the required velocity in the load. Thus a crane carries 5 tons at

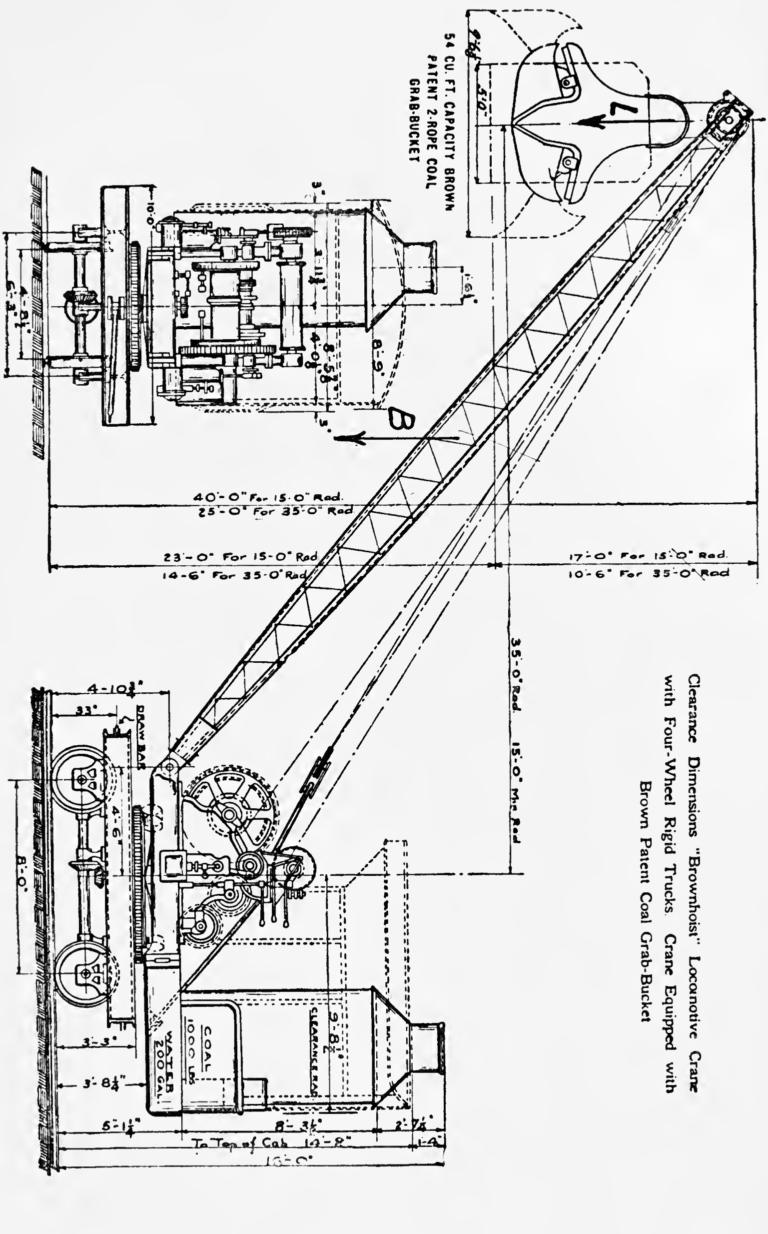



FIG. 312.

Clearance Dimensions "Brownhoist" Locomotive Crane with Four-Wheel Rigid Trucks. Crane Equipped with Brown Patent Coal Grab-Bucket

30 feet radius; if it is desired that at maximum radius the load in rotating shall attain a lineal velocity of 10 feet per second at the end of 2 seconds, the force acting at the load to produce this velocity would be

$$\text{Force} = \text{mass} \times \text{acceleration}.$$

$$F = \frac{10,000}{32.2} \times 5 = 1560 \text{ pounds}.$$

To provide strength in the boom to resist this force, it is customary to spread the sections forming the boom at the lower end.

The force required to rotate the crane will be that needed to overcome friction plus the accelerating force.

The following clauses are taken from a specification of a locomotive crane built by Wellman-Seaver-Morgan Company:

Purpose.—The crane will be designed especially for handling an automatic grab bucket, or can also be used for hoisting loads in ordinary yard service.

General Description.—The crane comprises a car on which is mounted a revolving platform carrying boiler, engines, drums, boom and operating mechanism. The crane is provided with the necessary mechanism for propelling it back and forth on tracks, raising its load, and rotating the revolving platform; also for raising and lowering the boom to vary the radius, or to give it the necessary clearance in doors or archways through which it is designed to pass.

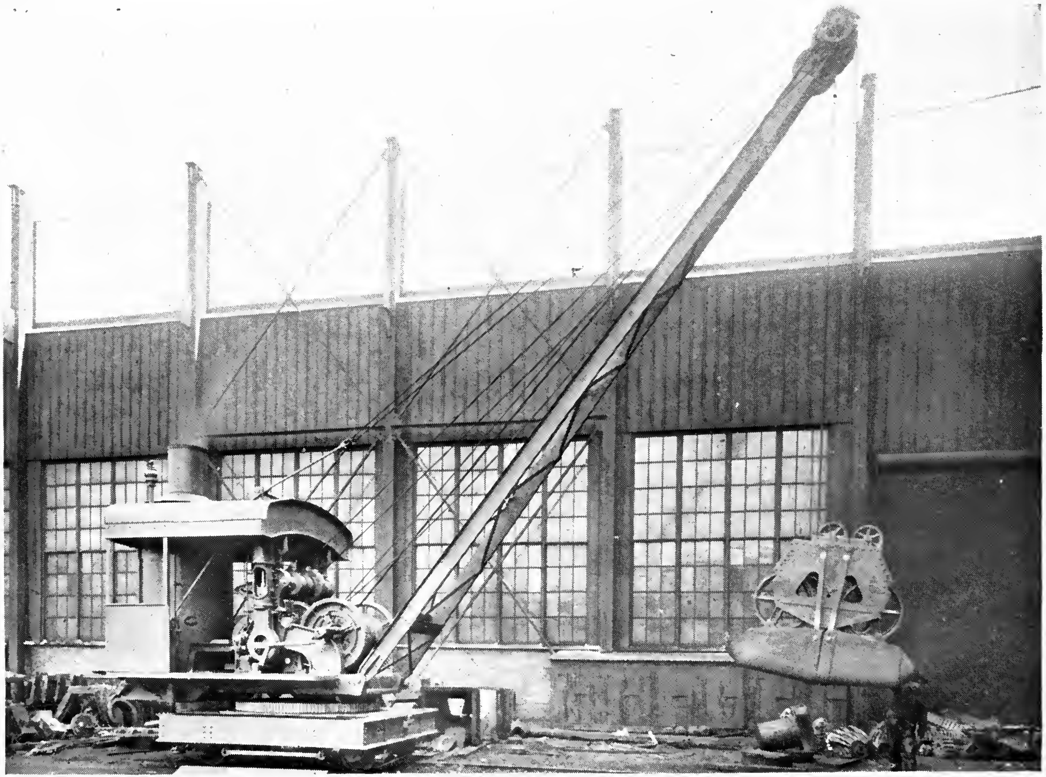
Capacities.—The crane will have the following capacities at the variable radii indicated:

| | Pounds. |
|---|---|
| At 30 feet maximum radius | 7,800 |
| At 25 feet radius | 10,000 |
| At 15 feet radius | 20,000 |
| At 10 feet radius | 20,000 |
| Draw-bar pull on straight level track | 7,000 |

The crane has power strength and stability to safely handle these loads at the given radii through a full circle without fastening the crane to the track, or will move along the track with these loads hanging from the boom.

The lifting capacities are based upon the track being in good condition and with the counter-weight box filled with 15,000 pounds of counter-weight; the counter-weight to be furnished by the purchaser. By using track clamps provided with the crane, these lifting capacities can be increased.

Fig. 313.



Locomotive Crane—10 Tons at 15 Feet Radius.

Speeds.—The crane will have the functions of hoisting, rotating and track-travel, which may be utilized simultaneously, or each function may be used independently, as desired, the speed being as follows:

| | Feet per Minute. |
|---|---|
| Hoisting with loaded bucket | 120 |
| Hoisting with load of 20,000 pounds | 60 |
| Hoisting with no load | 125 |
| | Revolutions per Minute. |
| Rotation of crane, no load | 6 |
| Rotation of crane, full load | 3 |
| | Feet per Minute. |
| Travel of crane on level track, full load | 400 |
| Travel of crane on level track, no load | 600 |
| | Per Cent. |
| Maximum grade crane will ascend, full load | 4½ |
| Maximum grade crane will ascend, no load | 7 |

Principal Dimensions:

Engine cylinders, 9-inch diameter × 9-inch stroke.
Speed with full load, 250 revolutions per minute.
Boiler diameter, 54 inches; height 8 feet 6 inches.
Number of 2-inch flues, 172.
Steam pressure, 100 pounds.
Wheel base of crane, 8 feet.
Clearance radius at rear of crane, 9 feet 8½ inches.
Length of boom, 36 feet.
Size of hoist rope, ⅝ inch in diameter.
Size of boom rope, ¾ inch diameter.
Diameter of axles, 6 inches.
Dimensions of journals, 5 inches × 8 inches.
Diameter of track wheels, 24 inches.
Capacity of water tank, 250 gallons.
Capacity of coal tank, 1500 pounds.
Extreme height of crane, 16 feet 8 inches.
Extreme width of crane, 10 feet.

The net weight of crane in running order, including the counterweight, is about 63,000 pounds.

Base.—The crane base will be a structural frame-work into which is bolted a turn-table. On this turn-table the rotating superstructure rests. This turn-table has teeth cast on its periphery for meshing with the pinion which revolves the super-structure.

The crane has cast-iron track wheels with chilled treads. These track wheels are forced on hammered steel axles, which run in bronze bearings fitted with oil boxes.

Revolving Platform.—On the base is mounted a revolving platform, consisting of a heavy casting carried on two conical cast steel rollers at the rear end, and four conical rollers in pairs, in equalizing frames, at the front end. This revolving platform is pinned to the main base at the center by a heavy hollow cast steel pivot pin. The heel of the boom is carried at the front end of this platform. The housings for the engines and the hoisting mechanism are connected to this platform so as to practically make one rigid piece. The rear extension of this platform forms a support for the boiler, and is hollow, thus forming a tank for the crane water supply.

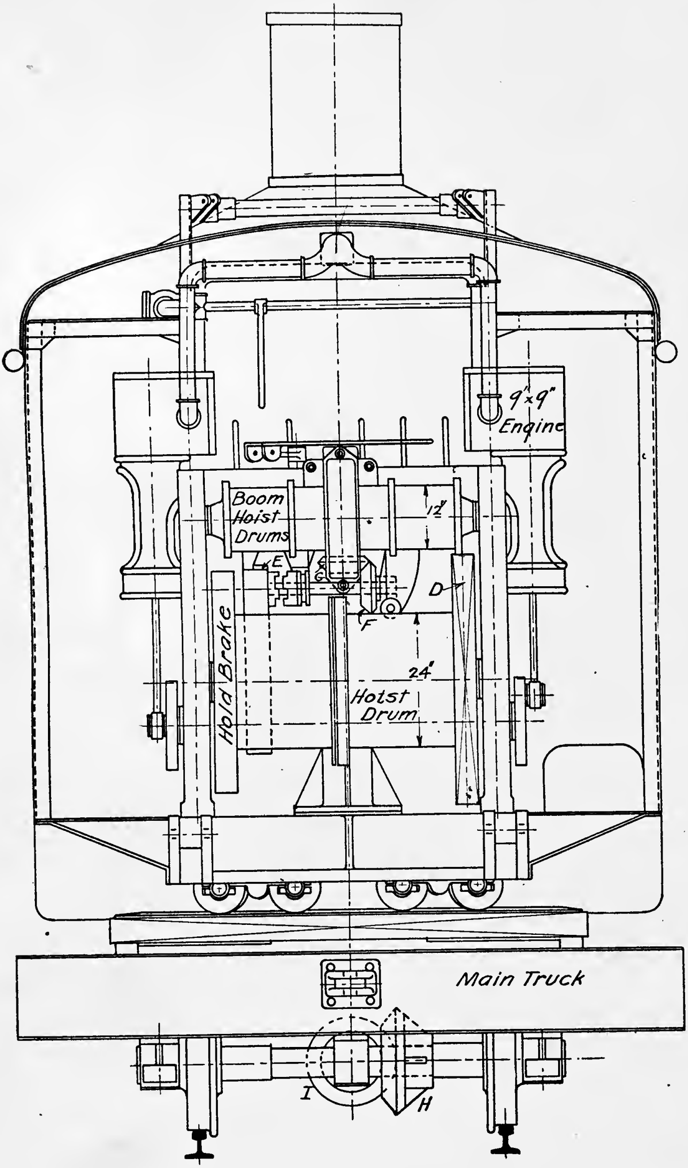
Boom and Mast.—The boom will consist of 2 heavy channels securely braced together. The mast carrying the tackle for supporting the outer end of the boom will consist of heavy channels securely braced to and supported on the engine housings.

Engines.—The engines will have two vertical cylinders with link-motion reversing gear. The engine shafts will be of forged steel and will have balanced disc cranks forced and keyed on. The engine shaft is geared to two intermediate shafts, one carrying the clutches for driving the drum shaft and for travelling the crane, the other carrying the clutches for the rotating mechanism.

Gearing.—The engine pinion will be of forged steel with cut teeth. The gears meshing with the engine pinion will be steel castings with cut teeth. All boom-raising gears, except bevels, will be of steel castings with cut teeth. All remaining gears will be steel castings with cast teeth, from metal patterns. The main axle-driving bevel gears are split. Bearings for main axles and for conical rollers will be of bronze shells. All bearings not otherwise specified will be lined with babbitt metal, poured in place and accurately scraped to fit.

A few points in the design of such a crane will now be considered. The given specification will be followed in a general way.

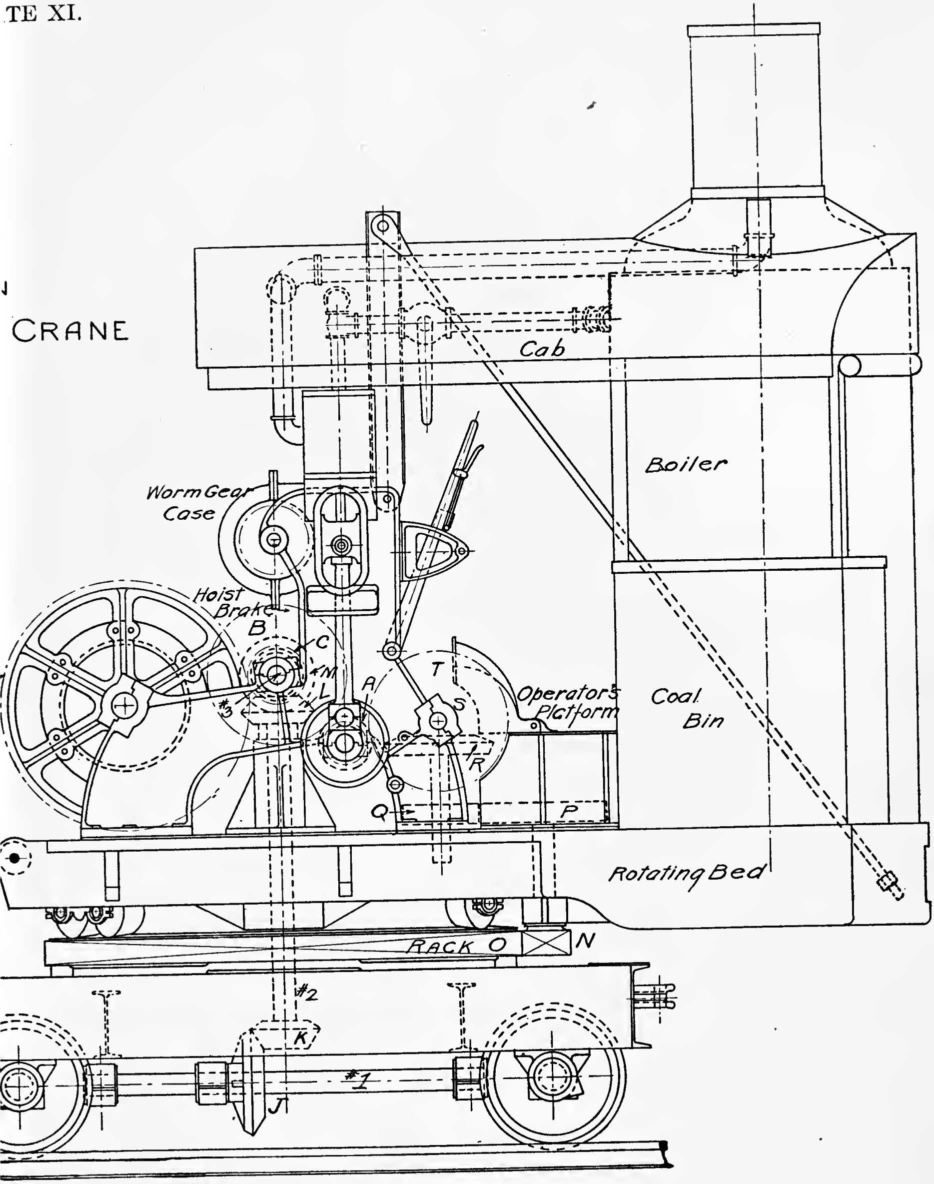
Rope.—A ⅝-inch diameter extra flexible plow steel rope running on a 24-inch diameter drum will be tried. The sheaves will be assumed at 21 inches diameter and the load of 20,000 pounds will be carried by 4 ropes.


9"×9" Engine
Boom Hoist Drums
12"
E
G
D
F
24"
Hold Brake
Hoist Drum
Main Truck
I
H

10
LOCOMOTI

N

CRANE


Cab
Boiler
Worm Gear Case
Hoist Brake
B
C
M
L
A
T
S
#3
Operator's Platform
Coal Bin
R
Q
P
Rotating Bed
RACK O
N
#2
K
#1
J

Allowing 500 pounds to cover the weight of the block, tackle and rope, the pull in each rope will be,

$$F=\frac{20{,}000+500}{4\times 0.07\times 0.98}=5{,}400 \text{ pounds.}$$

The total stress in the rope due to both tension and bending is given by the formula,

$$p_T=\left(\frac{S}{i\times\frac{\pi d^2}{4}}\right)+\left(\tfrac{3}{8}\times\frac{E\times\delta}{D}\right);\ \delta=\frac{d}{21}=\frac{0.625}{21}=0.03 \text{ inch,}$$

making the total stress,

$$p_T=\left(\frac{5400}{222\times\frac{3.14\times 0.030^2}{4}}\right)+\left(\tfrac{3}{8}\times\frac{30{,}000{,}000\times 0.030}{21}\right)=50{,}400 \text{ lbs.}$$

As plow steel rope has an ultimate strength of 220,000 pounds per square inch, the factor of safety with the above stress is

$$\frac{220{,}000}{50{,}400}=4.4.$$

Hoisting Drum.—As previously stated, this will be assumed as 24-inch pitch diameter. The pull on the circumference of this drum is that due to 2 ropes, *i.e.*, 10,800 pounds. The twisting moment on the drum shaft then is,

$$M=10{,}800\times 12=129{,}600 \text{ inch-pounds.}$$

Reduction Engine to Drum.—Assuming the speed of the engine at full load as 250 revolutions per minute, and that the load is raised 60 feet per minute.

The speed of rope winding on the drum is $60\times 2=120$ feet per minute; from this the revolutions of the drum per minute is

$$\text{Revolutions per minute of drum}=\frac{120}{2\times 3.14}=19.1.$$

The reduction from the engine to the drum is $\frac{250}{19.1}=13.1$ to 1.

This reduction will be accomplished by 1 reduction of 79 to 18 and another reduction of 3 to 1.

Engine.—The twisting moment on the drum due to the load has been found to be 129,600 inch-pounds. The reduction from the engine to the drum is 13.1 to 1. The boiler pressure is 100 pounds gauge. Cut-off assumed ½ card efficiency 90 per cent., engine efficiency 85 per cent., gearing efficiency 70 per cent. The ratio of the minimum turning moment of the engine to the mean turning moment is 75 per cent. The ratio of the stroke of the engine to its diameter is 1 to 1.

$$D=\sqrt[3]{\frac{2M_t}{\beta\,(p_m-p_b)\,R.\eta_1.\eta_2.\eta_3.\alpha}}$$

$$=\sqrt[3]{\frac{2\times 129{,}600}{1[(115\times 0.87)-18]\times 13.1\times .90\times .85\times .70\times .75}}$$

$D=8.45$ inches.

The engine used in the above specification was a 9-inch×9-inch. This size engine will be assumed here. The twisting moment developed at the crank by this engine, the conditions being the same as those just used, will be found as follows: Cut-off $\frac{8}{10}$.

$$M_t=(p_m-p_b)\,\eta_1.\eta_2\times\frac{d^3}{4}=95\times 0.90\times 0.85\times\frac{9^3}{4}$$

$$=13{,}250 \text{ inch-pounds.}$$

The maximum turning moment can be assumed at 1.4 times this, or $M_T$ (maximum) $=13{,}250\times 1.4=18{,}550$ inch-pounds.

The minimum pitch for a steel pinion A of 18 teeth, to carry the above twisting moment at an assumed velocity of 600 feet per minute, is given by

$$p=\sqrt{\frac{6.28\ M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}}$$

The allowable fiber stress for rolled steel pinion with cut teeth can be taken at 20,000 pounds per square inch; when properly reduced for the tooth velocity, this will be

$$s=20{,}000\left(\frac{600}{600+v}\right)=10{,}000 \text{ pounds.}$$ Use 8000 pounds per square inch.

$$p=\sqrt[3]{\frac{6.28\times 18{,}550}{18\times 8000\times 3\times 0.086}}=1.47 \text{ inches.}$$

The diametral pitch will be taken at 2 with a 4½-inch face.

Pinion A—18 teeth—9-inch pitch diameter. 2 pitch, 4½-inch face, steel.

Gear B—56 teeth—28-inch pitch diameter. 2 pitch, 4½-inch face, steel.

Drum Gear and Pinion.—Gear 79 teeth, pinion 18 teeth. The pinion will be cut steel, velocity assumed at 300 feet per minute, and the allowable fiber stress will be 12,000 pounds per square inch.

The twisting moment on the pinion is taken as that due to the load acting on the drum, and is

$$M_t = \frac{129{,}600}{0.92} \times \frac{18}{79} = 32{,}000 \text{ inch-pounds.}$$

The circular pitch is now found from the usual formula to be

$$p = \sqrt[3]{\frac{6.28 \times M_t}{n.s.c. \left(0.124 - \frac{0.684}{n}\right)}}$$

$$= \sqrt[3]{\frac{6.28 \times 32{,}000}{18 \times 12{,}000 \times 2.25 \left(0.124 - \frac{0.684}{18}\right)}} = 1.69 \text{ inches.}$$

1¾ diametral pitch with a tooth-face of 4 inches will be used.

Pinion C—18 teeth—10.29-inch pitch diameter. 1¾ pitch, 4-inch face, steel.

Gear D—79 teeth—45.14-inch pitch diameter. 1¾ pitch, 4-inch face, steel.

Boom Hoisting.—It not being intended to operate the boom excepting when empty or carrying light loads, and then infrequently, the sheaves can be made small for the rope diameter. Neglecting the bending and taking the stress as found in the stress diagram, the fiber stress upon 4 ¾-inch ropes is

$$p_T = \frac{S}{i \times \frac{\pi d^2}{4}}, \quad \delta = \frac{0.75}{15} = 0.05 \text{ inch.}$$

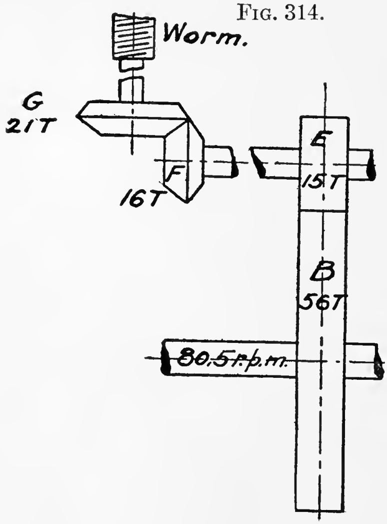
$$p_T = \frac{11{,}000}{114 \times \frac{3.14 \times 0.05^2}{4}} = 49{,}000 \text{ pounds.}$$

The factor of safety under these conditions is for plow steel,

$$\frac{220{,}000}{49{,}000} = 4\tfrac{1}{2}.$$

Boom Hoist Mechanism.—The boom is raised and lowered by a worm operating upon a worm wheel; the pitch of the worm is such that the weight of the boom and its load will not fall by driving the worm and gearing backwards. This avoids the necessity of having a brake upon this mechanism.

Fig. 314.




Gear B is the first gear in the hoisting train; the pinion E meshes with it and drives the worm moving the boom through the bevel gears F and G.

The speed of the worm is,

$$\text{R. P. M.} = \frac{250\times18}{56}\times\frac{56}{15}\times\frac{16}{21}\times\frac{1}{28} = 8.16.$$

The pitch of the worm wheel will be determined to carry the pull from the 2-¾-inch ropes secured to the drum. This drum will be assumed 12-inch pitch diameter.

The twisting moment on the drum is 22,000×6=132,000 inch-pounds.

Assuming the width of the tooth as twice the circular pitch of the worm wheel tooth,

$$p_c = \sqrt[3]{\frac{6.28\times M_t}{n.s.c.\left(0.124-\frac{0.684}{n}\right)}} = \sqrt{\frac{6.28\times132{,}000}{28\times12{,}000\times4\times0.1}}$$

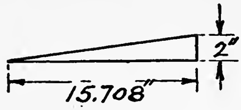
$$= 1.84 \text{ inches.}$$

Hence any circular pitch not less than 1.84 inches with 28 or more teeth will answer. We will assume a 2-inch circular pitch and the pitch diameter of such a worm wheel will be,

$$\text{Pitch diameter} = \frac{28\times2}{3.14} = 17.825 \text{ inches.}$$

The angle of the helix must now be examined to see that the worm will not permit the boom to drop. Assuming a 5-inch pitch diameter of the single threaded worm, the teeth being 2 inches circular pitch, the angle is,

$$\text{tang}^{-1} = \frac{2}{5 \times 3.14} = 0.1274 \approx 7° \, 20'.$$




The equation of a worm and wheel (see p. 32) when the worm wheel is the driver is,

$$Pa = Kr \text{ tang } (\alpha - \rho) - \mu K r_2 - \mu P r_1.$$

If this is not to run down $Pa \leqq 0$. Assuming $\mu$ at rest as 0.08 and making $\mu P r_1 = 0$, we have

$$Pa = K \, (r \text{ tang } (\alpha - \rho) - \mu r_2).$$

Assuming $\rho = \text{tang}^{-1}, \, 1/10 = 5° \, 45'$.

$$Pa = K \, [2.5 \text{ tang } (7° \, 20' - 5° \, 45') - (0.10 \times 1.5)]$$
$$= -0.08 \, K.$$

Since $Pa < 0$, the worm cannot be driven by the wheel.

The turning moment on the worm to raise the load must now be found.

$$K = \frac{\text{Twisting moment on the worm wheel}}{\text{Radius of worm wheel}}$$

$$K = \frac{132{,}000}{8.9} = 14{,}800 \text{ pounds.}$$

The twisting moment on the worm is

$$Pa = Kr \text{ tang } (\alpha + \rho) + \mu K r_2 + \mu P r_1.$$
$$Pa = (14{,}800 \times 2.5 \text{ tang } 13° \, 5') + (1/10 \times 14{,}800 \times 1.5) + \text{——}.$$
$$Pa \approx 10{,}800 \text{ inch-pounds.}$$

Bevel Gears.—Assuming the mean diameter of the bevel pinion as 7 inches, the force on the face of the pinion is $P = \frac{M_t}{a}$.

$$P = \frac{14{,}800}{3.5} = 4250 \text{ pounds.}$$

As it has been assumed that the boom would not be raised by this gearing when carrying the full load, these gears will be calculated for one-half this load. The formative number of teeth will be assumed as 20. The pinion will be a steel casting, cast from a cut metal pattern.

$$W = s.p.f.y.\frac{d}{D} \therefore p = \frac{WD}{s.f.y.d.}$$

$$y = \left(0.124 - \frac{0.684}{n}\right) = \left(0.124 - \frac{0.684}{20}\right) = 0.09.$$

$$p = \frac{2000 \times 9.2}{9000 \times 2.5 \times 0.09 \times 7} = 1.3 \text{ inches, say } 1\tfrac{3}{8} \text{ inches.}$$

Bevel Gear G, 21 teeth, 9.19-inch pitch diameter—1.375-inch pitch, 2.5-inch face.

Pinion F, 16 teeth, 7-inch pitch diameter—1.375-inch pitch, 2.5-inch face.

The twisting moment on the pinion shaft is $\frac{2000 \times 2.75}{0.92} \backsim 6000$ inch-pounds.

The spur pinion on this shaft meshes with the gear driven by the engine pinion, so that its pitch will have to be 2 pitch, the same as engine pinion. It will be necessary to find what width will be required so that this pinion will transfer to the bevel gears the full load that they will transmit. This face width can be found from the formula,

$$f = \frac{2\pi M_t}{n.s.p.^2\left(0.124 - \frac{0.684}{n}\right)} = \frac{6.28 \times 6000}{15 \times 9000 \times 1.57^2 \times 0.08} = 1.41 \text{ inches.}$$

The pinion will be made 2½-inch face, which is more than ample.

Conical Rollers.—The maximum load will come upon the front rollers. This force can be determined from the force diagram Fig. 318. The mean distance of the rollers from the center of the crane has been taken as 3 feet. The load upon these rollers will be a maximum when 20,000 pounds is carried at a radius of 15 feet. The total load on the rollers will then be,

| | Pounds. |
|---|---|
| Live load | 20,000 |
| Boom | 2,000 |
| Machinery, etc | 18,000 |
| Boiler | 16,000 |
| Total | 56,000 |

Load per roller 56,000/4 = 14,000 pounds.

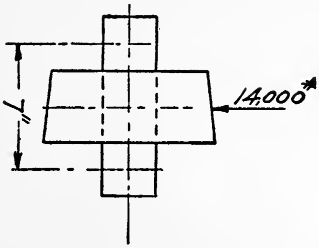
The formula for rollers (see p. 71) is

$$Q = 850 \times D \times b.$$

Making the rollers a mean diameter of 9 inches, the width must be at least,

$$b = \frac{Q}{850 \times D} = \frac{14,000}{850 \times 9} = 1.83 \text{ inches.}$$

FIG. 315.




Pin or axle for rollers.

$$d = \sqrt[3]{\frac{10\,M}{p}}.$$

$M = 7000 \times 3.5 = 24,500$ inch-pounds.

$$d = \sqrt[3]{\frac{10 \times 24,500}{9000}} \backsim 3 \text{ inches.}$$

Two bearings 3 inches × 3 inches give a projected area of 18 square inches. The maximum pressure per square inch of projected area then is $\frac{14,000}{18} = 780$ pounds. As this pressure is only occasional it is satisfactory.

TRACTIVE FORCE TO OPERATE CRANE.—Full load.

$$F = \frac{W}{R}(f + \mu r) = \frac{83,000}{12}\left(0.003 + \frac{2.25}{10}\right) = 1575 \text{ pounds.}$$

Crane empty.

$$F_1 = \frac{63,000}{12} \times 0.228 = 1200 \text{ pounds.}$$

If a force of 7000 pounds acting at the circumference of the driving wheels is furnished by the engine, this will be ample for accelerating the crane, either empty or carrying full loads, and will leave practically 6000 pounds for moving cars. The track wheels will be taken 24 inches in diameter.

According to the diagram, about ¾ of the tractive force may develop at one axle.

The twisting moment at the pinion shaft is

$$\frac{7000 \times 0.75 \times 12 \times 20}{24 \times 0.92} = 57,000 \text{ inch-pounds.}$$

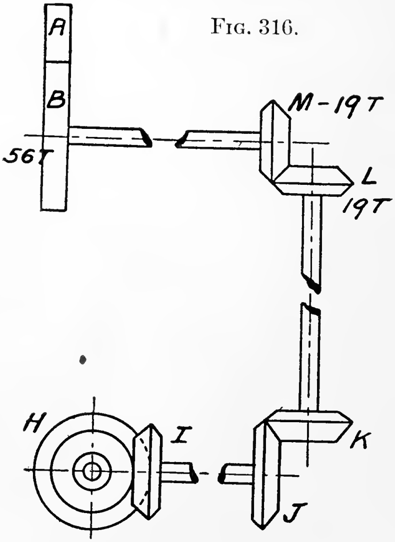
Assuming that the pinion has 20 teeth, and the gear 24 teeth, the formative number of teeth is found as follows:

$$\propto = \text{tang}^{-1}\frac{20}{24} = \text{tang}^{-1}\,0.834 = 39°\ 50'.$$

Sec. 39° 50′ = 1.302. cosec 39° 50′ = 1.561.
N = $n$ × sec ∝ = 20 × 1.302 = 26.

The mean circular pitch is given by the formula

FIG. 316.




$$p_m = \sqrt[3]{\frac{2\pi\,M_t}{n.s.c.\left(0.124 - \frac{0.684}{N}\right)}}$$

$$= \sqrt[3]{\frac{6.28 \times 57{,}000}{20 \times 900 \times 20 \times 0.098}}$$

$$= 2.16 \text{ inches.}$$

$$d_m = \frac{20 \times 2.16}{3.1416} = 13.75 \text{ inches.}$$

$$p_c = 2.16 \times \frac{(13.75 \times 1.561) + 4}{13.75 \times 1.561}$$

= 2.56 inches, say 1¼ diam. pitch.

Pinion I, 16-inch O. D.—20 teeth, 1¼ diametral pitch,—4½-inch face.

Gear H, 19.2-inch O. D.—24 teeth, 1¼ diametral pitch,—4½-inch face.

TWISTING MOMENT ON THE VERTICAL SHAFT.—Assume 16 teeth on the pinion and 24 teeth on the gear.

$$M_t = \frac{7000 \times 12 \times 20 \times 16}{24 \times 0.92 \times 24 \times 0.92} = 55{,}000 \text{ inch-pounds.}$$

$$\propto = \text{tang}^{-1}\frac{16}{24} = \text{tang}^{-1}\,0.667 \therefore \propto = 33°\ 40'.$$

sec ∝ = 1.20. cosec. ∝ = 1.804.

N = $n$ sec ∝ = 16 × 1.20 = 19.2, say 19.

$$p_m = \sqrt[3]{\frac{6.28 \times 55{,}000}{16 \times 10{,}000 \times 2.5 \times .088}} = 2.14 \text{ inches.}$$

Face $=2.14\times2\frac{1}{2}=5.35$ inches, say $5\frac{1}{2}$ inches.

$$d_m=\frac{16\times2.14}{3.14}=10.90 \text{ inches.}$$

$$p_c=2.14\times\frac{(10.90\times1.8)+5.20}{10.90\times1.8}=2.70 \text{ inches, say } 1\tfrac{1}{4} \text{ diam. pitch.}$$

$1\frac{1}{4}$ diametral pitch will be used, as the gear has to be kept small so that the gear will clear the track.

Pinion K, 16 teeth, $1\frac{1}{4}$ pitch, $5\frac{1}{2}$-inch face, steel.

Gear J, 24 teeth, $1\frac{1}{4}$ pitch, $5\frac{1}{2}$-inch face, steel.

Mitre Gears.—Assumptions, $n=20$, $N=n.\sec.\alpha=20\times1.41=28$.

$$\text{Twisting moment } M_t=\frac{7000\times12\times20\times16}{24\times0.92\times24\times0.92}=55{,}300 \text{ inch-pounds.}$$

The circular pitch of the mean pitch circle of the smallest 20-toothed pinion is

$$p_m=\sqrt[3]{\frac{6.28\ M_t}{n.s.c.\left(0.124-\frac{0.684}{N}\right)}}=\sqrt[3]{\frac{6.28\times55{,}300}{20\times10{,}500\times2.5\times0.10}}=1.88 \text{ inches.}$$

$$f=c.p_m=2.5\times1.88=4.70 \text{ inches, say } 4\tfrac{3}{4} \text{ inches.}$$

$$d_m=\frac{n\times p_m}{\pi}=\frac{20\times1.88}{3.14}=12 \text{ inches.}$$

$$p_c=p_m\times\frac{(d_m \operatorname{cosec}\alpha)+f}{d_m \operatorname{cosec}\alpha}=1.88\times\frac{(12\times1.41)+4.70}{12\times1.41}=2.4 \text{ inches.}$$

Both gears L and M, 20 teeth, $1\frac{1}{4}$ diametral pitch, O. D. 16 inches, face 4.75 inches.

Slewing.—It is first necessary to determine the force acting at the rack that will overcome the frictional resistance the crane offers to being rotated, and also supply sufficient additional force to produce the required acceleration. To determine the needed accelerating force the mass of the several portions of the crane will be first reduced to an equivalent mass concentrated at the pitch circle of the rack. This will be done by assuming the ratio between a mass and its equivalent mass as inversely proportional to the squares of their respective radii.

The desired acceleration will be assumed as 1 foot per second at a crane radius of 20 feet. The rack radius being 43 inches, the acceleration at the rack is $\frac{1\times 43}{20\times 12}=0.18$ foot per second.

Masses reduced to the rack pitch circle.

Live load $M_r=M_1\times\frac{r_1^2}{r^2}=\frac{20,000}{32.2}\times\left(\frac{15\times 12}{43}\right)^2=10,900$ pounds.

Boom radius ⅔ $(15\times 12)=120$ inches; $M_r=\frac{2000}{32.2}\times\frac{120^2}{43^2}$
$=484$ pounds.

Boiler $M_r=\frac{19,500}{32.2}\times\frac{88^2}{43^2}=2540$ pounds.

Machinery $M_r=\frac{18,000}{32.2}\times\frac{12^2}{43^2}=44.$

Total reduced mass $=10,900+484+2540+44=13,968$ pounds.

Accelerating force = mass × acceleration = $13,968\times 0.18=2510$ pounds.

In a similar way the acceleration must be determined for the crane when carrying the maximum load at its greatest radius.

The several reduced masses then become,

| | Pounds. |
|---|---|
| Live load $M_r=\frac{7600}{32.2}\times\left(\frac{40\times 12}{43}\right)^2$ . . . . . . . . . | $=29,500$ |
| Boom, effective radius, $(⅔\times 40)=\frac{80}{3}$, | |
| $M_r=\frac{2000}{32.2}\times\left(\frac{80\times 12}{43\times 3}\right)^2$ | $=$ 3,420 |
| Boiler, water, coal and machinery . . . . . . . . . . . | 2,574 |

Total reduced mass, $29,500+3420+2530+44=35,494$ pounds.

Accelerating force = Mass × acceleration $=35,494\times 0.18=6400$ pounds.

Frictional Resistance:

$$F=\frac{W}{R}(\mu\rho+f).$$

$\rho$ = axle radius. $f=0.003$

W = live load + machinery + boiler + coal + water
$=20,000+18,000+16,000+1,500+2,000=57,500$ pounds.

$$F=\frac{57,500}{4.5}\left[(1/10\times 1.5)+0.003\right]=1950 \text{ pounds.}$$

The total force required then is $1950+6400=8350$.

PITCH OF THE RACK TEETH:

$$W=s.p.f.\left(0.124-\frac{0.684}{n}\right).$$

Assuming a 12 toothed pinion,

$$p=\sqrt{\frac{8350}{12{,}000\times2\times0.067}}=2.27 \text{ in.; use } 2\tfrac{1}{4}\text{-in. circular pitch.}$$

The diameter of the pinion is 8.6 inches and the force acting on the tooth being 8350 pounds, the twisting moment on the pinion shaft is

$$M=8350\times4.3=35{,}900 \text{ inch-pounds.}$$

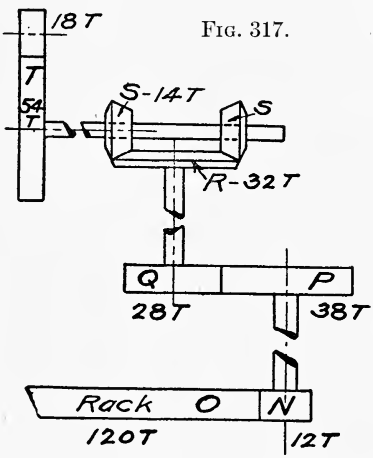
The twisting moment on the next shaft towards the engine is

$$M_t=\frac{35{,}900\times28}{38\times0.92}=28{,}800 \text{ inch-pounds.}$$

From this twisting moment the pitch of the smallest 28-toothed pinion is found to be,

$$p=\sqrt{\frac{6.28\times28{,}800}{28\times8000\times2\times0.10}}=1.59 \text{ inches, use } 1\tfrac{3}{4} \text{ pitch.}$$

FIG. 317.




The twisting moment on the pinion shaft of the bevel gearing is

$$\frac{35{,}900\times28\times14}{38\times0.95\times32\times0.92}=13{,}250 \text{ inch-pounds}=M_t.$$

$$\alpha=\text{tang}^{-1}\frac{14}{32}=23^\circ\ 40'$$

$$N = n \sec \alpha = 14 \times 1.09 = 15.25.$$

$$\text{Cosec } \alpha = \text{cosec } 23° \, 40' = 2.491.$$

$$p_m = \sqrt[3]{\frac{6.28 \times 13{,}250}{14 \times 8000 \times 2 \times 0.078}} = 1.68 \text{ inches.}$$

$$\text{Face} = 1.68 \times 2 = 3.36 \text{ inches, say } 3\tfrac{1}{2} \text{ inches.}$$

$$d_m = \frac{n \times p_m}{3.1416} = \frac{14 \times 1.68}{3.1416} = 7.49 \text{ inches.}$$

$$p_c = p_m = \frac{(d_m \text{ cosec } \alpha) + f}{d_m \text{ cosec } \alpha} = 1.68 \, \frac{(7.49 \times 2.49) + 3.36}{7.49 \times 2.49} = 1.98 \text{ inches.}$$

Use 1½ diametral pitch, 2.1-inch circular pitch.

Pinion—14 teeth, 9.33 inches O. D., face 3.50 inches. Steel.

Gear—32 teeth, 21.33 inches O. D., face 3.50 inches. Steel.

Boom.—To determine the stresses in the rope used to raise the boom, see Fig. 318. The boom rotates about the point C. Draw the boom in the several positions 1, 2, 3, etc., and draw the arms $a$ and $b$ to the several center lines of the wire rope. The upper portion of the rope is fastened to the top of the crane; it then passes over sheaves D and thence to the drum E, by whose rotation the boom is raised or lowered. Now if a load is at D and the boom and load are to be raised the drum E will have to be rotated clockwise. This will make the stress in the ropes running on E greater than the stress in the ropes secured to the top of the crane, owing to friction at the pulleys D. These stresses can be assumed in the ratio of about 1.04 to 1. Hence if P is the stress in one pair 1.04P will be the stress in the other pair. Taking moments about C for position No. 2 we have $L \times l = (P \times b) \times (1.04P \times a)$.

The measured values are $l = 36$ feet, $a_1 = 6.4$ feet, $b_1 = 8.8$ feet.

The load L is made up of ½ the weight of the boom + live load + weight of sheaves, tackle, etc., concentrated at D.

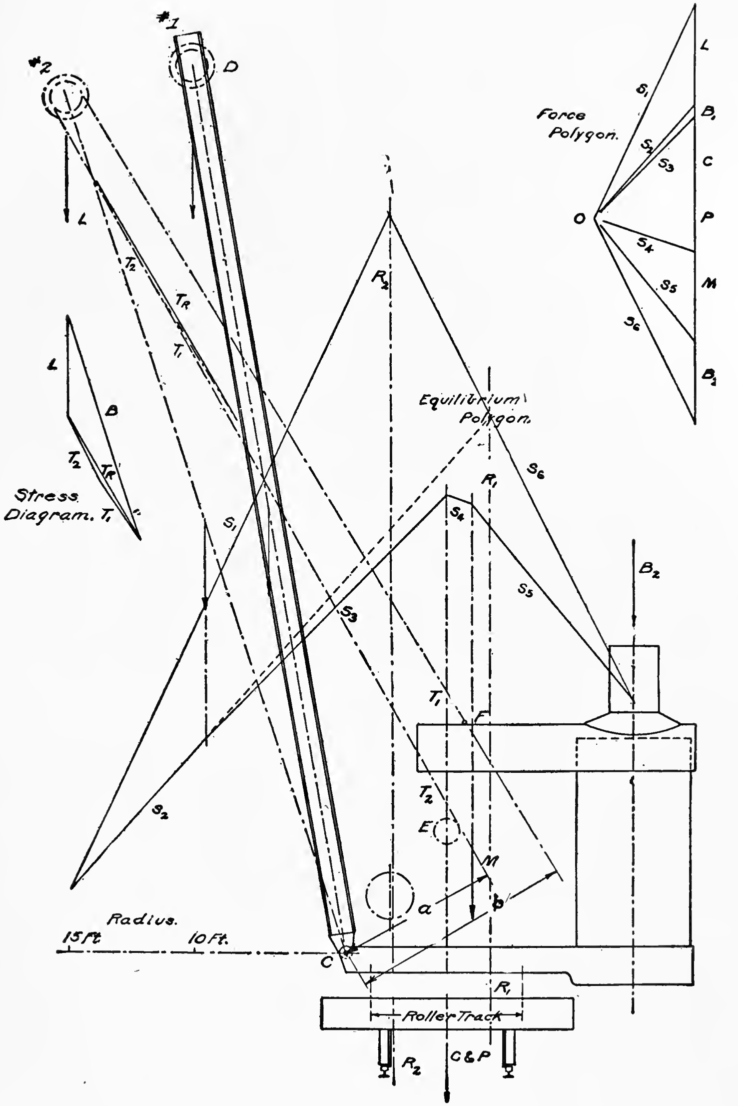
$$L = 20{,}000 + 1800 + 300 = 22{,}100 \text{ pounds.}$$

$$22{,}100 \times 11 = (P \times 8.8) + (1.04P \times 6.40).$$

$$P = \frac{22{,}100 \times 11}{15.5} = 15{,}700 \text{ pounds.}$$

Or if the two ropes are used, 7850 pounds per rope.

FIG. 318.


#1
#2
D
L
Force
Polygon.
B₁
C
P
M
B₂
O
Stress
Diagram.
Equilibrium
Polygon.
R₁
R₂
B₂
T₁
T₂
F
E
M
a
b
Radius.
15 Ft.
10 Ft.
C
Roller Track
C & P

The stress on each of the lower ropes is

$$P_1 = 7850 \times 1.04 = 8165 \text{ pounds.}$$

The analysis could be made with sufficient accuracy by assuming the stress in the ropes as equal; the diagrams upon this assumption are shown in Fig. 318. The loads at the several positions are taken approximately the maximum load the crane will carry at that radius on an 8-foot gauge track. The maximum stresses given by any of these boom positions should be used to determine the rope diameter.

Boom Stresses.—The force acting in the boom can be found for the several positions by taking moments about the point F, or may be determined graphically, as shown in the stress diagram.

Assume the tensions in the boom ropes as equal, $T_1 = T_2$. On one rope lay off $T_2$ and from one extremity of $T_2$ draw $T_1 = T_2$, and parallel to the other rope. The resultant $T_R$ will connect the ends of $T_1$ and $T_2$.

Now for the stress diagram lay-off,

L = live load carried by the boom in this position + weight of block, tackle, etc., + ½ weight of the boom.

The diagram is closed by drawing B parallel to the boom axis and $T_R$ parallel to the previously found direction of the resultant $T_R$.

The boom is in compression and should be designed for the highest stress found for any one of the several positions.

Design of the Boom.—Maximum compression = 65,500 pounds. Length, 36 feet.

Maximum allowable stress, due to all loadings, 14,000 pounds per square inch.

Trying two 12-inch channels weighing 25 pounds per foot, the calculation will be made as follows:

The bending on the boom due to its own weight is,

$$\text{Weight of boom} = 36 \times 50 = 1800 \text{ pounds.}$$

$$M = \frac{Wl}{8} = \frac{1800 \times 36 \times 12}{8} = 97{,}200 \text{ inch-pounds.}$$

Maximum fiber stress due to bending $p = \frac{Me}{I} = \frac{97{,}200}{2 \times 24}$
$= 2020$ pounds.

Direct stress, compression $= \frac{65{,}500}{14.7} = 4450.$

Force required at rack to slew boom and load (acceleration).

$$F = (29{,}500 + 3420) \times 0.18 = 5925 \text{ pounds.}$$

Assuming the separation of the boom channels where secured to the car body as 4½ feet, the force in these channels due to slewing will be

$$F = \frac{5925 \times \text{rack radius}}{\text{Channel separation}} = \frac{5925 \times 43}{4.5 \times 12} = 4720 \text{ pounds.}$$

$$\text{Unit stress} = \frac{\text{Force in 1 channel}}{\text{Area of 1 channel}} = \frac{4720}{7.35} = 640 \text{ pounds.}$$

The total combined stress in one channel then is

$$4450 + 2020 + 640 = 7110 \text{ pounds.}$$

It must now be determined from Ritter's column formula what load per square inch the boom acting as a column will carry.

$$\frac{l}{r} = \frac{36 \times 12}{4\ 4} = 98.$$

$$p = \frac{14{,}000}{1 + \frac{1}{10{,}000}\left(\frac{l}{r}\right)^2} = 7150 \text{ pounds.}$$

As the total stress of 7110 pounds is below the allowable of 7150 pounds, these two channels will be satisfactory.

Stability.—The stability of the crane under a given load can be determined by means of force and equilibrium polygons or by the method of moments.

Method of Moments.—Take the center of moments at the left-hand rail. Call clock-wise moments plus and counter-clockwise moments minus. To insure stability the resulting moment must be positive, and this moment divided by the sum of all the crane weights and the live load must give a lever-arm less than the distance between the rail centers.

Graphical Method.—Draw the force polygon, laying the several forces off to scale. (See Fig. 318.)

$L$ = live load carried at the boom position.
$B_1$ = weight of the boom.
$C$ = counter-weight.
$P$ = weight of platform.
$M$ = weight of the machinery.
$B_2$ = weight of boiler.

Take the pole about as shown and draw the several rays $S_1$ to $S_6$. $B_2$ is held in equilibrium by rays $S_5$ and $S_6$ (force polygon), hence draw the equilibrium polygon by taking any point in $B_2$ and draw the rays $S_5$ and $S_6$ parallel to $S_5$ and $S_6$ in the force polygon. In this manner complete the equilibrium polygon and the resultant $R_2$ must pass through the intersection of $S_1$ and $S_6$. This resultant produced vertically downward is seen to fall between the rails.

Had the crane carried no load in this position L would not have appeared in either the force polygon or in the equilibrium polygon. $S_2$ would be produced as shown in the dotted line, and the resultant $R_1$ would pass through the intersection of $S_2$ and $S_6$, which is seen to fall between the rails, again indicating stability.

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